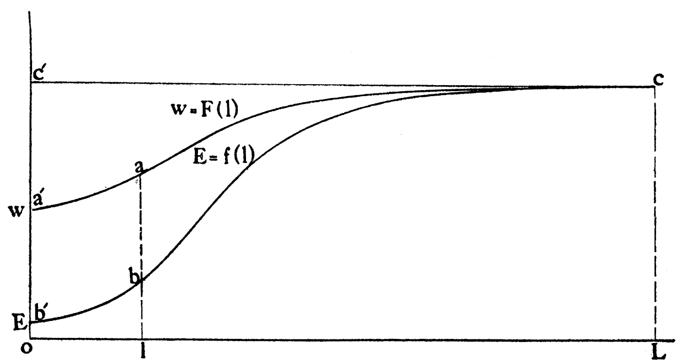
THE THERMAL CONDUCTIVITIES OF TUNGSTEN, TANTALUM AND CARBON AT INCANDESCENT TEMPERATURES BY AN OPTICAL PYROMETER METHOD.

By A. G. Worthing.

Introduction.

THE present paper is the direct outcome of a study of the energy losses in incandescent lamps by Hyde, Cady and Worthing.[1] With a single exception the question of thermal conductivity at incandescent temperatures seems not to have been attempted heretofore. Angell,[2] using a method suggested by Mendenhall, investigated aluminum and nickel up to temperatures of 600° C. and 1200° C. respectively. The temperature measurements were made by means of thermocouples located on the inner and the outer surfaces of hollow cylindrical rods which were electrically heated. Extremely large heating currents and fairly large specimens of material were, of course, necessitated. The question of incandescence did not enter in the method. In the present paper a method of measuring thermal conductivity depending on the incandescence of small filaments mounted in evacuated chambers, such as filaments in ordinary lamp bulbs, is described and made use of.



Distance along filament.

Fig. 1.

Diagram to illustrate method for detemining thermal conductivities.

[1] Trans. Illum. Eng. Soc. (U.S.), 6, p. 238 (1911); Illum. Eng. (Lond.), 4, p. 389 (1911).

[2] Phys. Rev., 33, p. 421 (1911).

THEORY.

Consider the energy supply and radiation intensity distributions per unit of filament surface for a small filament of uniform surface and cross section, mounted in an evacuated bulb and heated to a steady state of incandescence by an electric current. These quantities, w and E respectively, as functions of the distance along the filament are diagrammatically represented by the curves $a'ac$ and $b'bc$ respectively of Fig. 1. The meanings of the symbols here used together with others are given in Table I. Evidently at the center of the filament loop where there is

TABLE I.

Symbols and Quantities Used.

Symbol.	
w	Rate of heat production electrically at a given filament cross-section per unit of radiating surface.
E	Radiation intensity, or rate of radiation of energy per unit of radiating surface.
l	Distance measured along the filament.
L	One half of the filament length.
dH/dt	Rate of conduction of heat along the filament.
r	Radius of filament.
k	Thermal conductivity.
T	Temperature.
i	Current density.
ρ	Resistivity.
χ	Electrical conductivity.
w_m, E_m, T_m	Values of w, E and T at center of filament loop.
μ, σ, β, γ, A, and B	Constants defined by equations (8), (9) and (10).

no temperature gradient along the filament

$$(1) \qquad [w = E = w_m = E_m]_{l=L}.$$

It is easily seen that the areas $lacLl$, $lbcLl$ and $bacb$ respectively represent quantities proportional to the rate of heat production in the filament length lL, the rate of radiation of energy from the filament length lL, and the rate of heat conduction along the filament at l. For the last named quantity, *i. e.*, $\frac{dH}{dt}$, we may write

$$(2) \qquad \frac{dH}{dt} = 2\pi r E_m \int_l^L \left(\frac{w}{w_m} - \frac{E}{E_m}\right) dl.$$

This together with the general equation

$$(3) \qquad \frac{dH}{dt} = \pi r^2 k \frac{dT}{dl}$$

enables one to write for the thermal conductivity at l

$$k = \frac{2E_m}{r\frac{dT}{dl}} \int_l^L \left(\frac{w}{w_m} - \frac{E}{E_m} \right) dl. \tag{4}$$

This assumes that the radial component of the temperature gradient at l is negligible in comparison with the axial component. Determinations of k for different positions l and hence for different temperatures T lead to the determination of k as a function of T.

Apparatus and Method.

The arrangement of apparatus and the general method of procedure were the same as those described in the study of the temperature distribution in an incandescent lamp filament near a cooling junction.[1] The filaments studied were U-shaped lamp filaments mounted with welded junctions in large well-exhausted lamp bulbs. The lamps which were initially well aged were used as backgrounds. The pyrometer filaments were of tungsten, sufficiently small, usually about 0.025 mm. in diameter, so that even when seen projected against the background at a place where the axial temperature gradient was a maximum, there was no apparent difference in the brightness of the background at the two edges of the projected pyrometer filament.

The precautions noted for optical pyrometric methods of this type by Worthing and Forsythe[2] have been carefully observed in the work on tungsten. The work on tantalum and carbon is less accurate; and while at the time of the experimental work on them all of these precautions were not known, it is believed that the conditions were not such as to cause very great errors.

The experimental work consisted in determining the following three relations,

$$E = \varphi(T), \tag{5}$$

$$E = f(l), \tag{6}$$

$$w = F(l). \tag{7}$$

Equation (5) was obtained by balancing the pyrometer filament first against a central portion of the loop of the filament under investigation when operated successively at various currents, and then against a black body similarly placed and operated at various temperatures. In the former case the brightness of the background image was so reduced by sectored disks that the pyrometer filament currents were of the same order of magnitude in both instances. With the aid of ordinary electrical

[1] PHYS. REV., Vol. IV, No. 6.

[2] PHYS. REV., II, 4, p. 163 (1914).

and length measurements on the background lamp filament, one may then express E as a function of the black body temperature. The temperature relations determined by Mendenhall and Forsythe[1] were used to convert these into the desired functions of the true temperatures.

Equations (6) and (7) were determined in much the same manner that T as a function of l was determined in the paper on temperature distribution already mentioned. In fact (5) enables one to obtain (6) from the temperature distribution there given. The simple determination of the relation between ρ and E with the aid of (6) leads to (7), since obviously w is proportional to ρ everywhere in a uniform filament heated to a steady state by an electric current. The relations (6) and (7) are those represented in Fig. 1 respectively by the curves $b'bc$ and $a'ac$.

The effects of a Thomson electromotive force located in the filaments near the junction were eliminated by averaging the results obtained with the heating current flowing first in one direction through the filament and then in the reversed direction. In determining (5) account was taken of the fact that the average rate of energy supply electrically per unit surface of filament does not equal (less than, as represented in Fig. 1) the rate of energy radiation per unit surface of filament at a point remote from the filament junction. The correction factor is the ratio (area $a'c'ca'$)/(area $Oa'cLO$).[2]

Once given (5), (6) and (7) a method of applying (4) in order to determine k is obvious. Considerable difficulty was experienced in obtaining satisfactory relations for (6), due for the most part to the inability to realize the fundamental assumptions of uniform filament surface and cross-section. The fact that the glass of the bulb enclosing

[1] Astrophys. Jour., 37, p. 380 (1913).

[2] The area $a'c'ca'$ represents an input loss. For many purposes this loss may be considered as producing an apparent decrease in the effective length of the filament ΔL_1 given by (area $a'c'ca'$)/(length oc'). Similarly from the standpoint of the energy radiated, there is an apparent decrease in the effective length ΔL_2, given by (area $b'c'cb'$)/(length oc'). In a like manner one may obtain from the standpoint of the luminous flux from the filament, the apparent decrease ΔL_3. For uniform tungsten filaments, in loops sufficiently long so that there is no sensible heat conduction along the filament at the centers of the loops, there have been found in practice an approximately constant ratio between the junction temperatures, when fused with copper, and the maximum filament temperatures. For such cases general formulæ may be derived for ΔL_1, ΔL_2 and ΔL_3. Thus

$$\Delta L_1 \cdot i = 2700 \frac{\text{amps}}{\text{cm}}$$

where i is the current density L_1 and L_2 are approximately thereafter determined by

$$\Delta L_1 : \Delta L_2 : \Delta L_3 = 3:7:10.$$

For the lamp mentioned in the main article, operating at 2315° K., the apparent decreases for each junction are respectively 0.21 cm., 0.50 cm. and 0.71 cm.

the background was not optically good was also a source of error. However, as nearly as possible the law was empirically determined for the finite range $0.2 < (E/E_m) < 0.85$ and then assumed to hold also for the range $0.85 < (E/E_m) < 1.00$. Slight errors in pyrometer balancing in this region were always productive of comparatively great errors in the constants of the law sought. Measurements made there, however, always on the whole justified the assumption. Below $E/E_m = 0.2$ deviations from the empirical law found for the middle region always occurred in a definite direction, hence the limits chosen. No values obtained for k were based on the application of the empirical equation to regions where $E/E_m < 0.2$. How well the empirical law fills the requirements may be seen in Fig. 2, in which there have been platted the data

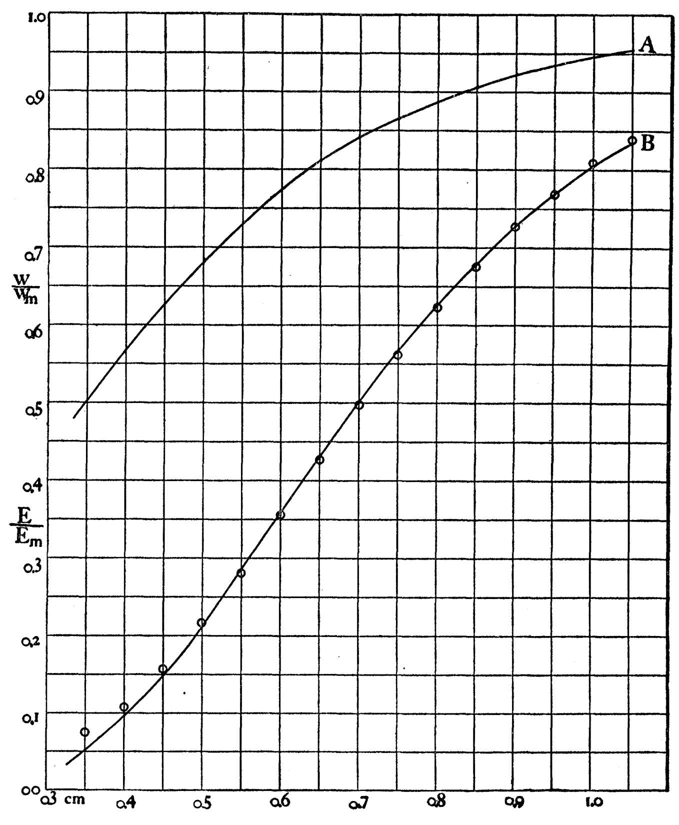


Fig. 2.

The relative input (A) and the relative radiation intensity (B) distributions for a tungsten filament ($r = 0.01045$ cm.) heated to a maximum temperature of 2315° K. The circles represent experimental values of radiation intensities obtained in a particular set of measurements. The curves represent distributions given by equations (9) and (10).

for a single experimental determination of the E and w distributions for the same lamp operated at the same maximum temperature as the one for which the temperature distribution has been given in a paper already referred to. The deviations, with the exception of the ones for small values of E/E_m, are accidental and in the average of many curves are eliminated.

RESULTS.

For tungsten the empirical relations found which determine the character of the functions in (5), (6) and (7) are of the types

$$E = \sigma T^{\beta}, \tag{8}$$

$$\frac{E}{E_m} = [1 - e^{-\mu(l+l_0)}]^{\gamma}, \tag{9}$$

$$\log \frac{w}{w_m} = \log \frac{E}{E_m}\left(A + B \log \frac{E}{E_m}\right). \tag{10}$$

Equations (8) and (10) seem to be quite exact. The values of various constants which enter in connection with the data platted in Fig. 2 in the application of (4) to the determination of k are given in Table II.

TABLE II.

Constants for a Particular Application of (4) to a Tungsten Filament.

Quantity.	Value.	Quantity.	Value.
r	0.01045 cm.	β	5.35
i	$12600 \frac{\text{amp}}{\text{cm}^2}$	σ	$6.2 \times 10^{-17} \frac{\text{watts}}{\text{cm}^2 \times \text{deg.}^{5.35}}$
E_m	$62.1 \frac{\text{watts}}{\text{cm}^2}$	γ	10.
T_m	2315° K	A	0.257
μ	$3.80 \frac{1}{\text{cm}}$	B	0.015

The value obtained for β, while probably fairly exact, is to be regarded as preliminary. It is hoped that a future paper from this laboratory will consider it along with other similar determinations in detail. In Fig. 3 there are given the results obtained by the application of (4) to the particular filament when operated at four different maximum temperatures. For each such maximum temperature four determinations have been made of k at various temperatures corresponding respectively to values for E/E_m of 0.8, 0.6, 0.4 and 0.3. That the assumption made in the derivation of (4), of a negligible difference between the temperature gradient and its axial component is justified for these positions, may be seen from an inspection of Table III. of the paper on temperature distribution. In the case of $l = 0.7$ cm., or $E/E_m = 0.808$, at the

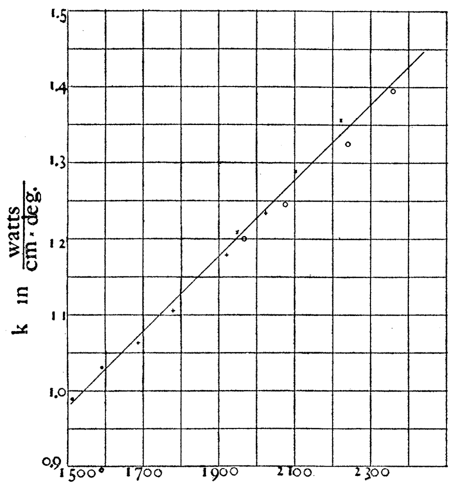


Fig. 3.

Thermal conductivity of tungsten as a function of temperature.

o	14280 $\frac{\text{amp}}{\text{cm}^2}$	2468° K
x	12600	2315
x	10520	2111
	8420	1890

surface of the filament the temperature gradient is only 0.6 per cent. greater than the axial component. The average over the filament cross section would naturally give a still smaller variation. In case the empirical distributions (9) and (10) are correct, no errors are introduced in the values of k thus determined, due to the fact that as E/E_m approaches unity, the axial component of the temperature gradient is no longer approximately equal to the whole gradient. The general agreement of the results is quite satisfying.

Values of k at various temperatures for tungsten, tantalum and untreated carbon are included in Table III. It is of interest to note that the order in which the materials are given represents their order as to thermal conductivity, tungsten having the highest values; that for carbon there is only a very small temperature coefficient, and that for both tungsten and tantalum there are fairly large positive temperature coefficients. With the exception of carbon no published values with which to compare results have been found. Of these only the rough determination of Hansen[1] on carbon electrodes (these carbon electrodes probably contain carbon in much the same form as do the untreated carbon filaments used by the writer) are directly comparable, since the

[1] Trans. of Amer. Electrochem. Soc., 16, p. 351 (1909).

TABLE III.

Thermal Conductivities k, and Values of $\frac{k}{xT}$ *for Tungsten, Tantalum and Carbon in the Range 1500° K. to 2500° K.*

Temperature, ° K.	Tungsten		Tantalum.		Carbon (Untreated).	
	k, $\frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units.	k, $\frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units	k, $\frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units.
1500	0.98	0.283		1.62		
1700	1.08	.322	0.73	3.3	0.084	11.9
1900	1.18	.362	.78	3.4	.086	10.1
2100	1.28	.405	.83	3.6	.088	8.8
2300	1.38	.450				
2500	1.48	.502				

others have experimented on the crystalline forms graphite and diamond. Hansen's results give as an average about 0.06 watts/cm. × deg. for the neighborhood 0° C. to 400° C., which is qualitatively in good agreement with the writer's values. He also mentions a value of about 0.008 watts/cm. × deg. for carbon at 3000° C. This does not check at all with the writer's values.

Because of the theoretical interest, determinations have been made of the function k/xT. Its values for all three substances for certain temperatures have been included in Table III. It is of interest to note that k/xT is not approximately constant for any one of them, there being a large negative temperature coefficient for carbon and large positive coefficients for both tungsten and carbon. Variations of a similar character for nickel and aluminum at high temperatures are indicated by the work of Angell,[1] although attention has not been called to them in his paper. Aluminum and nickel respectively have a large positive and a large negative temperature coefficient for k/xT. These deviations have an important theoretical bearing. The classic work of Jaeger and Diessehorst[2] on this function between 18° C. and 100° C. has usually been accepted as indicating its probable constancy at least for pure metals at higher temperatures. This conclusion has fitted in well with electronic theories. The work of Lees[3] and later particularly that of Meissner,[4] experimenting at temperatures as low as 20° K., indicated deviations from constancy such as would be consistent with a zero value at absolute zero. Such deviations were found to disappear, however,

[1] PHYS. REV., 33, p. 421 (1911).

[2] Wissensch. Abh. d. Phys. Techn. Reichsanstalt, 3, p. 269 (1900).

[3] Phil. Trans. (A), 208, p. 381 (1908).

[4] Verh. d. Deut. Phys. Gessell., 16, p. 262 (1914)

at the temperatures used by Jaeger and Diesselhorst. Modifications of the electron theories have been made in accord with these deviations. Further modifications are evidently necessary. Some data obtained by the method here used seem to indicate that the thermal conductivity of tungsten may be a function of the current density in the filament. The author hopes to consider this point in more detail.

Summary.

1. A method based on optical pyrometry for obtaining the thermal conductivity of certain solid substances at incandescent temperatures has been described.

2. The thermal conductivities of tungsten, tantalum and carbon in the region 1500° K. to 2500° K. have been obtained. The results on carbon check well with results by Hansen.

3. Values for the function k/xT (k thermal conductivity, x electrical conductivity, T absolute temperature) have been determined for the same substances and for the same temperature ranges. Large temperature coefficients of the function were found, in contradiction to what would be expected from electronic theories.

4. As a preliminary determination of the relation between the radiation intensity E and the absolute temperature T for tungsten, the equation

$$E = \sigma T^{\beta},$$

where $\beta = 5.35$ and $\sigma = 6.2 \times 10^{-17}$ watts/cm.^{2} $\times$ deg.$^{5.35}$, has been found to hold.

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