Investigations in the Field of the Ultra-short Electromagnetic Waves. IV. On the Dependence of the Ultra-short Electromagnetic Waves upon the Heating Current and upon the Amplitude of the Oscillations*

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An investigation has been made of the dependence of the length of the ultra-short electromagnetic waves generated by vacuum tubes on the magnitude of the heating current. It is shown that the normal waves (i.e., waves the frequency of which corresponds to the frequency of the oscillations of the electrons about the grid of the tube) differ from the dwarf waves (i.e., waves with frequency exceeding several times the frequency of electronic oscillations) in their dependence on the heating current. The length of the normal waves increases, while the length of the dwarf waves of all orders decreases as the heating current is decreased. Therefore, as the heating current is decreased, there is a better agreement between the observed and computed values of wave-lengths of the normal and of the dwarf waves. At low heating currents the product $\lambda^2 E_g$ for dwarf waves of different orders is very close to the theoretical values

$$\lambda^2 E_g = \text{const.}_0/n^2 \qquad n=2,\ 3,\ 4,\ \cdots$$

where const.$_0$ corresponds to the value of the product $\lambda^2 E_g$ for normal waves. The results of the experiments are in agreement with the theory of P. S. Epstein, which explains the discrepancy between the calculated and the observed length of the normal and the dwarf waves being due to the influence of the alternating potentials appearing on the electrodes of the tube during oscillations. In the limit, at infinitely low amplitudes of these potentials, the ratio of the values of the products $\lambda^2 E_g$ for normal and for dwarf waves approaches the theoretical value within the limits of the experimental error. Neglecting these alternating potentials introduces the largest error in the results of the existing theories of the generation of ultra-short waves. At sufficiently intensive oscillations, this error exceeds all the others taken together, including the error introduced through the simplifying assumption of plane electrodes. The limiting value of const.$_0$ corresponding to infinitely low amplitudes of the alternating potentials, approaches very closely the value of the product $\lambda^2 E_g$ calculated from the formulas of H. Barkhausen and A. Scheibe. This shows that normal waves really do correspond to a complete period of oscillations of the electrons about the grid, i.e., to the time the electrons take to pass from the filament to the grid and back to the filament.

§1. Introduction

IN THE previous papers[1] it was shown that ultra-short waves of several kinds can be obtained when large positive potentials with respect to the filament and the plate are impressed on the grid of a vacuum tube. The wave-

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[1] G. Potapenko, Phys. Rev. **39**, 625, 638, **40**, 988 (1932); these articles will be referred to as I, II and III.

length λ_0 of some of them is found to approximately correspond to the period of oscillations of the electrons about the grid of the tube, or in other words, to the time τ of the passage of the electrons from the filament of the tube to the plate and back to the filament; i.e.,

$$\lambda_0 = c_0\tau \text{ (normal waves)} \tag{1}$$

where c_0 is the velocity of light and the period τ can be approximately determined from the well-known formula:

$$\tau = d_a \times 10^{-7}/3(E_g)^{1/2} \tag{2}$$

where d_a is the diameter of the plate of the tube (in cm) and E_g is the constant positive potential (in volts) of the grid with respect to the filament and the plate. We called these waves normal or normal Barkhausen waves, since Eq. (1) is a fundamental equation of the theory proposed by Barkhausen. The lengths $\lambda_1, \lambda_2, \lambda_2, \cdots$ of the waves of the other kind are approximately $2, 3, 4, \cdots$ times shorter than the length of the normal waves, calculated from Eq. (1), i.e.,

$$\begin{aligned} \lambda_1 &= c_0\tau/2 \text{ (dwarf waves of the 1st order)} \\ \lambda_2 &= c_0\tau/3 \text{ (" " " " 2nd ")} \\ \lambda_3 &= c_0\tau/4 \text{ (" " " " 3rd ")} \end{aligned} \tag{3}$$

We called these waves dwarf waves of the 1st, 2nd, 3rd etc. orders.

We have seen that Eqs. (3) are fairly well satisfied experimentally and that Eq. (1) gives values which differ systematically from the observed values by some 15 percent (II, §9).

While investigating the problem of the motion of the electrons within the tube, we have seen that the peculiarities of the character of this motion completely explain the causes of the generation of dwarf waves and lead us to expect waves with periods equal to integral multiples of the period of the oscillation of electrons about the grids of the tube (III, §3). In other words, the character of electronic motion permits us to expect waves satisfying Eqs. (1)–(3). These investigations gave no direct indications of the possibility of a systematic discrepancy between these equations and the observations.

Thus, so far, we do not know the causes of this discrepancy. Its systematic character shows, however, that it cannot be ascribed to experimental errors and must possess a definite physical meaning. The purpose of the present paper is to elucidate the causes of this discrepancy.

§2. The Relation between the Length of Normal and Dwarf Waves and the Magnitude of the Heating Current

All of the previously made observations refer to three tubes of the same type R5. First of all we must compare them with each other. Every one of these observations, as, for example, the measurement of the wave-length, or the determination of the amplitude of oscillations, refers to a certain length of oscillating circuits, to a certain grid voltage, etc. Therefore, unless all of these factors are taken into account, a simple comparison of observations cannot be of any great value and will generally be avoided.

The measurements of the wave-lengths were made at such points of the working diagrams of the tubes (II, §3) which corresponded to the maximum current in the plate circuit of the generator (i.e., approximately corresponded to the maximum of the energy of oscillations). Moreover, observations were made at a constant heating current and only at such grid voltages which corresponded to the region of saturation current and were not in the neighborhood of the upper bend of the static characteristic of the tube. In this case the lengths of the normal and of the dwarf waves were satisfying Barkhausen's equation

$$\lambda^2 E_g = \text{const.} \tag{4}$$

An approximate estimate of the value of the constant could be made by means of Eqs. (1)–(3). If Eq. (4) holds true, we can compare the average values of the product $\lambda^2 E_g$ instead of comparing individual observations. This relieves us of the necessity of taking into a separate account the grid potentials as well as the lengths of the oscillating circuits. The latter follows from our having selected points of observation in such a manner that the length of the oscillating circuits already enters the value λ, in view of the definite relationship existing between these two magnitudes (II, §3).

Let us suppose that for the case of normal waves the observations give

$$\lambda_0^2 E_g = \text{const.}_0 \text{ (normal waves)} \tag{5}$$

If relations (1) and (3) hold true, we must have the following for the case of dwarf waves:

$$\begin{aligned} \lambda_1^2 E_g &= \text{const.}_1 = \text{const.}_0/4 \quad \text{(dwarf waves of the 1st order)} \\ \lambda_2^2 E_g &= \text{const.}_2 = \text{const.}_0/9 \quad (\text{`` `` `` `` 2nd ``}) \\ \lambda_3^2 E_g &= \text{const.}_3 = \text{const.}_0/16 \quad (\text{`` `` `` `` 3rd ``}). \end{aligned} \tag{6}$$

A general form of Eqs. (5) and (6) can be written thus:

$$\lambda^2 E_g = \text{const.}_0/n^2 \tag{7}$$

where for normal waves $n = 1$ and for dwarf waves of the 1st order, 2nd order, etc., we have $n = 2$, $n = 3$, $n = 4$, etc. respectively.

If we know the average values of const.$_0$ for the tubes investigated, then by means of Eqs. (6) we can compute the corresponding values of const.$_1$, const.$_2$, const.$_3$, · · · and compare them with the observed values. Table I gives the results of these computations; the columns under the heading *Obs.* give the average values of the products $\lambda^2 E_g$ for dwarf waves of the first four orders, which we had obtained previously (II, §§10, 13). The table also gives the mean observed values of the product $\lambda^2 E_g$ for normal waves and the values of the emission current. From a comparison of the mean observed values of $\lambda^2 E_g$ it is easily seen that for the case of dwarf waves of the 1st, 2nd and 3rd orders the values of these products diminish systematically as the heating of the filament is decreased. At the same time, for normal waves and for dwarf waves of the 4th order, the change in the value of $\lambda^2 E_g$ has no systematic character and is generally very small.

It must also be mentioned that Barkhausen's Eq. (4) was satisfied for all three tubes in spite of the fact that they were investigated at different absolute values of the heating current. Hence, it follows that under our conditions of observation the product $\lambda^2 E_g$ remains constant, regardless of the absolute value of the heating current.

TABLE I.

Tubes R5

	Tube No. 5 Const$_0$ = 5.81 × 10^5 I_e = 41 mA			Tube No. 20 Const$_0$ = 5.78 × 10^5 I_e = 19 mA			Tube No. 7 Const$_0$ = 5.85 × 10^5 I_e = 12 mA		
	Obs.	Calc.	Obs./Calc.	Obs.	Calc.	Obs./Calc.	Obs.	Calc.	Obs./Calc.
Const.$_1$	2.01 × 10^5	1.45 × 10^5	1.39	1.97 × 10^5	1.45 × 10^5	1.36	1.90 × 10^5	1.46 × 10^5	1.30
Const.$_2$	0.856 "	0.646 "	1.33	0.846 "	0.642 "	1.32	0.800 "	0.650 "	1.23
Const.$_3$	0.495 "	0.363 "	1.36	0.481 "	0.361 "	1.33	0.466 "	0.366 "	1.27
Const.$_4$	0.303 "	0.232 "	1.31	0.300 "	0.231 "	1.30	0.300 "	0.234 "	1.28
		Average	1.35		Average	1.33		Average	1.27

The ratios of the mean observed values of $\lambda^2 E_g$ to those computed from Eq. (6), which are given in Table I, indicate a systematic discrepancy between them. The mean observed values of $\lambda^2 E_g$ are systematically exceeding the calculated values by approximately 30 percent. This is as should be expected, according to what we had before. It is very significant, that the average values of these ratios Obs./Calc. decrease appreciably as we pass to tubes which worked with a lower heating of the filament. A decrease in the ratios Obs./Calc. mean, evidently, a decrease in the discrepancy between the observed wave-lengths and those computed from Eqs. (1)–(3).

§3

As there could be a difference in the vacuum of the tubes and consequently in their working regime, it is necessary to test the validity of the above relationship between the value of the ratio Obs./Calc. and the heating of the tube and to see whether it might be due to a possible difference in the vacuum of the tubes.

The simplest way of avoiding the effect of a difference in the vacuum is to make observations using only one tube at different heating currents. Such observations were made with tube No. 5. As far as could be seen, this tube possesses the greatest symmetry in the arrangement of the electrodes, its dwarf waves had a greater intensity than those generated by the two other tubes; it was therefore used for these tests. The tests were made by selecting some point on the lines of maxima of the working diagram (II, §3) and measduring the corresponding wave-length, intensity of oscillation, etc., at different heating currents I_h beginning with the largest possible values and ending with those at which the intensity of oscillations was very low and measurements were becoming uncertain. Each time the grid voltage was selected according to the maximum current I_a in the plate circuit of the generator.

The selection of the points of observations was made so as to obtain monochromatic oscillations. This circumstance was very important, since the intensity of waves of different length varies differently as the heating current is changed. Therefore, it frequently happens that waves, which at large heating currents play the role merely of an "admixture," become relatively intensive at low heating, thus completely distorting the results obtained.

As the intensity of dwarf waves of the 4th order was generally low and as the precision of the measurements of their wave-length fell off rapidly with decreasing heating current, it was decided not to include them in the tests, making observations for normal waves and dwarf waves of the first three orders only.

TABLE II. *Tube R5* (*No. 5*).

I_h (Amp.)	E_g (volt)	I_a (mA)	λ (cm)	$\lambda^2 E_g$
		normal waves; $L=23$ cm		
0.701	162	4.40	60.0	5.83×10^6
0.684	161	3.75	60.6	5.91 "
0.662	160	2.90	61.0	5.95 "
0.651	161	2.25	61.3	6.05 "
0.640	162	1.75	61.6	6.15 "
0.632	165	1.30	61.8	6.30 "
0.625	167	0.90	62.0	6.42 "
0.610	169	0.30	62.4	6.58 "
		dwarf waves, 1st order; $L=13$ cm		
0.701	152	2.70	36.6	2.04×10^5
0.684	146	2.20	37.0	2.00 "
0.662	138	1.80	37.3	1.92 "
0.651	136	1.30	37.2	1.88 "
0.640	133	0.85	37.0	1.82 "
0.625	127	0.30	36.8	1.72 "
		dwarf waves, 2nd ord.; $L=6$ cm		
0.701	240	2.00	18.85	0.852×10^5
0.684	234	1.60	18.9	0.836 "
0.662	228	0.80	18.8	0.806 "
0.651	224	0.50	18.7	0.783 "
0.640	221	0.20	18.55	0.760 "
0.625	217	0.02	18.4	0.735 "
		dwarf waves, 3rd ord.; $L=8$ cm		
0.701	305	0.95	12.7	0.492×10^5
0.684	300	0.65	12.55	0.473 "
0.674	288	0.35	12.5	0.450 "
0.662	278	0.10	12.5	0.434 "

The results of the measurements[2] are presented in Table II. Figs. 1 and 2 show the relation between the wave-length and the heating current, and also the relation between the value of the grid potentials, corresponding to a maximum plate current, and the heating current. As to the normal waves, their length increases very appreciably with decreasing heating current. At

[2] In Table II the observed values are rounded off to 0.1 cm for normal waves and dwarf waves of the 1st order. They are rounded off to 0.05 cm for dwarf waves of the 2nd and 3rd orders, as the accuracy of the measurements was somewhat greater for shorter waves.

the same time, the value of the grid voltage corresponding to a maximum plate current passes through a small minimum and increases rapidly, as the heating current is further diminished. In connection with this, there is, in the

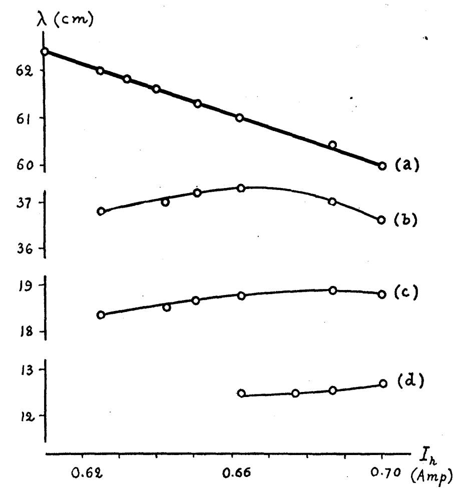

Fig. 1. Curves showing the relation between the length of the normal and dwarf waves and the heating current of the tube. Curve (a)—normal waves; curves (b), (c), (d)—dwarf waves of the 1st, 2nd and 3rd orders (E_g-variable).

case of normal waves, a rapid increase in the value of the product $\lambda^2 E_G$ as the heating current is decreased. This agrees well with the observations of W. Wechsung.[3]

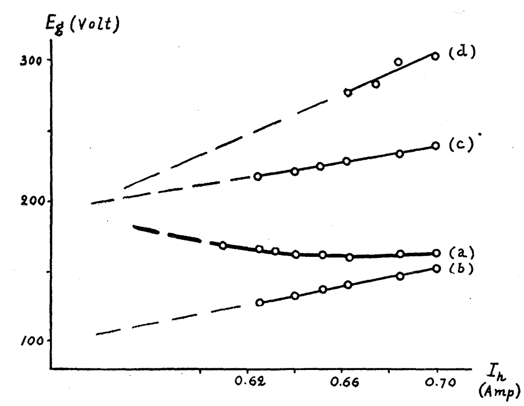

Fig. 2. Curves showing the relation between the grid voltage, corresponding to the maximum amplitude of oscillations, and the heating current. Curve (a)—normal waves; curves (b), (c), (d)—dwarf waves of the 1st, 2nd and 3rd orders.

A comparison of the results obtained for normal and for dwarf waves leads to an unexpected conclusion. It shows that *the normal waves depend on the heating current in a different way than the dwarf waves*. This is especially

[3] W. Wechsung, Zeits. f. Hochfr. **32**, 64 (1928).

clearly seen from the curves of Figs. 1 and 2 which show that the grid voltage corresponding to a maximum plate current, and also the length of the dwarf waves decrease as the heating current is decreased. As a result, in the case of dwarf waves the product $\lambda^2 E_g$ decreases as the heating current is decreased, i.e., it varies in a manner quite opposite to that of normal waves.

Table III gives a comparison of the values of const.$_1$, const.$_2$ and const.$_3$ calculated from Eq. (6) with those obtained from direct observations. For the sake of brevity the table gives the results of computation only for three values of the heating current. It is seen from Table III that the value of

TABLE III. *Tube R5* (*No. 5*).

	I_h=0.701 amp. Const.$_0$=5.83×10^5 I_e=36 mA			I_h=0.684 amp. Const.$_0$=5.91×10^5 I_e=28 mA			I_h=0.662 amp. Const.$_0$=5.95×10^5 I_e=18 mA		
	Obs.	Calc.	Obs./Calc.	Obs.	Calc.	Obs./Calc.	Obs.	Calc.	Obs./Calc.
Const.$_1$	2.04×10^5	1.46×10^5	1.40	2.00×10^5	1.48×10^5	1.35	1.92×10^5	1.49×10^5	1.29
Const.$_2$	0.852 "	0.648 "	1.32	0.836 "	0.657 "	1.27	0.806 "	0.661 "	1.22
Const.$_3$	0.492 "	0.364 "	1.35	0.473 "	0.370 "	1.28	0.434 "	0.372 "	1.17
		Average	1.36		Average	1.30		Average	1.23

the ratio Obs./Calc. decreases as the heating and emission current is decreased, exactly in the same manner as we have seen from Table I. Hence it follows that *the magnitude of the discrepancy between the observed wave-lengths and those calculated from Eqs.* (*1*)–(*3*) *does really depend on the heating current* and that there is no reason to ascribe it to a difference in the degree of vacuum of the tubes. Of course the mere fact that the magnitude of the discrepancy depends on the heating current, is not an explanation of the cause of these discrepancies. It indicates, however, a way of discovering this explanation. This question will be treated later.

From Table III it is seen also, hat the product $\lambda^2 E_g$ varies quite uniformly as the heating current is varied. Therefore, when the heating current is still lower, the divergence between the observed and calculated values must be

TABLE IV. *Tube R5* (*No. 5*).

I_h (Amp.)	Const.$_0$/Const.$_1$	Const.$_0$/Const.$_2$	Const.$_0$/Const.$_3$
0.701	2.86	6.84	11.8
0.684	2.96	7.07	12.5
0.662	3.10	7.38	13.7
0.651	3.22	7.73	—
0.640	3.32	8.09	—
0.625	3.73	8.73	—

even smaller than it is seen from Table III. Without giving any detailed computations we shall determine the values of the ratios const.$_0$/const.$_1$, const.$_0$/const.$_2$ and const.$_0$/const.$_3$. Theoretically, according to Eqs. (5) and (6) these ratios must be equal to 4, 9 and 16. Table IV gives the values of these ratios calculated from the data of Table II. It is seen from the Table IV that as the heating current is decreased these ratios increase and rather closely approach the theoretical values. A poorer agreement is obtained for the third ratio which cannot be calculated for the low heating current in view of the low intensity of the dwarf waves of the 3rd order.

As we said before, when measuring the wave-length at different heating currents, the grid voltage was selected in such a way as to have a maximum plate current. However, a change in E_g must affect the length of the waves. Therefore, in determining the relation between the wave-length and the heating current, for the sake of greater accuracy we should have kept E_g constand and the maximum values of the plate current should have been selected by varying the length of the oscillating circuits. We have chosen the first method because of its convenience. Furthermore, the results obtained by this method permit easily to determine the results obtainable by the second method. Figs. 1 and 2 show that as the heating current is decreased the length of the normal waves increases, at the same time there is an increase in the grid voltage corresponding to the maximum plate current. Since increasing the grid voltage produces a decrease in the wave-length (see article II Tables III and VI) it is clear that, if the grid voltage had been kept constant, we would have a still more pronounced increase in the wave-length of the normal waves as the heating current was decreased. It is also easily seen that, if the grid voltage had been kept constant, decreasing the heating current would produce even a more noticeable decrease in the wave-length of the dwarf waves than it is seen from Fig. 1.

§4. On the Influence of the Amplitude of the Alternating Potentials

The data of the last three tables confirm the existence of a definite dependence of the magnitude of the discrepancy between the observed and the calculated wave-lengths (or the values of $\lambda^2 E_g$) on the heating current. The data give also a fairly complete idea as to the character of this relationship. With these data it is possible to attempt an elucidation of the causes of this relationship.

The changes in the heating current and the consequent changes in the emission current can be accompanied by the following three phenomena: (1) a change in the density of the space charges, (2) a change in the initial velocity of the electrons and (3) a change in the amplitude of the oscillations generated by the tube. We did not consider these phenomena and none of them is taken into account by formula (2).

The change in the density of the space charges is frequently used to explain the changes in the wave-lengths with changing heating current. Starting from the concept of the influence of the space charges on the motion of the electrons, H. G. Möller[4] has shown theoretically that a decrease in the emission current must produce an increase in the length of the generated waves. This does not contradict our observations on normal waves, for which the theory was devised. However, there are good reasons to maintain that in our case the change in the density of the space charges cannot have an appreciable influence on the motion of the electrons. In the first place, all of our observa-

[4] H. G. Möller, Electr. Nachr. Techn. **7**, 411–419 (1930).

tions were made at such grid voltages which lay far in the region of saturation current, even at the maximum values of the heating current used. Therefore the influence of the space charges on the motion of the electrons ought to be very small in our case. Furthermore, if the change of the space charges had any appreciable influence, its effect would be the same for all kinds of waves. Our results show, however, that with a change in the heating current the normal waves vary in a different way than the dwarf waves. For the same reason we must consider as unapplied to our case the explanation proposed by A. Rostagni[5] which is based on the possibility of the existence of a natural period of the space charges analogous to the natural period of an ionized gas, and which is dependent on the density of the space charges. There is still less ground to admit the possibility of an influence due to a change in the initial velocity of the electrons, in view of the fact that this velocity is relatively low. There remains the third factor—the effect of a change in the amplitude of the generated oscillations.

If there are oscillations in the plate and the grid currents of the tube then alternating potentials must appear on the electrodes of the tube, as on the plates of a condenser. If these potentials are not vanishingly small in comparison with the constant grid voltage, they necessarily must affect the character and the velocity of electronic motions and change the time the electrons take to pass from the filament to the grid. Thus the length of the generated waves will be changed. E. Gill and J. Morrell[6] had mentioned the possibility of an influence of the alternating potentials on the motion of electrons. H. E. Hollmann[7] was the first to make an estimate of the influence of these potentials on the time of passage of the electrons and on the length of the generated waves. The formula which he obtained shows that the wavelength must decrease as the amplitude of the oscillations is increased. The same conclusions were arrived at by N. Kapzov[8] and F. W. Sears[9] who made a graphical investigation of the electronic trajections within the tube, and also by H. G. Möller,[4] as a result of his theoretical investigations. All of these results refer to normal waves only and are in complete agreement with the results obtained for these waves, presented in Table II. From this table it is seen that the length of the normal waves decreases continuously, as the amplitude of the oscillations (or the current I_a) is increased. P. S. Epstein,[10] independently from the previous papers, made a theoretical investigation of the influence of the amplitude of the alternating potentials on the length of the normal and dwarf waves. He showed that for dwarf waves this influence must be opposite to that for normal waves.[11] To this difference he ascribed

[5] A. Rostagni, Atti della R. Acad. di Torino **66**, 123–130, 217–223, 383–395 (1931); see also Th. V. Ionescu, C. R. **193**, 575–577 (1931).

[6] E. W. B. Gill and J. H. Morrell, Phil. Mag. **44**, 161–178 (1922).

[7] H. E. Hollmann, Ann. d. Physik **86**, 129–188 (1928).

[8] N. Kapzov, Zeits. f. Physik **49**, 395–427 (1928).

[9] F. W. Sears, J. Frankl. Inst. **209**, 459–472 (1930).

[10] P. S. Epstein, Report at a Physics Research Conference, Pasadena, Mar. 5, 1931.

[11] According to his theory the above deduction is always true for dwarf waves of the 1st order. For dwarf waves of the second and higher orders this deduction depends on the dimensions of the tube.

our systematic discrepancy between the observed and the calculated wavelengths. So far we had no opportunity for a detailed verification of this theory. One of its results shows, however, that this theory not only explains the above discovered difference in the effect of the heating current on the normal and on the dwarf waves but also leads to quantitatively correct results.

We have seen above that the ratios $const._0/const._1$, $const._0/const._2$, $const._0/const._3$ must be equal to 4, 9 and 16 if Eqs. (1) and (3) are true. If the assumption of the influence of the amplitude of the alternating potentials is correct, then in comparing our observations it will be more correct to refer them to the same amplitude of oscillations and not to the same value of the

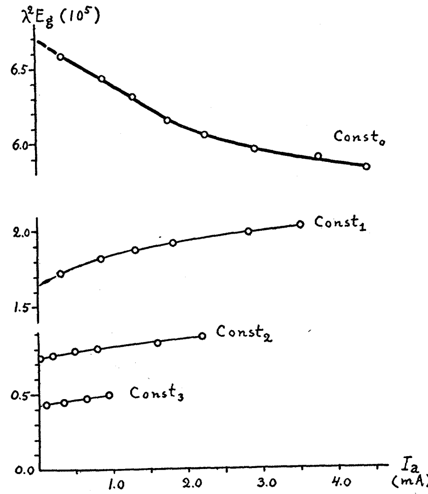

Fig. 3. Curves showing the dependence of the values of $const._0$, $const._1$ $const._2$ and $const._3$ on the amplitude of oscillations (amplitude of the alternating potentials which appear at the electrodes of the tube during oscillations).

heating current, as we have done it before. The ratios $const._0/const._1$, $const._0/const._2$, $const._0/const._3$ must be equal to 4, 9 and 16 at infinitely low amplitudes of the alternating potentials, i.e., in the limit at $I_a = 0$.

The values of $const._0$, $const._1$, $const._2$, and $const._3$ at $I_a = 0$ can be determined graphically from the data of Table II, by constructing curves showing the relations between these constants and the plate current which, as we have seen before (I, §6) approximately corresponds to the amplitude of oscillations and, which is more important, appears only when oscillations are present. For this last reason the condition $I_a = 0$ does really correspond to the condition of infinitely small amplitudes of alternating potentials. These curves are shown on Fig. 3. We find by extrapolation

$$\left|\begin{array}{l} const._0 = 6.70 \times 10^5 \\ const._1 = 1.65 \quad \text{"} \\ const._2 = 0.735 \quad \text{"} \\ const._3 = 0.43 \quad \text{"} \end{array}\right| I_a = 0$$

where the value 0.735×10^5 is taken directly from Table II. Hence we obtain

$$\left(\frac{\text{const.}_0}{\text{const.}_1}\right)_{I_a=0} = 4.1 \quad \left(\frac{\text{const.}_0}{\text{const.}_2}\right)_{I_a=0} = 9.1 \quad \left(\frac{\text{const.}_0}{\text{const.}_3}\right)_{I_a=0} = 15.6.$$

The discrepancy between these figures and the theoretical values does not exceed the limits of the possible errors. This proves that as far as can be seen from the above investigations, the difference between the observed and calculated wave-lengths is to be completely explained by the dependence of the length of waves on the amplitude of the alternating potentials which appear on the grid and the plate of the tube during oscillations.

Evidently it does not follow that the dependence of the wave-length on the heating current can *always* be explained as being due to the influence of alternating potentials alone. When observations are made in the region $E_g < E_s$ a greater influence can be exercised by the space charges of which we have spoken previously (II, §12). On the other hand, our results permit us definitely to deny the widely spread opinion, according to which a decisive importance is attributed to the space charges in all cases when explaining the dependence of the wave-length on the emission and heating current. This opinion is usually connected with the notion that the amplitudes of the alternating potentials are vanishingly small in comparison with the constant grid potentials and that the influence of the alternating potentials on the period of electronic oscillations is insignificant. This conception is erroneous.

The dependence of the wave-length on the amplitude of oscillations and the heating current enables us to explain some of the other phenomena observed. We shall mention as an example the displacement of the regions of oscillations with a change in the heating current (II, §13). Since to each point of the diagram corresponds its own wave-length it is clear that a change in the heating current and a consequent change in the wave-length must produce a displacement of the region of oscillations. Furthermore, the regions of normal waves will be differently displaced than the regions of dwarf waves; for their wave-lengths change differently with varying heating current, as we have just seen. The fact that for normal waves of low intensity, when they are mixed with the dwarf waves (II, §13), the values of $\lambda^2 E_g$ are greater than usual will also be readily understood.

§5

In the previous paper we have discussed the results of a graphic integration of the equations of motion of the electrons in the tube, performed by G. Kreutzer. As we saw, the electronic trajectories showed that the alternating potentials at the electrodes of the tube must decrease the length of the dwarf waves. On the other hand, we have found above, that the length of the dwarf waves increases with an increase in the heating current and the amplitude of the alternating potentials. As a matter of fact, these two results are not contradictory. The graphic integration of the equations of the motion of the electrons was performed for the case $E_0/E_g = 1/10$, E_0 being the amplitude of the alternating potentials. Direct measurements of the amplitude of the alternating potentials, which we shall not discuss for the present, showed, however, that for our dwarf waves of the 2nd and 3rd order the maximum value of the ratio E_0/E_g is approximately equal to $1/30-1/45$, i.e., is considerably below the value assumed in our computations. For dwarf waves of the first order this ratio had a larger value being approximately equal to 1/14, as a maximum. Fig. 1 shows that it is exactly for these waves that the curve showing the relation between the wave-length and the heating current has a maximum. Hence it follows that in the case of dwarf waves the alternating potentials at the electrodes of the tube have the following effect: the wave-length increases at first, reaches a maximum and decreases as the amplitude of oscillations is further increased. It is evident that no maxima are observed on the curves relating to dwarf waves of the 2nd and 3rd orders simply because the amplitudes of the alternating potentials are very small in this case. On the other hand, all the calculations were made for relatively large amplitudes of alternating potentials. This explains the apparent discrepancy mentioned above.

§6. On the Limiting Value of Const.$_0$

There are several formulas expressing the length of the waves in terms of the voltage at the electrodes and the dimensions of the tube. The simplest one is the well-known formula of H. Barkhausen

$$\lambda^2 E_g = d_a^2 \times 10^6 \tag{8}$$

which is easily obtainable from Eqs. (1) and (2). This formula claims no particular accuracy, since in deriving Eq. (2) plane electrodes were assumed for the sake of simplicity.

S. Zilitinkiewitch[12] and A. Scheibe[13] obtained more accurate formulas which take into account the cylindrical shape of the electrodes. The formula of S. Zilitinkiewitch is expressed in a function of two rather slowly converging series. It is therefore less convenient than the formula of A. Scheibe. The latter formula can be written as follows for the case when the plate potential are low or equal to zero:

$$\lambda^2 E_g = \frac{2c_0^2 d_g^2}{e/m \times 10^8}[f(\ln d_g/d_f)^{1/2} + g(\ln d_a/d_g)^{1/2}]^2 \tag{9}$$

where d_a, d_g and d_f are the diameters of the plate, grid and filament respectively, e/m is the ratio of the electronic charge to its mass[14] and the two functions in the brackets are:

[12] S. Zilitinkiewitch, Drahtl. Tel. u. Tel. (Russ) **18,** 2–22 (1923); **19,** 166–175 (1923); Arch. f. Elektrot **15,** 470–484 (1926).

[13] A. Scheibe, Ann. d. Physik **73,** 54–88 (1924).

[14] The constant coefficient $[2c_0^2/(e/m)\times 10^8] = 1.017\times 10^6$. Strictly speaking, formula (8) also ought to have this coefficient. It had been omitted, as this formula is only approximate.

$$f(\ln d_g/d_f)^{1/2} = f(x) = xe^{-x^2}\int_0^x e^{n^2}dn$$
$$g(\ln d_a/d_g)^{1/2} = g(x) = xe^{x^2}\int_0^x e^{-n^2}dn. \quad (10)$$

The numerical values of $f(x)$ and $g(x)$ can be obtained from the tables given in the paper of A. Scheibe[13] or from more detailed tables computed by N. Kapzov and S. Gwosdower.[15]

As to the accuracy of Barkhausen's formula, we have already mentioned that it gives wave-lengths which differ by approximately 15 percent from the observed values. Of course, this figure refers only to the observations described above. From what we have said, it follows that the discrepancy between the observed and the calculated lengths of normal waves must become greater at greater amplitudes of oscillations. As an example, we can mention our preliminary measurements made with two tubes working simultaneously (II, §3). During these observations the plate current was greater than the one given by a single tube, $\lambda^2 E_g$ was equal to 5.44×10^5 and the discrepancy between the observed length of normal waves and that given by formula (8) was approximately 20 percent, as could be readily shown by simple computations.

The formula of A. Scheibe has been verified experimentally several times. As could be expected, it was in better agreement with the observations than formula (8). Nevertheless, the values it gave differed by some 10 percent from the observed wave-lengths.

Approximately the same agreement with the experimental data is obtained using the formula of S. Zilitinkiewitch, judging from the data of his paper. It must be mentioned that the obtained wave-lengths (normal waves) are always shorter than those computed from formulas (8) and (9).

In deriving formula (9), as well as in deriving formula (8), the dependence of the length of waves on the amplitude of the oscillations has also not been taken into account and the electrode potentials were assumed to be constant. From the preceding it is clear that for this reason alone there could be no good agreement between these formulas and the observations.

In the limiting value of $(\text{const.}_0)I_a=0$ the effect of the alternating potentials is eliminated. Hence, this value must be in better agreement with formulas (8) and (9) than the direct observations. We shall verify this statement.

Let us consider formula (8). As we have seen previously (II, §4), for the tubes of the type R5 this formula gives:

$$\text{const.}_{\text{Barkh}} = 7.6\times 10^5. \quad (11)$$

This value differs from our limiting value 6.70×10^5 by approximately 12 percent. This corresponds approximately to a 6 percent difference in the wave-lengths, since the constant contains the square of the wave-length.

[15] N. Kapzov u. S. Gwosdower, Zeits. f. Physik **45**, 114–134 (1927).

As previously this difference in the wave-length amounted to approximately 15 percent (the difference in the values of $\lambda^2 E_g$ being approx. 30 percent) it becomes evident that among the errors of this formula, the largest error is introduced by neglecting the effect of the alternating potentials. Of course, this statement refers only to waves of sufficient intensity, such as our normal waves. As the intensities of the waves falls off, there must be a decrease in the relative error due to the alternating potentials.

Using formula (9) we obtain:[16]

$$\text{const.}_{\text{Sch}} = 7.22 \times 10^5. \tag{12}$$

This value differs from the limiting value of $(\text{const.}_0) I_a = 0$ by approximately 7 percent, which corresponds to a difference in the wave-lengths of approximately 3.5 percent. It could be shown by computations that the difference between directly observed wave-lengths and formula (9) constitutes approximately 12 percent. By comparing this figure with the preceding one, we can see again how large is the error introduced by neglecting the influence of the alternating potentials.

Under the conditions of our experiments the possible errors in formulas (8) and (9) can roughly be estimated as thus: neglecting the alternating potentials at the electrodes introduces in the wave-lengths an error of about 9 percent; the simplification of the problem by assuming plane instead of cylindrical electrodes introduces an error of about 3 percent, finally, all the other possible causes,[17] including the effect of space charges and the initial velocity of the electrons, introduce an error of also approximately 3 percent.

Comparing the magnitudes of all these errors it can be seen that in developing a theory of the generation of ultra-short waves there is no particular need of taking into account the shape of the electrodes.

In order to simplify the problem, it is quite sufficient to assume the electrodes of the tube to be plane. At the same time, it is quite necessary to take into account the effect of the alternating potentials. This is true of all problems which aim at determining the length of the generated waves, the time it takes the electrons to pass inside the tube, etc. In other cases, for example, in deriving formulas determining the conditions of the appearance of oscillations, their energy, etc., the assumption of plane electrodes will be of greater importance.

§7.

In deriving formulas (8) and (9) the wave-lengths were determined from the time it takes the electrons to pass from the filament to the plate and back to the filament. The very small difference existing between the values of $(\text{const.}_0)_{I_a=0}$, $\text{const.}_{\text{Barkh}}$ and $\text{const.}_{\text{Sch}}$ shows that *the normal waves generated by the tubes do really correspond to the time of passage of the electrons from the*

[16] The tubes of the type R5, which we investigated, had d_a (internal) $=0.87$ cm; d_g (middle) $=0.37$ cm; $d_f=0.006$ cm.

[17] The error introduced through experimental errors is very small. For example, for observations given in Table V of article II the calculations give for normal waves $\lambda^2 E_g = 5.78 \times 10^5 \pm 0.017$.

filament to the plate[18] *and back to the filament,* in other words, they correspond to a complete period of electronic oscillations about the grid of the tube. This conclusion is of a very great importance for the theory of the generation of ultra-short waves. There exists a number of papers,[19] in which the length of waves is calculated from the time it takes the electrons to pass from the grid to the plate and back to the grid. This tendency of using a shorter path is quite understandable in view of the fact that the observed wave-lengths are always shorter than those obtained from formulas (8) and (9), as we have mentioned before. The agreement between the theory and the observations can be improved thereby. The foregoing considerations show, however, that another way must be chosen in order to obtain a better agreement between the theory and the observations. Namely, the effect of the alternating potentials on the motion of the electrons within the tube must be accurately estimated. At the same time, it is necessary to preserve the fundamental idea of H. Barkhausen's theory, according to which the length of the normal waves, generated by the tube corresponds to a complete period of electronic oscillations. This idea is unquestionably correct. The only possible exception is the case when waves are generated by a tube with a very closely wound grid (close mesh). In such a case, as it was shown by H. Hollmann,[6] it is possible to obtain, together with the usual oscillations, the so called "oscillations of a higher frequency." They will correspond to the shorter path and to a shorter time of passage of the electrons, which in this case will be easily captured by the grid on their way from the plate to the filament.

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[18] Strictly speaking to the point of reversal.

[19] See, for example, E. W. Gill and J. H. Morrell, Phil. Mag. **44**, 161–178 (1922), D. Pfetscher, Phys. Zeits. **29**, 449–478 (1928); E. W. Gill, Phil. Mag. **12**, 843–853 (1931).