Investigations in the Field of the Ultra-short Electromagnetic Waves III. Electronic Oscillations in Vacuum Tubes and the Causes of the Generation of the Dwarf Waves

By G. Potapenko*

Norman Bridge Laboratory of Physics, California Institute of Technology

(Received April 12, 1932)

The purpose of this paper is the elucidation of the causes of the generation of the dwarf waves i.e., of waves with frequencies exceeding many times the frequency of the oscillations of the electrons about the grid of the tube. The existing theories of the generation of ultra-short waves cannot give a satisfactory explanation of the origin of dwarf waves of different orders. This is due to the simplifying assumptions made by the theories in the conditions of the problem. The phenomena of generation of ultra-short waves are closely connected with the motion of electrons. Therefore, in order to obtain the explanation in question an investigation was made of the motion of the electrons in tubes generating ultra-short waves. The motion of the electrons was investigated graphically by means of special diagrams which simultaneously represented the trajectories of the electrons and the alternating potentials at the electrodes of the tube. In this manner it was shown that the reason a vacuum tube can generate dwarf waves is because the tube can transmit energy into the oscillating circuit coupled with it and transmit it periodically with a period equal to the natural period of the circuit not only when this period T is equal to the period of the electronic oscillations τ (normal waves), but also when $T=\tau/2$, $T=\tau/3$, $T=\tau/4, \cdots$ (dwarf waves). This property of the tube depends on the fact that in all the cases described a periodic division of the electrons into two groups is possible. According to the theory of E. Gill and J. Morrell this division determines the transmission of energy into the oscillating circuit. The above property of the tube means also that the dynamic resistance of the tube is negative under these conditions. The diagrams of the trajectories and the velocities of the electrons, constructed on the basis of the computations of G. Kreutzer, confirmed the above deductions.

§1. Introduction

IN THE two preceding papers[1] we described some of the results of the investigations of the generation of ultra-short waves by vacuum tubes on the grid of which is impressed a high positive potential with respect to the filament and the plate. In this case, as we know, the electrons issuing from the filament can oscillate about the grid of the tube as a position of equilibrium. The period of oscillation and the corresponding wave-length λ can be determined from Barkausen's approximate formula:

$$\lambda^2 E_g = d_a^2 \times 10^6 \tag{1}$$

where d_a is the diameter of the plate of the tube (in cm) and E_g is the positive grid potential (in volts).

* Carnegie Fellow.

[1] G. Potapenko, Phys. Rev. **39**, 625, 638 (1932). These articles will be referred to as I and II respectively.

We have shown that in reality, together with the waves of a length approximately corresponding to the Barkhausen's formula (normal waves), the vacuum tubes can also generate waves whose length is approximately 2, 3, 4, $\cdots$, times shorter than that given by this formula (dwarf waves). Furthermore, investigations have shown that all of these waves belong to the same type of *GM*-oscillations, i.e., they are oscillations of the oscillating circuits connected with the tube. Hence, it follows, that these circuits are excited in such a manner that during one period of electronic oscillations or, in other words, during the time τ the electrons take to pass from the filament to the plate and back, the circuits execute two complete oscillations (dwarf waves of the 1st order), three complete oscillations (dwarf waves of the 2nd order), etc.

We have not yet explained why dwarf waves can be generated by vacuum tubes. In other words, we did not explain why it is that in oscillating circuits connected with the tube oscillations can be maintained having integral relations between the period of the circuit and the period of electronic oscillations. This is one of the fundamental questions. In the present paper we shall attempt to answer it.

§2. On the Theories of Generation of Ultra-short Waves

Several papers can be mentioned where attempts were made theoretically to solve the problem as to what should be the relations between the period of the oscillating circuit T and the time τ of the passage of the electrons in order that the vacuum tube be capable of generating oscillations.

E. Gill and J. Morrell[2] were the first to attempt to give an energetic theory of the generation of ultra-short waves. They showed that the tube can give energy to the circuit and, consequently, maintain its oscillations at different relations between the period T and the time of passage τ. Among others they made the following two fundamental assumptions: (1) the electrons which had already passed the grid and are moving back toward the filament are completely detained by the grid and (2) all the electrons have the same speed when they are moving across the grid. It is easily seen that the first assumption confines the problem to the case of oscillations generated by tubes with very closely wound grids when electrons can actually have such trajections, as it was shown by H. Hollmann.[3] The second assumption is generally incorrect,[4] if only because it contains an assumption of the absence of an alternating field between the grid and the filament. Because of these two assumptions the results obtained by E. Gill and J. Morrell can be considered only as an indication of a theoretical possibility of oscillations at $T\neq\tau$. Unfortunately, it is not possible to make any numerical comparisons of these results with the results of our experiments. O. Pfetscher[5] made the same assumption of the equality of speeds of electrons passing across the grid. His theory showed two relations between the period of oscillations T and the

[2] E. W. Gill and J. H. Morrell, Phil. Mag. **44,** 161–178 (1922).

[3] H. E. Hollmann, Ann. d. Physik **86,** 129–188 (1928).

[4] See also E. W. Gill, Phil. Mag. **12,** 843–853 (1931).

[5] O. Pfetscher, Phys. Zeits. **29,** 449–478 (1928).

time τ_{ga} the electrons take to pass from the grid to the plate. One of them corresponds to the condition $T=2.94\tau_{ga}$, the other, with a lower energy of oscillations, corresponds to the condition $T=\tau_{ga}$. The first condition corresponds rather closely to normal waves, since $\tau\sim 3\tau_{ga}$ for the usual type of tubes. The second condition must in this case correspond to dwarf waves of the 2nd order, as it can be seen that the ratio of the wave-lengths of these two oscillations is equal to 2.94. O. Pfetscher's theory gives thus an indication of the possibility of dwarf waves of the 2nd order. The theory differs, however, from the results of experiments in so far as dwarf waves of the 1st, and 3rd orders are concerned. To these waves must correspond the conditions $T\sim 1.5\tau_{ga}$ and $T\sim 0.73\tau_{ga}$, when the generation of waves is impossible according to this theory.

J. Sahánck[6] gave also a theory of the generation of ultra-short waves, based on calculations of energy. According to his theory, oscillations can be maintained only when there exists one of the following three relationships between the period of oscillations T and the time of passage of electrons τ. In our usual notation[7] these relationships are:

$$\begin{aligned} 1.35\tau &\geqq T \geqq 0.50\tau \\ 0.34\tau &\geqq T \geqq 0.25\tau \\ 0.20\tau &\geqq T \geqq 0.165\tau. \end{aligned} \tag{2}$$

The middle points of these regions correspond to the conditions: $T=0.925\tau$, $T=0.295\tau$ and $T=0.183\tau$. The first of these relationships is near to that which must exist in the case of normal waves. The second relationship is near to the one existing in the case of dwarf waves of the 2nd order. Passing now to the wave-lengths and dividing the first equality by the second we get $\lambda_2=\lambda_0/3.14$, which is really close to the expression for dwarf waves of the second order, which we had mentioned previously (II, Eq. 3). The third relation is close to the one which must exist for the case of dwarf waves of the 4th order. Passing now to wave-lengths[8] and dividing the third expression by the first we get $\lambda_4=\lambda_0/5.05$, which is also close to the relationship which we found for the case of dwarf waves of the 4th order. Thus, J. Sahánck's theory permits us to expect dwarf waves of the first two even orders. It differs from the results of experiments, as far as the dwarf waves of odd orders are concerned. The results of our previous paper show that these waves are no less intensive than the others, while from the relationships (2) it is seen that the conditions $T=0.5\tau$ (dwarf waves of the 1st order), $T=0.25\tau$ (dwarf waves of the 3rd order) and $T=0.165\tau$ (dwarf waves of the 5th order) correspond to the boundaries of the possible regions of oscillations. Therefore, according to A. Sahánck's theory under these conditions there should be no waves of any

[6] J. Sahánck, Phys. Zeits. **26,** 368–376 (1925); **29,** 640–654 (1928); Zeits. f. Hochfreqn. **38,** 78–80 (1931).

[7] Sahánck uses the time of passage of electrons in one direction. We take the time of passage of electrons from the filament to the plate and back to the filament, which corresponds to the complete period of oscillation of electrons about the grid.

[8] In passing to wave-lengths we denote, as we usually did, $cT=\lambda_0$ for the case of normal waves and $cT=\lambda_2$ and $cT=\lambda_4$ for the case of dwarf waves of the 2nd and 4th orders.

appreciable intensity. However, these waves are not entirely "prohibited," as it is the case in the theory of O. Pfetscher. The theory of J. Sahánck differs still in another respect from the results of experiments. It concerns the dimensions of the tube. His theory shows that oscillations are possible only when

$$2.0 < r_a/r_g < 5.0 \tag{3}$$

where r_a and r_g are the radii of the plate and the grid respectively. However, oscillations are observed even at such values of r_a/r_g which lie outside these limits. For example, N. Kapzov[9] observed oscillations with a tube for which $r_a/r_g = 1.75$ and A. Scheibe[10] obtained oscillations with a tube for which $r_a/r_g = 5.81$. The disagreement between the theory of A. Sahánck and the experimental results[11] is apparently due to the fact that his formulas were derived for the case of a tube with two electrodes (diode), and their extension to the case of a tube with three electrodes is not entirely faultless. Moreover, the theory assumes plane electrodes in the tube and a uniform field between the electrodes, while the experimental results were obtained with tubes with cylindrical electrodes and therefore the field between them was not uniform.

Thus, so far, we have no theory which would enable us to compute the values of the relations between T and τ, for which oscillations can exist in tubes under actual experimental conditions. The lack of success of the attempts made so far can be attributed to the simplifications introduced into the conditions of the problem. The results of the above investigations are important, however, in that respect that they permit us to foresee the possibility of the generation of waves corresponding to the condition $T \ll \tau$, i.e., oscillations having frequencies considerably exceeding the frequencies of the electronic oscillations.[12*]

[9] N. Kapzor, Zeits. f. Physik **35**, 131 (1925).

[10] A. Scheibe, Ann. d. Physik **73**, 79 (1924).

[11] L. Tonks, Physik Rev. **30**, 501–511 (1927) in determining theoretically the conditions under which the so-called "negative reistance" appears in tubes, also indicates $r_a > 2.1$ as the condition under which oscillations can appear. He attributes the results of N. Kapzov to the presence of traces of mercury vapor in his tube. In evaluating the results of A. Scheibe it must be borne in mind that the grid of his tube had a square cross-section and the relation given above is equivalent.

[12] K. Schuster, Ann. d. Physik **7**, 54–64 (1930), has recently investigated the problem of the generation of ultra-short waves from the point of view of wave mechanics. His fundamental ideas are as follows. High potential gradients exist at the surface of the electrodes. Therefore, the de Broglie waves, corresponding to the oscillating electrons, must be totally reflected from the surfaces of plate and of the filament and must form standing waves between these surfaces. The equation of Schrödinger gives for this case discrete energy levels. Their differences determine the frequencies of the electromagnetic waves which, in principle, can be observed. Hence it follows that to any frequency of electronic oscillations must correspond a whole series of frequencies of electromagnetic waves. The theory shows that these frequencies are analogous to band spectra; the boundary of the "head line" of this series corresponds exactly to the period of electronic oscillations and the remaining bands have frequencies higher than this initial one. The initial conditions assumed by this theory correspond, strictly speaking, to the case of BK-oscillations and therefore we shall not examine them in detail. They show us, however, a new path toward the explanation of the origin of oscillations with frequencies higher than the frequency of the electronic oscillations.

§3. The Motion of Electrons in Vacuum Tubes

The phenomena of generation of ultra-short waves are most intimately connected with the motion of electrons in the tube. Therefore, a study of this motion is a most natural procedure in trying to find an answer to the question as to why dwarf waves appear in vacuum tubes. The simplest and the clearest way will be a graphic representation of the electronic trajectories by means of specially constructed diagrams. We shall discuss them now.

We have already mentioned that all the ultra-short waves which we had obtained belonged to the type of *GM*-oscillations. We shall examine them.

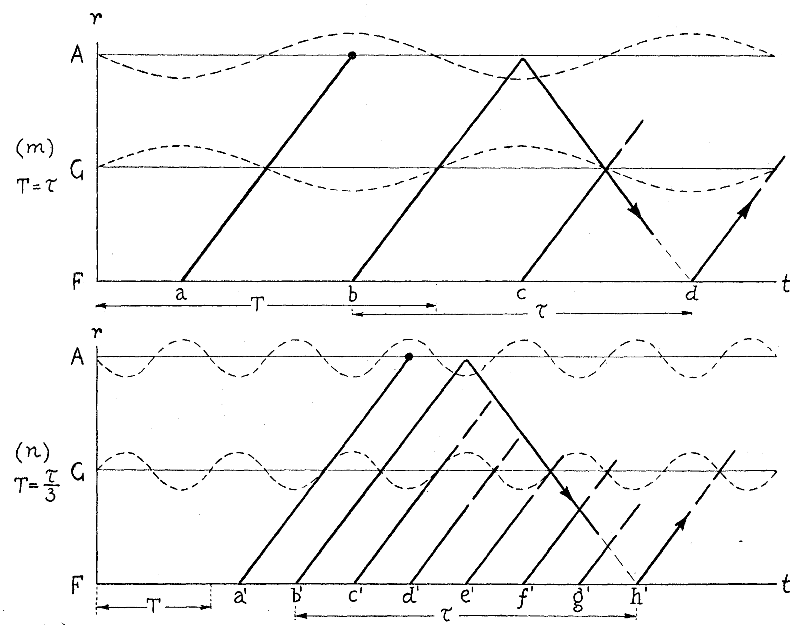

Fig. 1. Diagram of the motion of electrons in the tube; (m) the period of the generated oscillations T as equal to the time τ the electrons take to pass from the filament to the plate and back to the filament, (n) the period of the generated oscillations T is equal to one third of the time τ.

The characteristic feature of *GM*-oscillations is their dependence on the oscillating circuit connected with the tube (II, §4). Therefore the diagrams of electronic motion which we have to construct must give a simultaneous representation both of the electronic trajectories and of the alternating potentials of the electrodes of the tube which characterize the oscillations of the circuit.

Let us plot the time along the x-axis and the distances of the electrons

* *Note added to proof.* A Rostagni, Atti della R. Ac. di Torino 16, 217–223 (1931), has recently given a new theory of the generation of oscillations in which he took into account the natural period of the space charges in the interval grid-plate of the tube. The former he determined in a manner analogous to the determination of the natural period of an ionized gas. This theory led him to the relationship between the periods of the normal and of the dwarf waves which he had obtained.

from the filament—along the y-axis. Let FG be the distance (in arbitrary units) from the filament to the grid and FA be the distance (in the same units) from the grid to the plate (Fig. $1m$). Let the alternating potentials at the grid and the plate with respect to the filament be represented by two sinusoidal curves of an arbitrary period. Since a constant difference of phase of 180° must exist between the oscillations of potentials at the grid and at the plate, these two sinusoidal curves will be mirror images of each other. It must be also remembered that the constant grid potential is the larger quantity and therefore the grid will always remain positively charged. At the same time, the constant plate potential is always equal to zero and therefore during oscillations the plate will alternately have positive and negative potentials (with respect to the filament). The oscillations of the potential at the filament can be disregarded because in our generator the oscillating circuits were connected only to the plate and the grid. Furthermore, special chokes were connected in the heating circuit (see I, Fig. 1) which precluded appreciable alternating potentials at the filament.

Let us take two electrons a and b and plot their trajectories. Let us assume, moreover, that the constant grid potential E_g is so selected that the time τ the electrons take to pass from the filament to the plate and back to the filament, is equal to T, the natural period of the oscillating circuit, or in other words to the period of the alternating potentials at the grid and at the plate. The electron a will issue from the filament at the moment the grid potential will have its maximum value. On its path toward the grid its acceleration will also have its maximum possible value; the electron will be able to pass across the grid with a high velocity. On its path toward the plate the electron will be acted upon by a retarding field, which will be weaker than the accelerating field that had acted upon the electron on its path from the filament to the grid. This will be caused by two circumstances: the grid potential will no longer be so high as before and the plate will have a positive potential. Therefore, the electron will strike the plate and impart to it its remaining energy. The electron b will issue from the filament at the moment when the grid potential is at its minimum. On its path toward the grid the electron b will have an acceleration smaller than the one the electron a had on the same path. Therefore, it will pass across the grid with a lower velocity. The retarding field on its path from the grid to the plate will act on the electron more strongly than the accelerating field on the first part of its path, because now the grid potential will be higher than before and the plate-potential will be negative. Therefore, the electron b will not reach the plate and after coming to rest it will start moving toward the grid, will again be able to pass across the grid and so forth.

The same difference as between electrons a and b will exist between electrons c and d. Therefore, because of the presence of alternating potentials at the electrodes of the tube the electrons will be sorted out into two groups. One part of them will not be able to produce oscillations and coming to the plate will give rise to a current in the plate circuit of the generator. The other part will be able to oscillate and in passing from the plate to the grid will reduce the current flowing through the tube. Thus, there will be a certain order in the motion of the electrons and the current flowing through the tube will oscillate. Besides these two groups there will be other electrons which after issuing from the filament will come to the grid and will be captured by it. The majority of electrons will belong to this group, as it could be seen from a comparison of the currents flowing through the grid circuit and the plate circuit of the tube. These electrons, however, do not disturb the process of oscillations and we shall not consider them.

The electrons which lose their energy in coming from the filament to the plate and which return to the grid impart their energy to the oscillating circuit.[13] For this reason, the periodic oscillation of the current flowing through the tube indicates that small amounts of energy are periodically imparted to the oscillating circuit, the period being equal to the natural period of the circuit. The oscillations of the circuit will be maintained in this manner, if the energy received will be sufficient to cover the losses in the circuit.

Let us assume now a three-fold decrease in the natural period of the circuit and therefore, in the period of oscillations of the potentials of the grid and the plate, with which the circuit is connected. At the same time let the constant grid potential and therefore the time τ of the passage of electrons remain unchanged. In this case instead of the condition $T=\tau$ we shall have $T=\tau/3$. This case is represented on Fig. $1n$. Again, let us take two electrons a' and b'. It is easily seen that the electron a' will be under conditions analogous to those of electron a. It will come to the plate and will strike it. The second electron, b', will be analogous to the electron b. It will not reach the plate, will come to rest in the vicinity of it and will start moving toward the grid. In other words it will be capable of oscillating about the grid. For similar reasons electrons c', e', g', $\cdots$, will be captured by the plate and electrons d', f', h', $\cdots$, will start moving toward the grid and will be able to oscillate about it. Thus a perfectly definite order in the electronic motions will be established in this case, i.e., when $T=\tau/3$. As before, part of the electrons will settle on the plate, producing plate current, and another part will come back from the plate to the grid, thus decreasing the current flowing through the tube. It is easily seen that this current through the tube will oscillate periodically, performing three oscillations during each period τ. Hence, it again follows that periodically, three times during every period τ, energy will be imparted to the oscillating circuit connected to the tube. Due to this energy oscillations of the period $\tau/3$ will be maintained in the circuit. Of course the resistance of the oscillating circuit should not be too great, so that the energy supplied could cover its losses.

From the above, it follows that if oscillations are possible, i.e., if they can be maintained by the tube when $T=\tau$, then they are possible when $T=\tau/3$.

[13] See, for example, E. W. Gill and J. H. Morrell, Phil. Mag. **44**, 171 (1922). A certain amount of energy will be taken from the oscillating circuit and transformed into heat on the plate by electrons, the velocity of which will increase under the influence of the alternating potentials. However, this amount of energy will be less (under definite conditions) than that imparted to the circuit by electrons losing their velocity.

Using a similar argument it can be shown that oscillations are possible when $T=\tau/2$, $T=\tau/4$, $T=\tau/5$ etc. This is essentially an answer to the question as to why dwarf waves can appear. It is easily seen that this answer can be formulated as follows: *The reason a vacuum tube can generate dwarf waves is because the tube can transmit energy into the oscillating circuit connected with it and transmit it periodically with a period equal to the natural period of the circuit not only when this period T is equal to the period of the electronic oscillations τ (normal waves) but also when $T=\tau/2$, $T=\tau/3$, $T=\tau/4$, $\cdots$,* (dwarf waves). It is quite evident that the energy supplied periodically can maintain oscillations of a corresponding period.

We considered the process of sorting out of electrons in two groups, assuming that the constant potential at the plate with respect to the filament is equal to zero. Even if there had been a sufficiently large negative potential at the plate and the electrons were quite unable to reach the plate, the presence of alternating potentials at the electrodes would cause a division of the electrons into two groups. These two groups would differ in the amplitude of electronic oscillations and in the time τ of passage of electrons. This case, as well as the one considered above, was first indicated by E. Gill and J. Morrell.[2] It was recently examined by H. Möller[14] under the name of "sorting out by phases" ("Phasen Aussortierung"). It approaches very closely the scheme of oscillations, which was proposed by F. Harms.[15]

The division of electrons into two groups permits us to explain why oscillations can be observed at all in circuits connected with the tube. The mere presence of electronic oscillations cannot explain this fact. If the oscillations of electrons about the grid were disorderly, no oscillations or waves could, evidently, be observed outside the tube. If oscillations are observed, it means that the electronic oscillations about the grid are in some way regulated. From the above, we see that this regulation has a perfectly definite sense. When speaking of electronic oscillations it must be borne in mind that only an observer watching the separate groups of electrons would be able to observe the presence of such groups which actually oscillate periodically about the grid and are periodically replenished by an influx of new electrons (b, d, $\cdots$, etc., see Fig. 1m). At the same time, an observer watching what takes place at some given point between the electrodes of the tube, would have at any given moment electrons moving at different velocities from the filament to the plate and from the plate to the filament. Such an observer could easily call this motion disorganized (especially in the case of dwarf waves) and the only periodic process which he would observe would be the oscillation of the density of current flowing through the tube.

§4.

The oscillation of the density of the current flowing through the tube and the process of transmission of energy into the oscillating circuit could formally be ascribed to the presence of the so-called "negative resistance" between the

[14] H. Möller, Elektr. Nachr. Techn. **7**, 293–306 (1930).

[15] F. Harms, Verh. d. Deutsch. phys. mediz. Ges. **51**, 136–144 (1926).

electrodes of the tube. This idea was first expressed by V. Pagliarulo.[16] It is also consistently carried out in the works of K. Kohl.[17]

The concept of negative resistance is generally used in the radio engineering. It is applied in the cases of the presence of the so-called "falling characteristic" (for example, in the case of arc, dynatron, negatron, etc.). The concept of a negative resistance receives a different interpretation when applied to the description of the phenomena of the generation of ultra-short waves. In the first place, the presence in an oscillating circuit of a negative resistance, as it is usually understood, permits to obtain in this circuit oscillations corresponding to its natural frequency and within wide limits independently of what this frequency may be. On the other hand, in the case of ultra-short waves the vacuum tube can maintain oscillations only when there exists a very definite relation between the period of these oscillations and the time of passage of electrons. This distinction is very essential. Evidently, it is caused by the fact that in the region of ultra-short waves the periods of the generated oscillations are of the order of magnitude of the time of electronic passage. On the other hand, in schemes similar to the one with which we were obtaining ultra-short waves, the vacuum tube has no falling characteristic. Therefore, the term negative resistance must in our case be used only in the sense of a dynamic negative resistance. A. Sahánck,[6] as far as we know, was the first one to indicate the possibility of the existence of such a resistance, in other words, the possibility of a transition of the usual static characteristic into a falling dynamic characteristic. L. Tonks[18] and O. Pfetscher[5] gave a detailed theoretical investigation of the conditions under which the resistance of the tube may become negative.

Applying the concept of negative resistance to our results we can say that *a vacuum tube has the character of a dynamic negative resistance if the period of the oscillating circuit connected with it and the time the electrons take to pass from the filament to the plate and back to the filament, satisfy approximately the conditions* $T=\tau$, $T=\tau/2$, $T=\tau/3, \cdots$, When we explain the phenomena of generation of ultra-short waves from the point of view of electronic processes we give a physically correct explanation as to why under precisely these conditions the resistance of the tube becomes negative.

§5.

The above considered scheme of electronic motions is, of course, merely an illustration of the fundamental idea which explains the causes of the experimentally observed fact of the generation of ultra-short waves with different relations between T and τ. For example, this scheme does not make it clear whether the electrons, issuing at the time the grid potential is at its maximum (electrons b, d, b', d', etc.,) will have the maximum velocity, or whether it will be the electrons which issue from the filament a little before that moment. The same can be said, of course, of the electrons having the

[16] V. Pagliarulo, Phys. Rev. **23**, 300 (1924).
[17] K. Kohl, Ann. d. Physik **85**, 1–62 (1928); Erg. d. ex. Naturwiss. **9**, 324 (1930).
[18] L. Tonks, Phys. Rev. **30**, 501–511 (1927).

lowest velocity. Our scheme gives no indications as to how great will be the number of electrons which can oscillate about the grid as compared with the number of electrons which fall on the plate. Finally, it is not clear whether only those oscillations are possible for which $T<\tau$ or also those for which $T>\tau$. The answer to these questions must be given by the theory.

To obtain a more exact picture of the motion of the electrons we must construct more accurate diagrams capable of presenting in some definite units both the trajectories and the speeds of electrons for different working conditions of the tube. These diagrams can be constructed if we determine the trajectories and the speeds of the electrons from their equations of motion. We can make use of the results of computations made at our suggestion by G. Kreutzer in 1929.

All the vacuum tubes with which we succeeded in obtaining ultra-short waves had cylindrical electrodes, symmetrically arranged (I, §5). The equations of motion of the electrons will take into account this fact. Let r_f, r_g and r_a be the radii of the filament, grid and plate respectively. The constant potentials of the filament and the plate will be assumed equal to zero. Let E_g be the constant positive potential of the grid with respect to the filament. In the presence of oscillations alternating potentials must appear on the grid and the plate. Let E_0 be their amplitude. Since a constant phase difference of 180° must exist between these two potentials, the actual potentials of the grid and the plate will be

$$[E_g] = E_g + E_0 \sin \omega t \quad [E_a] = -E_0 \sin \omega t \tag{4}$$

where ω is the angular frequency of oscillations. According to what we said before, we neglect here the alternating potentials at the filament.

The field between the electrodes of the tube will be logarithmic and the equations of motion of the electrons can be written as follows:[19]

$$\begin{aligned} \frac{d^2r}{dt^2} &= K_1 \frac{1+\alpha \sin \omega t}{r} \qquad \text{(for the interval filament-grid)} \\ \frac{d^2r}{dt^2} &= -K_2 \frac{1+\beta \sin \omega t}{r} \qquad \text{(for the interval grid-plate)} \end{aligned} \tag{5}$$

where

$$K_1 = \frac{eE_g/m}{ln(r_g/r_f)} \quad K_2 = \frac{eE_g/m}{ln(r_a/r_g)} \quad \alpha = \frac{E_0}{E_g} \quad \beta = \frac{2E_0}{E_g}$$

and r is the distance of the electron from the filament. In deriving these equations neither the space charge nor the initial velocity of electrons issuing from the filament were taken into account.

It is easily seen that the above equations have two groups of parameters. Some of them —r_f, r_g, r_a—are given by the dimensions of the tube; others —ω, E_g, E_0—can be selected quite arbitrarily. Also, the frequency ω corresponds to the natural period T of the oscillating circuit coupled with the tube;

[19] N. Kapzov, Zeits. f. Physik **49**, 395–427 (1928).

E_g determines the period τ of electronic oscillations and the number of electrons falling on the plate depends on E_0. E_0 can be selected in agreement with the experimental data. As to the selection of ω and E_g, we shall distinguish several cases, depending on the resulting relations between T and τ: $T=\tau$ (normal waves) and $T=\tau/2$, $T=\tau/3$, $T=\tau/4$, $\cdots$, (dwarf waves).

The equations of motion of the type written above cannot be integrated in known functions. When expanded they give very slowly converging series.[20] Therefore, numerical or graphical integration is the only method of solving them.

(a). Normal waves, $T=\tau$. For this case, the numerical integration of the equations was performed by N. Kapzov.[19] His results show that those electrons do not reach the plate which issue from the filament at moments corresponding to the phases of oscillation of the grid potential within the limits from $-80°$ to $+60°$. Here the phase 0° corresponds to the beginning of the positive half period of oscillation. The electrons which correspond to the phase $-80°$ have a minimum velocity in passing across the grid; the electrons which correspond to the phase $+80°$ have in this case a maximum velocity and fall on the plate.[21] Computations have also shown that electrons corresponding to the phases $-70°$ and $+70°$ do not fall upon the filament on their motion toward it and can, therefore, perform several oscillations about the grid.

F. Sears[22] has recently performed a graphic integration of equations (5) by means of an integraph. His results are in agreement with those of N. Kapzov.

(b). Dwarf waves; $T=\tau/2$, $T=\tau/3$, $T=\tau/4$, $T=\tau/5$. These cases are of particular interest to us. At our suggestion G. Kreutzer performed the integration of Eq. (5) for all of these cases,[23] using the method of Runge-Kutta.[24] This work is very fatiguing and demands a great deal of attention. It was assumed that: $r_f=0.006$ cm, $r_g=0.31$ cm, $r_a=0.8$ cm, $E_g=100$ v, $E_0=10$ v. N. Kapzov and later F. Sears used the same conditions. Therefore, all of these results are easily comparable. The time of passage of electrons under these conditions was calculated from the formula of A. Scheibe[10] which takes into account the cylindrical shape of the electrodes and is more accurate than formula (1). This time of passage was found to be equal to 0.507×10^{-8} sec. T, the period of oscillations was, therefore, taken as being equal to $\frac{1}{2}$, $\frac{1}{3}$, $\cdots$, of this magnitude. Computations were made in such a manner that for each of the cases indicated above the trajectories and the velocities of the electrons issuing from the filament were successively determined for different moments of time. For dwarf waves of odd orders computations were made for moments corresponding to the phases 0°, 90°, 180°, 270°

[20] S. Zilitinkewitsch, Arch. f. Electrotech. **15**, 470–484 (1926).

[21] Compare Fig. 1*m*, electrons *a* and *b*. No computations were made by N. Kapzov for electrons corresponding to the phases $-90°$ and $+90°$.

[22] F. Sears, Journ. Franklin. Inst. **209**, 459–472 (1930).

[23] See preliminary report: G. Potapenko, Zeits. f. Physik **10**, 542–548 (1929).

[24] See C. Runge und H. König, "Numerisches Rechnen," pp. 286, 311 (1921)

and for dwarf waves of even orders computations were made only for 0° and 180°. The latter was done to shorten the computations as it was found that these two electrons represented the two groups in which we were interested.

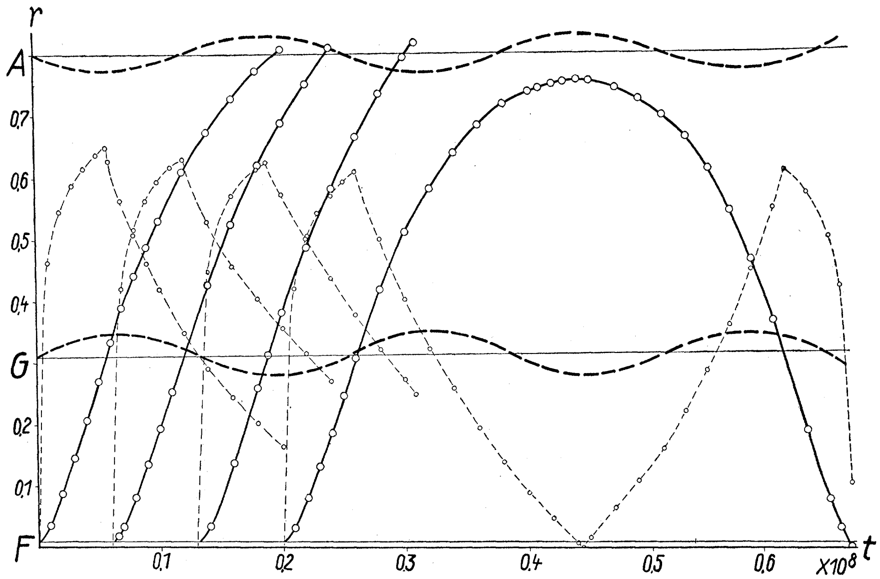

Fig. 2. Trajectories and velocities of the electrons for the case $T=\tau/2$.

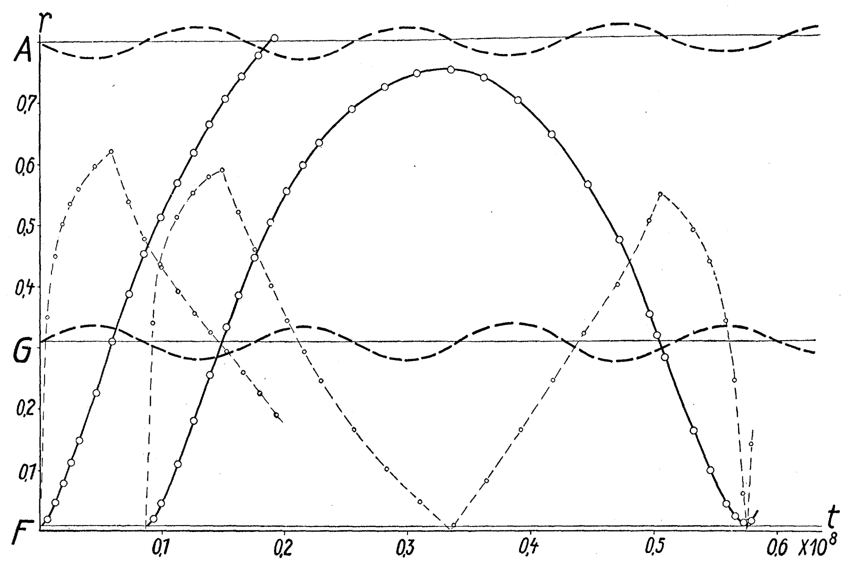

Fig. 3. Trajectories and velocities of the electrons for the case $T=\tau/3$.

The results of computations are given in Figs. 2–5. The continuous lines represent the paths of the electrons and the dotted lines—their velocities.

Figs. 2–5 show that in all cases without exception there actually exists a division of electrons into two groups; some of them reach the plate and fall on it, others lose their velocity before reaching the plate and start moving back toward the plate. This fully confirms the scheme of electronic motions,

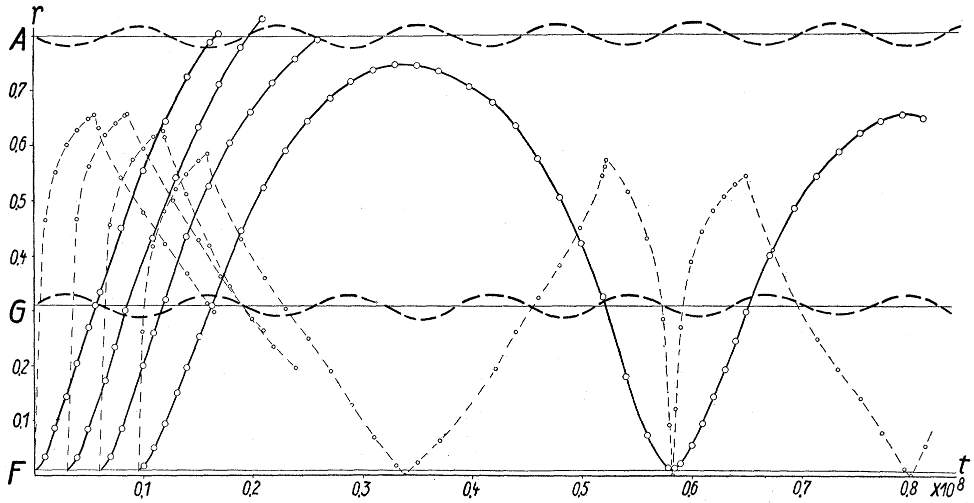

Fig. 4. Trajectories and velocities of the electrons for the case $T=\tau/4$.

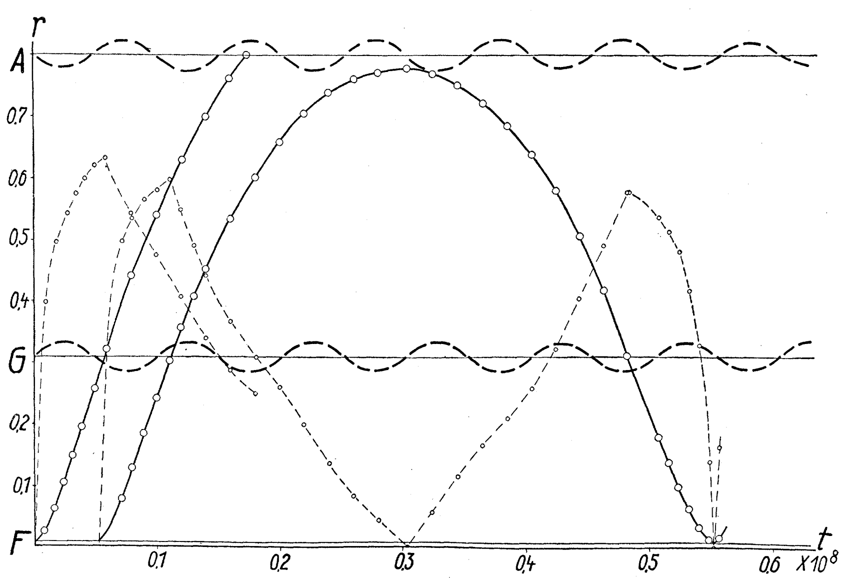

Fig. 5. Trajectories and velocities of the electrons for the case $T=\tau/5$.

given above, and thus fully confirms our basic idea as to the causes of the origin of dwarf waves.

The diagrams show also that the group of electrons moving from the plate toward the filament corresponds in different cases to different phases ω. At $T=\tau/2$ and $T=\tau/4$ only those electrons will move back toward the filament which issue from it at $\omega=270°$ or, which is the same thing, at $\omega=-90°$. When $T=\tau/3$ and $T=\tau/5$ the electrons issuing from the filament at $\omega=180°$ will also be able to move back toward the filament. The alternating potential at the grid is of decisive importance in the division of electrons into two groups. This is particularly clearly seen when $T=\tau/2$. In this case the electrons issuing at $\omega=0°$ and $\omega=270°$ approach the plate when it has approximately the same positive potential. Nevertheless, only the first electron will strike the plate, the second does not reach the plate and starts moving back. The trajectories and also the velocity curves show, moreover, that the electrons after passing the grid for the second time can fail to reach the filament and will thus be able to oscillate several times about the grid. It is of interest, that under the conditions which we selected, the period of the electronic oscillations, as determined from the trajectories of the electrons, will be a little shorter than the one previously computed from A. Scheibe's formula. Their ratio is 1.02–1.05. This difference must be ascribed to the influence of the alternating potentials which is not taken into account by the formula of A. Scheibe. It must be mentioned that in the case $T=\tau$ the difference between these two periods is considerably greater. According to the results of N. Kapzov[19] their ratio is 1:15 under the same amplitudes of oscillations. Hence it follows that the influence of the alternating potentials decreases as the difference between T and τ increases. This is easily understood, for the more often the field alternates during, the time of passage of the electrons, the more there will be of a summation and mutual cancellation of the influence of the separate phases of oscillations.

In conclusion the author wishes to express his gratitude to the Carnegie Corporation of New York for the grant of a Fellowship and to Professor R. A. Millikan for the facilities of the Norman Bridge Laboratory.