Temperature and the Compton Effect of Sylvine

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From the theory of the diffuse scattering of x-rays by single crystals it appears that the ratio of the intensity of the incoherent to that of the total scattering at a given angle should be a function of the temperature. Accordingly x-rays of wave-length 0.40A were scattered from sylvine at temperatures of 300°K and 90°K. Aluminum was transferred from the primary to the scattered beam and from the observed change of absorption coefficient of the x-rays the above ratio was calculated. It was found that this ratio increased as the temperature decreased.

I. INTRODUCTION

A. H. COMPTON[1] has shown that the intensity of x-rays scattered by a monatomic gas is given by

$$S = 1 + (Z - 1)f'^2/Z^2 \tag{1}$$

where S is the scattered intensity relative to the Thomson[2] value at the same angle and f' is an average atomic structure factor which has been discussed by Jauncey[3] and Herzog.[4] More recently Jauncey and Harvey[5,6] have shown that the intensity of x-rays diffusely scattered by a simple cubic crystal consisting of atoms of one kind is given by

$$S = 1 + (Z - 1)f'^2/Z^2 - F^2/Z \tag{2}$$

where F is the atomic structure factor including the effect of thermal agitation and the other symbols have the same meaning as in Eq. (1). Independently Woo[7] has considered the diffuse scattering of x-rays from crystals and obtained an expression for the intensity which may be written in the form

$$S = \frac{1 - f'^2/Z^2}{(1 + \alpha \operatorname{vers} \phi)^3} + (f'^2 - F^2)/Z \tag{3}$$

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[1] A. H. Compton, Phys. Rev. **35**, 925 (1930).
[2] J. J. Thomson, Conduction of Electricity through Gases. 2nd Ed., p. 325.
[3] G. E. M. Jauncey, Phys. Rev. **38**, 1 (1931).
[4] G. Herzog, Zeits. f. Physik **69**, 207 (1931).
[5] G. E. M. Jauncey, Phys. Rev. **37**, 1193 (1931).
[6] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **37**, 1203 (1931).
[7] Y. H. Woo, Phys. Rev. **38**, 6 (1931).

where $\alpha = h/mc\lambda$ and ϕ is the angle of scattering. Woo's formula reduces to our formula, Eq. (2), when α in Eq. (3) is equal to zero. According to the theory of Wentzel,[8] Woo's formula consists of two parts—the first part applying to the incoherent and the second to the coherent radiation. The incoherent part contains the Breit-Dirac factor $1/(1+\alpha \text{ vers } \phi)^3$. Hence the ratio of the intensity of the incoherent to that of the total scattering is

$$\eta = \frac{1 - f'^2/Z^2}{(1 + \alpha \text{ vers } \phi)^3} \bigg/ S. \tag{4}$$

Since F is a function of the temperature, S is a function of the temperature as has been shown experimentally by Jauncey[9] and Claus[10] for rocksalt and

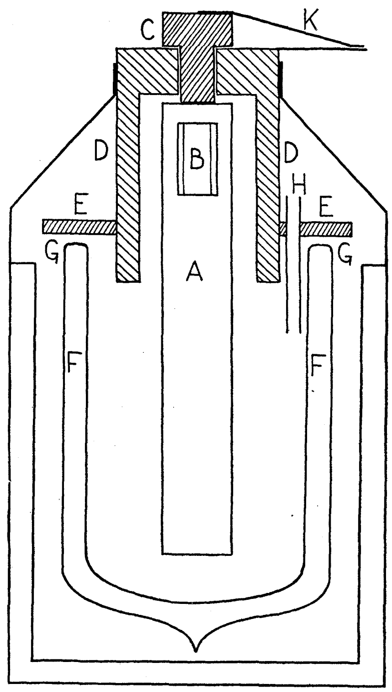

Fig. 1. Diagram of cooling chamber and crystal mounting.

by Jauncey and Harvey[11] for sylvine. It is reasonable to suppose that the incoherent radiation is not affected by temperature so that the ratio η is a function of temperature in virtue of S. If S_1 and S_2 are values of S for a given angle of scattering ϕ but for different temperatures T_1 and T_2 respectively, and if η_1 and η_2 are the corresponding values of η, then

$$\eta_1/\eta_2 = S_2/S_1. \tag{5}$$

[8] G. Wentzel, Zeits. f. Physik **43**, 1 and 779 (1927).
[9] G. E. M. Jauncey, Phys. Rev. **20**, 421 (1922).
[10] W. D. Claus, Phys. Rev. **38**, 604 (1931).
[11] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **38**, 1925 (1931).

The object of the present experiment was to find whether or not η is a function of the temperature.

II. Apparatus and Procedure

The cooling chamber is shown in Fig. 1. *A* is a copper plate 0.4 by 2.5 by 15 cm. This plate has a hole which contains the crystal of sylvine *B* whose dimensions are 0.11 by 1.0 by 2.5 cm. The plate *A* is supported by the hard rubber rod *C* which fits into an inverted micarta cup *DD*. A pointer *K* is attached to the top of the rod *C*. This pointer moves over the scale of a protractor. The inverted cup *DD* is supported above the Dewar flask *FF* and is attached to an insulating ring *EE*. Cotton is placed between *EE* and *FF* at *GG*. Liquid air may be admitted to the Dewar flask through the tube *H*. Cellophane windows in the inverted cup *DD* provide for the ingress and egress of x-rays. The temperature of the crystal may be measured by a copper-iron thermocouple in contact with the crystal.

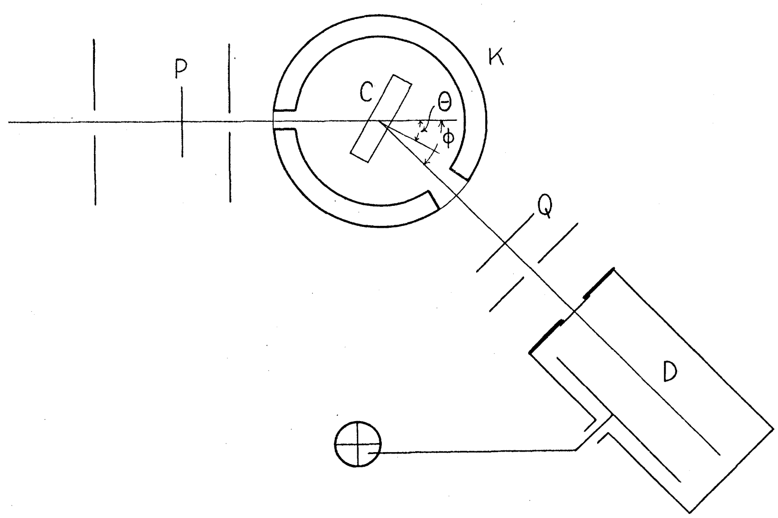

Fig. 2. Diagram of apparatus.

The x-rays from a tungsten target tube pass into the cooling chamber *K* in Fig. 2 and are therein scattered by the crystal *C* to the ionization chamber *D*. This chamber is connected with a reservoir containing ethyl bromide which is maintained at a constant temperature below that of the room during the course of an experiment.

The ionization chamber was first set to receive the primary x-rays through the crystal. Different thicknesses of aluminum were placed in the path of the x-rays and the logarithm of the intensity of the x-rays entering the ionization chamber was plotted against the thickness of the aluminum. It was found that the curve was practically a straight line so that the x-rays coming through the crystal may be considered as homogeneous. The mass absorption coefficient of the x-rays in aluminum was 1.03 corresponding to a wave length of 0.40A as given by Compton's tables.[12] The ionization chamber was

[12] A. H. Compton, X-Rays and Electrons. p. 184.

next placed so as to receive the x-rays scattered by the crystal at an angle ϕ. The crystal was rotated by means of the pointer K into a position such that only diffusely scattered and not regularly reflected x-rays entered the ionization chamber. A thickness d of aluminum was placed in the primary beam at P, Fig. 2, and a reading of the ionization current i_1 taken. The aluminum was then transferred to the scattered beam at Q and the ionization current i_2 measured. Liquid air was then admitted to the flask F and the currents i_1 and i_2 again measured.

III. Experimental Results

At first it was intended to measure the ratio i_1/i_2 at various angles of scattering. However, it will be seen by reference to Eq. (5) that the ratio η_1/η_2 is greatest where the ratio S_2/S_1 is greatest; also by reference to the paper by Jauncey and Harvey[11] it is seen that the ratio of S at room temperature to S at liquid air temperature is greatest for small values of $(\sin\phi/2)/\lambda$, that is, at small angles. But the change of wave-length in the Compton effect is small at small angles, so that the change of the absorption coefficient in going from the primary to the scattered beam is small and the difference between i_1 and i_2 is therefore small also. At large angles, although the Compton change of wave-length becomes greater, S_2 approaches S_1 as seen by reference to the curve in the paper by Jauncey and Harvey.[11] There is an optimum position where the ratio S_2/S_1 does not too nearly approach unity and where the change of wave-length is not too small. Accordingly we abandoned the idea of measuring the ratio i_1/i_2 at different angles and concentrated our efforts on measuring the value of i_1/i_2 at 50°, which is close to the optimum angle.

Three different runs were made. Each run consisted of three sets of readings, the first and third sets being made at room temperature and the second set at liquid air temperature. Each set consisted of twenty-five readings of i_1 and twenty-five readings of i_2 the readings of i_1 alternating with those of i_2. In each set the readings of i_1 and i_2 were respectively averaged and the average of i_1 divided by the average of i_2. These average values of i_1/i_2 are

TABLE I. *Change of i_1/i_2 with temperature at* $(\sin\phi/2)\lambda = 1.06$

Run	i_1/i_2			
	300°K	90°K	300°K	Average at 300°K
1	1.062	1.098	1.023	1.042
2	1.027	1.088	1.024	1.026
3	1.032	1.122	1.028	1.030
Average		1.103		1.033

shown in Table I. Comparing the third and fifth columns of Table I, it is quite obvious that the ratio i_1/i_2 increases when the temperature is decreased from 300°K to 90°K.

IV. Discussion of Results

In order to compare the experimental results with Eq. (5), it is necessary to obtain values of the ratio of incoherent to total scattering from the experimental values of i_1/i_2. The case where the scattered radiation is assumed to consist of coherent and incoherent radiation, the wave-length of the latter differing from that of the former by an amount $\delta\lambda=0.0242$ vers ϕ,[13] has been discussed by Jauncey and Defoe.[14] However, the formula of Jauncey and Defoe only applies when $\theta=\phi/2$, where θ is the angle between the normal to the crystal and the direction of the primary beam. The formula for the case $\theta\neq\phi/2$ may easily be shown to be

$$\eta = \left[1+C\,\frac{1-ye^{-kd}}{y-1}\right]^{-1} \tag{8}$$

where

$$C = e^{-g'}\frac{1-e^{g'}e^{\mu_3 t[\sec(\phi-\theta)-\sec\theta]}}{1-e^{\mu_3 t[\sec(\phi-\theta)-\sec\theta]}}\cdot\left[1-\frac{(\mu_4-\mu_3)\sec(\phi-\theta)\cos\theta}{\mu_3[1-\sec(\phi-\theta)\cos\theta]}\right]^{-1}$$

$$g' = (\mu_4-\mu_3)t\sec(\phi-\theta)$$

$$y = i_1/i_2$$

the notation being that used in the paper of Jauncey and Defoe.[14] In the present experiments $\lambda=0.40$A, the crystal thickness $t=0.11$ cm, $\phi=50°$, $\theta=20°$, and the thickness of aluminum transferred, $d=0.3$ cm. Using the average values of i_1/i_2 in the last row of Table I and correcting for the lack of complete absorption in the ionization chamber, which was 43 cm long, we obtain $\eta_{300°K}=0.39$ and $\eta_{90°K}=1.01$. There is obviously something the matter since η at 90°K comes out greater than unity. The error is very likely in the readings since by reference to Table I it is seen that the three values of i_1/i_2 at 90°K differ considerably from each other and we have used the average value. Also it must be remembered that Eq. (6) is based on the assumption that the incoherent radiation at a given angle of scattering ϕ consists of a single wave-length, whereas according to the theories of Jauncey[15] and Wentzel[8] the incoherent radiation consists of a band of wave-lengths, the center of gravity of the band being on the long wave-length side of the Compton modified line. If the average change of wave-length of the incoherent band is greater than the Compton change of wave-length, the calculated value of η from the experimental value of i_1/i_2 will be smaller, and so it is very probable that on this account $\eta_{90°K}$ can be made less than unity. At the same time, however, $\eta_{300°K}$ will also be reduced. Lacking knowledge to the contrary, we shall suppose that the ratio $\eta_{300°K}/\eta_{90°K}$ will be practically unaltered by assuming a change of wave-length which differs slightly from the Compton change. The ratio $\eta_{300°K}/\eta_{90°K}$ is equal to

[13] A. H. Compton, Phys. Rev. **21**, 483 (1923).
[14] G. E. M. Jauncey and O. K. Defoe, Phil. Mag. **1**, 711 (1926).
[15] G. E. M. Jauncey, Phys. Rev. **25**, 314 and 723 (1925).

0.39, which by Eq. (5) should equal $S_{90°K}/S_{300°K}$. The value of $S_{90°K}/S_{300°K}$ at $(\sin \phi/2)/\lambda = 1.06$ is 0.76 according to Jauncey and Harvey.[11]

V. Conclusion

We have shown definitely that the ratio i_1/i_2 increases when the temperature of sylvine decreases from 300°K to 90°K. In 1929 Jauncey and Bauer[16] examined the scattered radiation from carbon, aluminum and copper for an effect of this type and found that the ratio i_1/i_2 is independent of the temperature. Ordinary carbon, aluminum and copper may be considered as made up of powdered crystals, and Woo[7] has suggested that for powdered crystals the ratio i_1/i_2 should not be a function of the temperature. Our present result for the diffuse scattering of x-rays from a single crystal is thus distinct from the result found for the scattering of x-rays from amorphous substances. From our values of i_1/i_2 at 300°K and 90°K it may be inferred by means of Eq. (6) that the ratio of incoherent to total scattered radiation from sylvine at 90°K is greater than the ratio for sylvine at 300°K. We are not at present prepared to say whether or not $\eta_{300°K}/\eta_{90°K}$ is equal to $S_{90°K}/S_{300°K}$ as it should be according to Eq. (5).

[16] G. E. M. Jauncey and H. Bauer, Phys. Rev. **34**, 387 (1929).