

mg of radium was allowed to pass through a hole in a thick lead block and strike a rotating aluminum disk. As the disk turned in the anti-clockwise direction, the irradiated spot went past a second lead tube leading to a Geiger counter, in  $7 \times 10^{-5}$  sec., so that fluorescence persisting that long might be detected. The corresponding time for clockwise rotation was  $3.3 \times 10^{-2}$  sec. Extended counts, made with the disk alternately rotating clockwise and anti-clockwise, agreed within the limits of statistical error, indicating that there was no fluorescence detectable by the means used. In addition to aluminum, beryllium (sulphate), carbon (paraffin), and lead (litharge), were successively waxed onto the disk and the experiment repeated, always with negative results. The sensitivity of the arrangement was sufficient to detect the gamma-ray equivalent of  $1.2 \times 10^{-5}$  mg of radium on the disk.

In another series of experiments, thick sheets of lead, molybdenum, tungsten, tantalum, and bismuth were placed 25 cm from the target of a tungsten x-ray tube, operated at 5 m.a. and 50 to 100 k.v.p., and irradiated for periods of 30 min. to an hour. After each exposure, the specimen was placed as near as possible to the counter, within 10 sec., and the subsequent counting rates were recorded at intervals of 5, 15, 30 and 60 min. There was no evident change of the counting rate with time, although the sensitivity was at least  $2 \times 10^{-6}$  mg. The tests were repeated under the same conditions until a sufficiently large number of impulses were recorded to make the statistical error in each interval small.

Long-time fluorescence of two x-ray tube targets was also investigated. The intense ra-

diation incident on the targets of the tubes seemed to be a promising source of the fluorescence. A molybdenum tube was tried many times with a counter of about 44 cc volume, after running at 25 m.a. and 30 k.v.p. Lately, a tungsten tube was used with the small counter previously mentioned (volume 0.32 cc), after operation at 5 m.a., 50 to 100 k.v.p. The tubes were run for intervals of 5 sec. to an hour before each test. It was found that the counters recover their usual characteristics of response in about 0.02 sec., after being paralyzed by the intense radiation. The sensitivity of the small counter was about  $5 \times 10^{-5}$  mg, but no fluorescence was detected in any of the tests.

The small Geiger counter used here is a special one which also responds to ultraviolet light. A full description of its construction and operating characteristics will be published very soon. It was found that the intense discharge produced in the counter by strong x-rays temporarily sensitized it to visible light, so that it responded to the light from the x-ray tube filament, unless an opaque screen was interposed.

Clearly, the tests described here do not exclude the existence of induced gamma-ray emission, or nuclear fluorescence. But they indicate that if these effects do exist, their intensity, or duration, or both, must be below the limits of detection by the methods I have used.

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#### A Comparison of the Theoretical Results of Sugiura and Sommerfeld on the Production of X-rays

Recently A. Sommerfeld<sup>1</sup> has treated the theoretical problem of the production of x-rays by a method somewhat different from the usual quantum mechanical method used by Oppenheimer<sup>2</sup> and Sugiura,<sup>3</sup> which considerably simplifies the analytical treatment.

<sup>1</sup> Sommerfeld, *Ann. d. Physik.* **11**, 257 (1931).

<sup>2</sup> Oppenheimer, *Zeits. f. Physik* **55**, 725 (1929).

<sup>3</sup> Scientific Papers of the Institute of Physical and Chemical Research **17**, 99 (1931).

The basic model is the same in both cases, namely an electron, whose original velocity and direction are known, is retarded and deflected by a positive nucleus, the problem being the determination of the intensity of the radiation resulting from this process. From the results of Sugiura's calculations and those of Sommerfeld, it is not immediately evident whether or not their results are in agreement, and therefore at Professor Sommerfeld's suggestion, the following comparison has been made.

The only explicit formula given by Sugiura

which is of interest in this connection is the expression for the absolute intensity of x-ray radiation at the short wave-length limit (formula 11). When terms in  $\beta_1^2$  are neglected, it can be put in the following form:

$$I(\delta) = \frac{8e^2 h^2 a_1^4}{3R_0^2 m^2 c^3 (e^{2\pi a_1} - 1)} \quad (I)$$

$$\left[ \begin{array}{l} \sin^2 \delta + (1\frac{1}{4} \sin^2 \delta \cos \delta \\ - 2 \sin^3 \delta) \beta_1 + (2 - \sin^2 \delta) a_1^2 \\ + (4 \cos \delta + 3 \sin^3 \delta \\ - 2 \cos \delta \sin^2 \delta) \beta_1 a_1^2 \end{array} \right]$$

The notation of Sugiura has been retained in which  $\delta$  is the angle between the initial direction of the oncoming electron and the direction of observation of the emitted radiation;  $\beta_1 = v_1/c$  where  $v_1$  is the initial velocity of the oncoming electron;  $a_1 = hZ/2\pi m a c \beta_1$ , where  $Z$  is the atomic number of the nucleus and  $a$  is the radius of the first Bohr orbit in hydrogen, and finally  $R_0$  is the distance from the nucleus to the observer.

It is not possible to compare Sommerfeld's result with the above expression directly, but the comparison can be made by proceeding as follows: In Sommerfeld's notation the total intensity corresponding to I above is

$$A^2 = A_\theta^2 + A_z^2$$

where in turn

$$A_\theta^2 = (1/4\pi) \int A_\theta A_\theta^* \sin \alpha d\alpha d\beta$$

and

$$A_z^2 = (1/4\pi) \int A_z A_z^* \sin \alpha d\alpha d\beta \quad (1)$$

and  $\alpha, \beta$  are the angles denoting the asymptotic direction of the outgoing electron.  $A_\theta$  is the component of the vector potential of the emitted radiation perpendicular to the direction of observation and lying in a plane determined by this direction and that of the incident electrons for a given  $\alpha$  and  $\beta$ . Correspondingly  $A_z$  is the  $z$  component. The complete expressions for  $A_\theta$  and  $A_z$  for the case of the short wave-length limit are given by formulae 106a in Sommerfeld's paper apart from a multiplicative factor which contains only universal constants. These expressions were expanded according to ascending powers of  $n_1$  and  $\beta_1$  where  $|n_1| = a_1$ , and the integrations over  $\alpha$  and  $\beta$  were performed as indicated in (1) above, and finally the expression for  $A^2$  was obtained which contains the universal con-

stants expressly omitted in Sommerfeld's formulae. The final expression is

$$A^2 = \frac{8e^2 h^2 n_1^4}{R_0^2 m^2 c^3 (e^{2\pi |n_1|} - 1)} \quad (II)$$

$$\left[ \begin{array}{l} \sin^2 \theta + 4 \sin^2 \theta \cos \theta \cdot \beta_1 \\ + \frac{1}{3}(7 \sin^2 + 2) |n_1|^2 \\ + \frac{1}{3}(28 \sin^2 \theta \cos \theta + 4 \cos \theta) |n_1|^2 \beta_1 \end{array} \right]$$

A comparison of expressions I and II shows several significant differences. In the first place, there is a factor 3 in the denominator of the multiplicative factor in I, not present in II; secondly the coefficients of the terms in  $\beta$ ,  $|n_1|^2$ ,  $|n_1|^2 \beta_1$ , are quite different. The intensity distribution curves therefore have a different shape, formula I yielding about one-third the intensity at  $\theta = \pi/2$  given by II, while, on the other hand, the two expressions give the same result at  $\theta = 0$  and  $\theta = \pi$ . A comparison of the intensity distribution given by I and II with the experimental result obtained by Kulenkampf<sup>4</sup> can not be strictly made because his results do not correspond exactly to the short wave-length limit and the theoretical intensity distribution changes very rapidly as the short wave-length limit is approached. It appears however, that II agrees with the observed intensity distribution obtained by Kulenkampf a little better than I, especially in regard to the position of maximum intensity. No attempt has been made to check all of the extremely complicated computations of Sugiura so that the source of the above mentioned discrepancies can not be pointed out.

It should be borne in mind, when applying Sommerfeld's formulae that terms in  $n_1^2$  and  $\beta_1^2$  have been neglected in most cases, as he explicitly states, both of which, however, cannot be small simultaneously, since  $|n_1|^2$  and  $\beta_1^2$  are connected by the reciprocal relation  $|n_1|^2 = Z^2/13n^2\beta_1^2$ . Thus in making comparisons with experimental results where terms in  $\beta_1^2$  are negligibly small, terms containing  $n_1^2$  should be retained. It is of little use to retain terms in  $\beta_1^2$  as Sugiura has done unless the calculations are made using Dirac's wave functions. A paper giving the results of such calculations is now in the course of preparation at this institute.

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<sup>4</sup> Kulenkampf, Ann. d. Physik 57, 597 (1928).