Elastic Electron Scattering in Helium

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The scattering of electrons by helium atoms was measured experimentally over an angular range from 15° to 150° for electrons varying in speed from 25 to 700 volts. The scattering curves all fall steeply with angle, especially so for the smaller angles, provided that the speed is above 100 volts. For speeds below 100 volts, the steep fall at small angles is followed by a gentle rise beginning near 90°. Within the range cited, no maxima in the angular scattering curves were found. The theory of Mott accounted for the 700 and 500 volt curves accurately over the whole range, and for the 350 volt curve, over the range 20° to 100°. Below 20° the experimental curve was steeper than the Mott curve, and above 100° the experimental curve was flatter than the Mott curve. As we go to lower energies, the departures from the Mott curve increase progressively. For small angles the theoretical curve is not steep enough and for large angles it is too steep. The results were also compared with the theory developed by Massey and Mohr taking into account certain factors omitted by Mott. While the newer theory accounts qualitatively for the upturn in the angular scattering curve, it fails to give its correct shape. It is shown how atomic structure factors, or F values, which are employed in the theory of x-ray scattering, can be inferred from these experiments in electron scattering.

THE scattering of electrons by helium atoms has been investigated by a number of physicists. Dymond and Watson,[1] who studied the angle distribution of 210 volt electrons between 5° and 60°, established the fact that the scattering curves for elastic collisions were considerably less steep than they would have been had the scattering been due solely to the attraction of the nucleus of the atom for the electron passing by it. Results of the same general type were obtained by Harnwell,[2] who investigated the scattering of electrons at three angles, 0°, 8°, and 16°, with electron energies varying between 75 and 300 volts. McMillen[3] obtained curves for the scattering of electrons by helium atoms, the angle range being from 7° to 60° and the electron energies varied from 50 to 150 volts. According to his results, the scattering curves fall off more steeply with the angle the higher the energy of the electrons. A discussion of the results obtained by McMillen, Harnwell, and by Dymond and Watson, and their connection with the theories put forward by Sommerfeld, Mott and Mitchell, will be found on page 1043 of McMillen's paper. Investigations of the scattering of electrons at a fixed angle (90°), as the energy of the electrons was varied, have been made by Kollath[4] and by

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[1] E. C. Dymond and E. E. Watson, Proc. Roy. Soc. **A22**, 571 (1929).
[2] G. P. Harnwell, Phys. Rev. **33**, 559 (1929).
[3] J. H. McMillen, Phys. Rev. **36**, 1034 (1930).
[4] R. Kollath, Ann. d. Physik **87**, 259 (1928).

Werner.[5] Kollath's measurements and Werner's measurements were for the ranges 1 to 36 volts, and 40 to 300 volts, respectively. A considerable extension in the angular range was effected by Bullard and Massey[6] who studied the elastic scattering of electrons by helium, between 20° and 140° for energies varying from 4 to 50 volts. The new features in the scattering curves found by them are that, when the electrons have energies between 20 and 50 volts, the curves become practically flat between 90° and 140°, and that, when the energies are reduced below 10 volts, there is a decided rise in the curves from about 70° to 140°. Ramsauer and Kollath[7] made very extensive measurements of the scattering over a range extending from 15° to 167° for slow electrons (1.8 to 19.2 volts). In marked contrast to the scattering for faster electrons (e.g., above 50 volts), they found but little scattering at small angles (0° to 30°), then a gradual rise to a maximum (at 90° for 19 volt-electrons, and at 110° for 1.8 volt electrons) and after that a slow decrease. The theoretical papers which we have found useful in discussing our experimental results are those of Mott[8] and of Massey and Mohr.[9]

Mott's theory leads to the result that the number of electrons scattered elastically at any angle should decrease monotonically with increasing angle. However, Arnot[10] found that in many cases this was not true; the scattering curves fall steeply as the angle is increased to somewhere between 70° and 100°, but for larger angles the curves often pass through well-marked maxima and minima. These maxima and minima fade out as the speed of the electrons is increased. Now the speed at which these maxima and minima fade out appears to be higher, the greater the atomic number of the atom under investigation. It would appear therefore that the maxima and minima should vanish at comparatively low electron energies when helium is used to scatter the electrons. As Arnot did not include helium in his comprehensive study of electron scattering in gases and as Bullard and Massey did not use electrons of energy higher than 50 volts, it appeared to be very desirable to make a study of electron scattering in helium over a wide range of energies and angles.

Experimental Method

The method of measuring the scattering of electrons by helium was very much the same as that employed by us in our studies of scattering in argon[11] and in hydrogen.[12] However, a new apparatus was constructed in which the analyzer and the collision chamber were made small enough to allow both of them to be put inside a glass tube 4.25 inches in diameter, so that they could be well outgassed if necessary. The new apparatus was designed chiefly to allow us to investigate another aspect of the general problem of scattering,

[5] S. Werner, Proc. Roy. Soc. **A134,** 202 (1931).
[6] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133,** 637 (1931).
[7] C. Ramsauer and R. Kollath, Ann. d. Physik **9,** 756 (1931); **12,** 529 (1932).
[8] N. F. Mott, Proc. Roy. Soc. **A127,** 658 (1930).
[9] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A132,** 605 (1931); **A136,** 289 (1932).
[10] F. L. Arnot, Proc. Roy. Soc. **A133,** 615 (1931).
[11] A. L. Hughes and J. H. McMillen, Phys. Rev. **39,** 585 (1932).
[12] A. L. Hughes and J. H. McMillen, Phys. Rev. **41,** 39 (1932).

but it happened also to be very suitable for investigation of elastic scattering in helium. A detailed description of the apparatus is postponed to a later paper. Pure helium was prepared by passing helium from a tank through charcoal in liquid air into a reservoir. From the reservoir the helium passed through a fine capillary tube and then through another tube of charcoal in liquid air into the apparatus, from which it was pumped out continuously. It was possible to use pressures as high as 0.02 mm because of the two factors which worked together, (1) the small dimensions of the apparatus and (2) the long mean free path of electrons in helium.

The experimentally-measured scattering coefficients are given in Table I. By the scattering coefficient we mean, as in our previous papers, the number of electrons scattered in a direction θ from that of the original beam, through unit solid angle, per unit length of path of the original beam, per single electron in the beam, per single atom in unit volume.

TABLE I. *Scattering coefficients* $\times 10^{+20}$.

700 volts		500 volts		350 volts		200 volts	
7°	1597.0	9.5°	1195.0	11°	2088.0	11°	3192.0
9.5°	1307.0	12°	1047.0	16°	1183.0	13.5°	2367.0
12°	914.0	22°	467.0	21°	717.0	16°	1930.0
17°	541.0	27°	284.5	26°	469.0	21°	1285.0
22°	272.0	37°	148.2	31°	334.0	26°	728.0
27°	182.0	47°	60.5	41°	160.0	31°	624.0
37°	65.8	57°	30.76	51°	90.1	41°	292.3
47°	30.7	67°	15.88	61°	53.1	51°	176.6
57°	15.5	77°	11.41	71°	36.7	61°	113.9
72°	8.12	87°	6.14	81°	23.4	71°	82.0
87°	5.10	107°	5.56	91°	27.1	81°	55.85
102°	3.97	127°	4.36	106°	8.3	91°	50.4
117°	3.56	137°	2.87	136°	7.4	101°	39.45
132°	3.44	147°	1.51	151°	8.4	111°	31.65
147°	1.54					121°	29.00
						131°	28.50
						141°	27.9
						151°	25.8

100 volts		75 volts		50 volts		25 volts	
14°	3121.0	17°	3010.0	12°	3570.0	15°	2430.0
19°	2097.0	22°	2145.0	17°	2505.0	20°	1852.0
24°	1550.0	27°	1547.0	22°	1732.0	25°	1325.0
29°	1055.0	37°	844.2	27°	1320.0	35°	1012.0
39°	660.2	47°	557.0	37°	827.0	45°	791.0
49°	451.0	57°	402.0	47°	615.0	55°	693.0
59°	307.5	67°	301.0	57°	410.2	65°	588.0
69°	201.5	77°	233.0	67°	347.5	75°	560.0
74°	169.7	87°	189.0	77°	280.2	85°	513.0
79°	159.7	97°	159.2	87°	242.0	95°	513.0
89°	116.8	117°	160.5	97°	208.0	105°	523.0
99°	103.2	127°	161.0	107°	205.0	115°	545.0
109°	100.2	137°	162.0	117°	205.0	125°	576.0
119°	86.0	142°	159.5	127°	214.5	135°	609.0
129°	85.4	147°	174.0	137°	222.2		
139°	83.7			142°	210.7		
149°	74.8						

The results are also shown graphically in Figs. 1 and 2. Two diagrams are necessary to avoid undesirable crowding of the curves. The heavy black lines are the smoothed experimental curves and the experimental points through which they were drawn are indicated in an appropriate manner. When the heavy line is continuous it means that the experimental curve coincides with Mott's theoretical curve. When the heavy line is broken, it indicates that Mott's theoretical curve and the experimental curve no longer coincide. In this case the theoretical curve is shown as a *light continuous* line. Finally, where comparison is possible with the recent theoretical curves of Massey and Mohr,[13] these theoretical curves are indicated by a *light broken* line. The Mott curves are given by the formula

$$\alpha(\theta,\, v) \;=\; [(e^2/2mv^2)(Z - F)\ \mathrm{cosec}^2\,\theta/2\,]^2$$

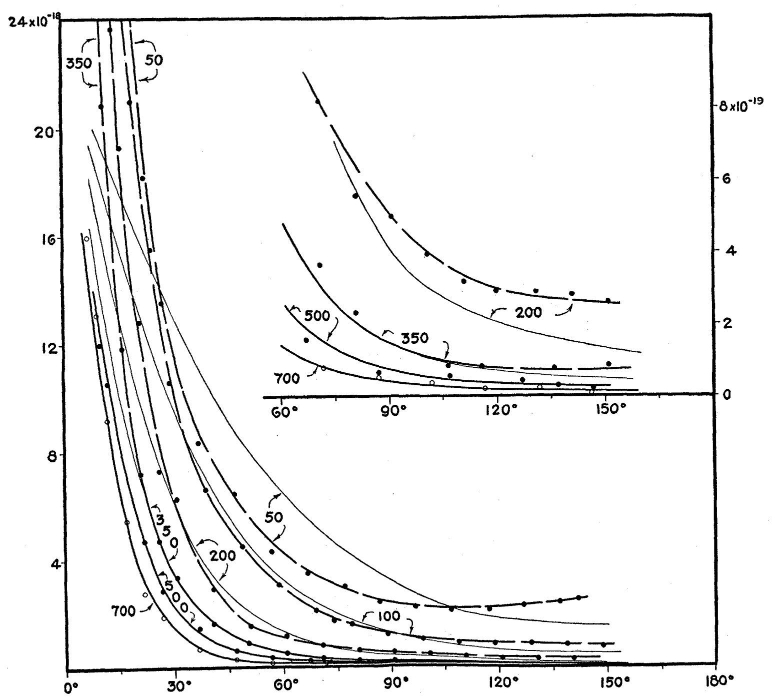

Fig. 1. Scattering of electrons by helium atoms. Angular distributions for 700 to 50 volt electrons. *Heavy continuous* lines: for experimental curves coinciding with Mott's theoretical curves. *Heavy broken* lines: for experimental curves not coinciding with Mott's theoretical curves. *Light continuous* lines: Mott's theoretical curves. *Inset:* Magnified diagram for the range 60 to 150 degrees.

where e, m, v are the charge, mass, and velocity of the electron, Z the atomic number of helium, θ the angle of scattering and F the atomic structure factor. F values for use in computing the formula were kindly supplied to us by Mr. Mott. Since then, they have been published in a paper by James and Brindley.[14] It was found that the 700 and 500 volt experimental curves fitted

[13] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A132**, 605 (1931); **A136**, 289 (1932).

[14] R. W. James and G. W. Brindley, Phil. Mag. **12**, 81 (1931). The F values above 1.1μ were obtained by extrapolation according to the method outlined by D. K. Froman (Phys. Rev. **36**, 1339 (1930)).

Mott's formula exactly over the whole range. It will be seen that there is a slight departure from Mott's formula for the 350 volt curve at small angles and at large angles. For angles between 20° and 100° the coincidence is good. As we go to lower energies, the departure from the Mott formula becomes more and more marked. We do not show the Mott curve for the smallest electron energies, because the disagreement becomes too great to allow us to regard the theory as more than a very rough approximation to the experimental facts.

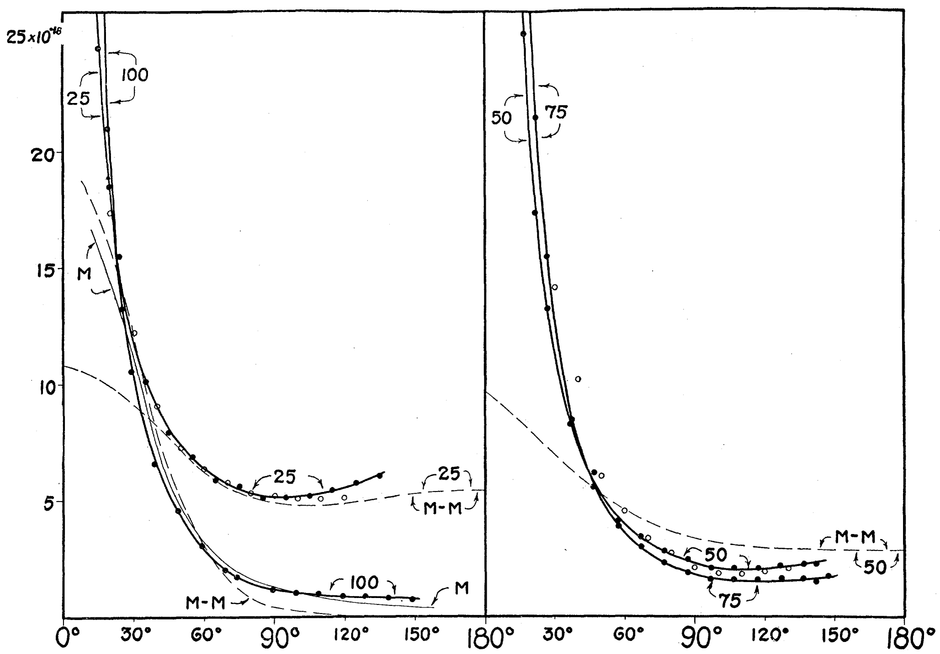

Fig. 2. Scattering of electrons by helium atoms. Angular distributions for 100 to 25 volt electrons. *Heavy continuous* lines: experimental curves. *Dots*: our experimental points. *Circles*: Bullard and Massey's experimental points (for the 50 and 25 volt curves only). *Light continuous* line: Mott's theoretical curve (for 100 volts). *Light broken* lines: Massey and Mohr's theoretical curve (for 25, 50 and 100 volts).

It should be emphasized that all the experimental curves and all the Mott curves shown in Figs. 1 and 2 are accurately drawn to the same scale. There was but one arbitrary adjustment (since our experimental arrangement did not give us accurate *absolute* values) and that was to make the 700 volt experimental curve fit the corresponding Mott curve. (The scattering coefficients, given in Table I, are obtained in this way.) When this was done it was found that the 500 volt curves fitted each other and that a departure from a perfect fit began to be discernable for 350 volts. The character of the departure of the experimental curves from the Mott curves is always of the same kind (at least down to 50 volts): the experimental curve is always steeper at small angles and always flatter at large angles. Moreover, the experimental and theoretical curves cut each other twice as is evident from the diagrams; alternatively, at two points for each electron speed the Mott formula happens to give just the right value for the scattering.

In Fig. 1, an enlargement of the curves, for the region above 60°, is shown in the inset. In Fig. 2 we have assembled the experimental results and the theoretical curves for the lower electron energies. The 50 volt theoretical curve is taken from the more recent paper of Massey and Mohr[13] and the 25 volt theoretical curve is an interpolation between the curves they give for 20 and 30 volts. The 100 volt theoretical curve attributed to Massey and Mohr is taken from their earlier paper, as they do not give a 100 volt curve in their later paper. This procedure is of course open to question, since they believe that the method of calculation developed in the second paper is more dependable than the one used earlier.

In view of the fact that theoretical physicists are continually improving their methods of calculating electron scattering, it is therefore desirable to discuss briefly the accuracy of the experimental curves. The work of Bullard and Massey[15] overlaps our work in the region between 25 and 50 volts. We have therefore superposed their values on the corresponding curves in Fig. 2. It is clear that the agreement is very satisfactory. When one considers how different their experimental arrangements were from ours, one may now feel confident in assuming that both sets of results are free from serious error arising in some way or another from the peculiarities of the apparatus. Although we did not investigate the scattering for electrons moving with speeds less than 25 volts, it is possible to make the same sort of comparison between the results of Ramsauer and Kollath and those of Bullard and Massey in the region below 20 volts. Both investigations agree well as to the shapes of the 10 and 20 volt curves. However, there is no agreement between their 4 volt curves; all we can find in common between them is an upward trend as we go from 50° to 120°. The final conclusion to be drawn from the comparisons discussed in this paragraph is that, over the range 10 to 50 volts, the experimental angular distribution curves now available may be regarded as fairly exact, and free from systematic errors due to the particular type of apparatus used. This conclusion is based on the significant fact that there is very satisfactory agreement between the final results, where they overlap, although each investigation was carried out with an entirely different type of apparatus from that used in the other two investigations.

Our results were compared with those of Werner.[16] It will be remembered that Werner measured the scattering of electrons by helium, at a fixed angle 90°, and varied the energy of impact of the electrons. To effect a comparison, we measured the number of electrons scattered, at a fixed angle 37°, as the electron energy was varied from 25 to 700 volts. (This, incidentally, was the method whereby the relative magnitudes of the experimental curves in Figs. 1 and 2 were determined.) Then, using our experimental curves shown in Figs. 1 and 2, we calculated the relative scattering, at 90°, of electrons having

[15] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133,** 637 (1931).

[16] S. Werner, Proc. Roy. Soc. **A134,** 202 (1931).

speeds ranging from 25 to 700 volts. The results were in very satisfactory accord with those given by Werner.

Werner concluded that Mott's theory held for electron scattering by helium atoms so long as the electron energy was above 100 volts. This is true for the angle at which he worked, *viz.*, 90°, but an examination of Fig. 1 suggests that the conclusion has no particular significance, for it so happens that, at 90°, for 100 volt-electrons, the experimental curve and the Mott curve cross each other. Had any other angle been chosen the conclusion would have been such as to exclude 100 volt electrons from the generalization. From the curves shown in Fig. 1, we should be entitled to conclude that Mott's theory adequately describes the scattering of electrons at 90° so long as their energy exceeds an amount somewhere between 200 and 350 volts.

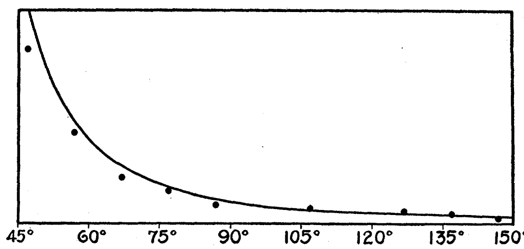

Fig. 3. Comparison of experimental scattering coefficient with values for a pure inverse square law scattering. *Continuous* line: graph of cosec $^4\theta/2$. *Dots*: experimental points for 500 volt electrons.

Mott's formula for electron scattering by atoms may be expressed in terms of μ, or in terms of v and θ, as follows,

$$\alpha(v, \theta) = [e^2/2mv^2(Z - F) \operatorname{cosec}^2 \theta/2]^2 \tag{1}$$

or,

$$\alpha(\mu) = [(e^2m/2h^2)(Z - F)(1/\mu^2)]^2 \tag{2}$$

where

$$\mu = [\sin(\theta/2)]/\lambda, \text{ and } \lambda = h/mv.$$

If the electrons within the atom contribute nothing at all to the scattering, the atomic structure factor F would be zero and the formula would then describe the scattering by a bare nucleus (Rutherford scattering, nuclear scattering, or inverse square law scattering). Now F is a function of μ which diminishes as μ increases, so that above a certain value of μ, F is negligible in comparison with Z. Evidently the larger μ is, (or, the larger θ is for a given electron speed), the more closely does the scattering approach Rutherford scattering. This is well illustrated in Fig. 3, which shows how closely the experimental points for 500 volt-electrons fit a cosec^4 $\theta/2$ curve (Rutherford scattering), so long as the angles are greater than about 70°. Below this angle, it is necessary to use the Mott theory to describe the experimental results. This means that we can no longer neglect F in comparison with Z in the formula.

We may here recall the parallelism between Mott's theory of electron scattering and the theory of x-ray scattering. Eq. (2) can be resolved into two factors, $[(e^2m/2h^2)(1/\mu^2)]^2$ which measures the scattering by the nucleus per unit nuclear charge, and $(Z-F)^2$ which gives the effective nuclear charge. Here Ze is the total nuclear charge, whose full effect is diminished by the presence of the electrons within the atom by an amount which is measured by F, the atomic structure factor. F depends on the number of electrons within the atom and the way in which they are distributed. The formula for the scattering of x-rays by gaseous atoms is

$$\alpha(\theta, \lambda) = \left(F^2 + \frac{Z^2 - F^2}{Z}\right)\left(\frac{e^2}{mc^2}\right)^2\left(\frac{1 + \cos^2\theta}{2}\right). \tag{3}$$

If we retain only that part associated with coherent scattering, that is, the part most directly comparable with the elastic scattering of electrons, we have left

$$\alpha(\theta, \lambda) = (F^2)(e^2/mc^2)^2(1 + \cos^2\theta)/2. \tag{4}$$

Here the factor $(e^2/mc^2)^2(1+\cos^2\theta)/2$ denotes the Thomson scattering per single free electron, and the factor F^2 may be regarded as the effective number of electrons in the atom. Thus, in both x-ray scattering and electron scattering, the way in which the electrons are distributed in the atom leads us to the notion of an effective number of electrons which is measured by F the atomic structure factor. If all the electrons were concentrated at the nucleus, F would equal Z the total number of electrons within the atom, but because they are distributed over a finite volume the phase differences between the scattered waves for each electron lead to a value of F necessarily smaller than Z.

It is instructive to change Eq. (2) into

$$\mu^2[\alpha(\mu)]^{1/2}(2h^2/e^2m) = (Z - F). \tag{3}$$

If we replace the theoretical scattering coefficient $\alpha(\mu)$ by the experimental values I_{exp}, and then plot $\mu^2[I_{\mathrm{exp}}]^{1/2}(2h^2/e^2m)$ against μ, we ought to get a graphical representation of $Z-F$. This is done in Fig. 4. If the F were always negligible in comparison with Z, then of course the graph would be a horizontal line parallel to the x-axis, whose height would be given by Z. The fact that it is not, indicates that F is, over a large range of μ's, of the same order of magnitude as Z. Indeed, we could say that the *points* plotted in Fig. 4 determine the atomic structure factor for helium atoms by means of electron scattering data. The *heavy continuous* line in Fig. 4 is a plot of $(Z-F)$ using the F values calculated by James and Brindley. (The *light continuous* line is a plot of F values obtained from x-ray data by Wollan[17] and Herzog[18].) The ordinates of the F curve and the $Z-F$ curve add up to Z as they should. The coincidence of the experimental points with the continuous line, over the range $\mu=0$ to $\mu=1.1$, of course shows that the Mott formula, in conjunction

[17] E. O. Wollan, Rev. Mod. Phys. **4**, 205 (1932).
[18] G. Herzog, Zeits. f. Physik **70**, 590 (1931).

with James and Brindley's F values, is adequate to describe electron scattering by helium atoms when the electron energy is not too low. It will be noticed that the experimental points for μ greater than about 1.1 are very irregularly distributed. This comes about from the peculiar way in which Fig. 4 involves the experimental results. Where the experimentally-measured quantities are small and difficult to measure accurately, the corresponding ordinates in the figure are large, but where they are large and easy to determine with precision the corresponding ordinates are small. Figs. 1 and 3 afford a fairer test of the relative accuracy of the results over the whole range of angle than does Fig. 4, but of course they do not bring out in so explicit a manner the shape of the curve showing the atomic structure factor as a function of μ.

The failure of Mott's theory to account for the experimental results when the electron energies are low has been attributed to the neglect of certain factors in the theory which are important only when the energies are relatively

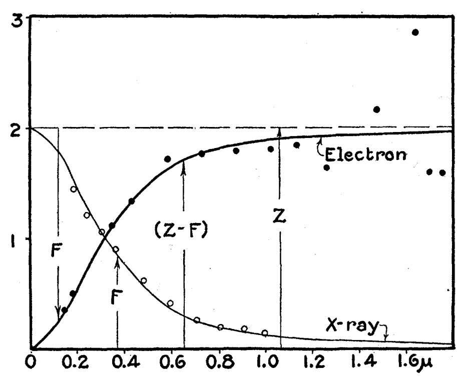

Fig. 4. Atomic structure factor for helium from electron scattering data. *Heavy continuous* line: From Mott's theory. Dots are experimental points. *Light continuous* line: F values according to Wollan.

low. Such factors are the distortion of the electron wave by the atom it passes over, electron exchange, and polarization of the atom by the electron. Massey and Mohr,[9] in the second of the papers referred to, computed the scattering when electron exchange and the distortion of the electron wave are taken into account. The calculations are difficult, and certain approximations have to be made in order to arrive at results. In Fig. 2 we show their 25 and 50 volt curves along with our experimental curves. It must be admitted that the agreement between theory and experiment is far from satisfactory. The upturn in the 25 volt curve is accounted for qualitatively by theory, but not the one in the 50 volt curve. There is also a serious difference at small angles (below about 50°) where the experimental curve is far steeper than the theoretical. In Fig. 2 we show the experimental curve for 100 volt-electrons, the Mott curve and the Massey and Mohr curve. (The Massey and Mohr curve for 100 volts is taken from their *earlier* paper in which they consider electron exchange effects, but not distortion of the electron wave. They do not give a 100 volt curve in their later paper.) It seems that, for 100 volt-electrons, the original Mott theory gives a somewhat better agreement with experiment than does the theory of Massey and Mohr, although the latter theory takes into account the exchange effect which was supposed to be a cause of the deviation between the Mott theory and experiment.

It will be noticed that nowhere do we find a maximum in the scattering curves, although it is possible that one may be found at an angle greater than 150° for the low velocities. This is in qualitative accord with the results of Arnot, which indicate that the maxima in the scattering curves at large angles were more marked the greater the atomic number of the scattering atom.

We should like to thank Professor G. E. M. Jauncey and Dr. G. G. Harvey for information they have supplied to us in connection with the atomic structure factors.