# A DETERMINATION OF AVOGADRO'S CONSTANT $N$ FROM MEASUREMENTS OF THE BROWNIAN MOVEMENTS OF SMALL OIL DROPS SUSPENDED IN AIR.

BY HARVEY FLETCHER.

## INTRODUCTION.

In a previous paper[1] equations were developed which showed the effect of the so-called Brownian movements upon a spherical particle moving under the influence of a constant outside force. In these formulæ there occurred a factor $k$ which was a function of $l/a$, the exact dependence being unknown at the time the paper was written. For this reason the experimental work was directed toward determining $Ne$, which process did not involve $k$. Since that time Prof. Millikan, by a series of careful experiments, has determined the form of the function[2] $k$; and has thus made possible a fairly exact determination of $N$ by the Brownian movement method as applied to gases.

The former equations have been modified so that the task of finding the Brownian movement effect is now much less laborious than before. Also the apparatus has been improved, making it possible to obtain from 1,000 to 6,000 observations on a single particle.

## § 1. DEDUCTION OF THE EQUATION FOR THE DETERMINATION OF $N$.

It was shown in the article cited above that the law of distribution of the times of fall through a constant distance of a small spherical particle which is subjected to the bombardment of gas molecules is

$$n = \frac{\mathcal{N}}{2}\sqrt{\frac{h}{\pi}}\int_{t_2}^{t_1}(bt^{-\frac{3}{2}} + Vt^{-\frac{1}{2}})e^{-h/t(b-Vt)^2}dt, \tag{1}$$

where

$$h = \frac{9\pi\mu ak}{4\epsilon} = \frac{3}{2}\frac{\pi\mu akN}{RT}. \tag{2}$$

In this formula $n$ is the number of times, out of $\mathcal{N}$ observations, that the particle will fall a constant distance $b$ in a time which lies between $t_1$ and $t_2$; $V$ is the constant velocity due to gravity and may be found

[1] PHYS. REV., Aug., 1911, p. 81.

[2] PHYS. REV., Vol. II., p. 109, 1913, gives the method and PHYS. REV., Vol. I., p. 219, 1913, gives the results on $k$. See also Le Radium, *10*, p. 15, 1913.

experimentally as shown later; $h$ is a constant depending upon the number of molecules $N$ in a gram molecule of the gas, the viscosity coefficient $\mu$, the gas constant $R$, the absolute temperature $T$, and the function $k$ as indicated in equation 2. The method of finding $a$ and $k$ will be outlined in another section.

Let the time of fall due to gravity,

$$(3) \qquad \frac{b}{V} = t_g.$$

Now the average value of all the observed times of fall below $t_g$ will be given by

$$t_a = \frac{1}{2}\sqrt{\frac{h}{\pi}}\int_{t_g}^{0} t(bt^{-\frac{3}{2}} + Vt^{-\frac{1}{2}})e^{-h/t(b-Vt)^2}dt.$$

Change the variable from $t$ to $u$ by the relation $b = Vt + ut$ and we get

$$(4) \qquad \bar{t}_a = 2\sqrt{\frac{h}{\pi}}\int_0^{\infty} \frac{2u^2 + 4bV - 2u\sqrt{u^2 + 4bV}}{4V^2} e^{-hu^2}du.$$

Similarly, the average value of all the observed times of fall above $t_g$ is given by

$$(5) \qquad \overset{+}{t}_a = 2\sqrt{\frac{h}{\pi}}\int_{-\infty}^{0} \frac{2u^2 + 4bV - 2u\sqrt{u^2 + 4bV}}{4V^2} e^{-hu^2}du.$$

Evaluating (4) and (5) we have

$$\overset{+}{t}_a = \frac{b}{V} + \frac{1}{4hV} + \frac{1}{V^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bV}e^{-hu^2}du,$$

$$\bar{t}_a = \frac{b}{V} + \frac{1}{4bV} - \frac{1}{V^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bV}e^{-hu^2}du.$$

Adding we have

$$(6) \qquad \frac{\overset{+}{t}_a + \bar{t}_a}{2} = \frac{b}{V} + \frac{1}{4hV^2}.$$

Theoretically this equation in connection with equation (2) could be used to determine $N$, but the difference between $\left(\frac{\overset{+}{t}_a + \bar{t}}{2}\right)$ and $b/v$ is so small that it is not well adapted for this purpose.

Let $\tau = \left(\frac{\overset{+}{t}_a - \bar{t}_a}{2}\right)$, then

$$(7) \qquad \tau = \frac{1}{v^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bv}e^{-hu^2}du.$$

If the variable is changed by the relation $hu^2 = x$, and the substitution

(8) $$z^2 = 4hbv$$

is made, there results

$$\tau = \frac{2}{\sqrt{\pi}} \frac{t_g}{z^2} \int_0^\infty \sqrt{x + z^2} e^{-x} dx.$$

If we expand the integrand and integrate the resulting series we obtain

$$\tau = \frac{t_g}{z^2} \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-x} \left( 1 + \frac{1}{2} \frac{x}{z^2} - \frac{1}{8} \left( \frac{x}{z^2} \right)^2 + \frac{1}{16} \left( \frac{x}{z^2} \right)^3 \cdots \right) dx$$

$$= \frac{t_g}{z} \frac{2}{\sqrt{\pi}} \left( 1 + \frac{1}{2z^2} - \frac{1}{4z^4} + \frac{3}{8z^6} + - \right)$$

or

(9) $$z = \frac{t_g}{\tau} \frac{2}{\sqrt{\pi}} \left( 1 + \frac{1}{2z^2} - \frac{1}{4z^4} + \frac{3}{8z^6} \right).$$

This expresses $z$ in terms of itself; but inasmuch as the term $1/2z^2$ is usually of the order .02, the formula

$$z = \frac{2}{\sqrt{\pi}} \frac{t_g}{\tau}$$

may be used to get an approximate value of $z$, and if then this value is substituted in the brackets a more accurate value may be obtained which may again be substituted in the brackets. We thus arrive at a value of $z$ in terms of $t_g$ and $\tau$ which can be determined from experiment. Comparing (9) with (8) and (2) we see that

(10) $$N = \frac{RT}{6\pi\mu} \frac{z^2}{akV^2 t_g}.$$

For air this reduces to

$$N = 7.12 \times 10^{12} \{1 + .0008(t - 20)\} \frac{p^2}{akV^2 t_g}.$$

Comparing (2), (6) and (9), we get

(11) $$\frac{\overset{+}{t}_a + \overset{-}{t}_a}{2} = t_g + \frac{1}{3} \frac{t_g}{z^2}.$$

This equation is of theoretical interest only and can be checked by experiment. In the observations recorded below, it is nearly always found that $(\overset{+}{t}_a + \overset{-}{t}_a)/2$ is greater than $t_g$ by a quantity of the order of magnitude indicated in this equation.

All of the quantities of the right-hand member of equation (10) can be measured by experimental methods to be described later.

## § 2. Method of Calculating $a$ and $k$.

Prof. Millikan gives[1] for the value of $k$,

$$k = \left\{1 + \frac{l}{a}(.874 + .32e^{-1.54a/l})\right\}^{-1} \tag{12}$$

and deduces $l$ from the Boltzman equation $\mu = .3506\rho\bar{c}l$. If we combine this with the pressure equation $p = \frac{1}{3}\rho c^2$ and remember that $\bar{c} = .921c$ we get for air the equation

$$l = \frac{\mu}{.919p\sqrt{\frac{3\rho_0}{p_0}}} = \frac{70.5}{p} \times 10^{-5}\{1 + .00446(t - 20)\}, \tag{13}$$

where $p$ is expressed in centimeters of mercury and $t$ is the number of centigrade degrees above 20° C.

When gravity is the only force acting, we have from Stokes's law of fall

$$mg = 6\pi\mu akV.$$

Write equation (12) in the form

$$k = \left(1 + \frac{B}{a}\right)^{-1} \tag{14}$$

where

$$B = l(.874 + .32e^{-1.54a/l}),$$

and substitute in the above and solve for $a$. Then

$$a = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \frac{9}{2}\frac{\mu V}{\rho g}} = -\frac{B}{2} + \sqrt{\frac{B}{2} + .921 \times 10^6 V}. \tag{15}$$

To determine $a$ from (14) and (15) first neglect the exponential term to get an approximate value of $B$; substitute this in (15) to obtain an approximate value of $a$; then substitute this in (14) thus giving a more accurate value of $B$, and repeat the process until the desired accuracy is obtained. In most cases this need only be repeated once and at most twice. $a$ and $B$ are then substituted in 13 to obtain $k$.

## § 3. Description of Apparatus.

The apparatus is similar to Professor Millikan's and to that used in my previous experiments on the Determination of Ne for Gaseous Ioniza-

[1] Phys. Rev., 1, p. 218, 1913. Also Le Radium, *10*, p. 15, 1913.

tion with some important modifications. It consists essentially of four parts, the lighting system, the observing system, the timing system, and the vessel containing the small oil drops suspended in air. The lighting system is the same as used before. The light from a 15-ampere direct current arc is passed through a condensing lens, then through a column of water to absorb the heat, and then between the condenser plates to illuminate the drop (See Fig. *B*, p. 93, PHYS. REV., Aug., 1911). The arc is held in position by means of two tangent screws at right angles, which enables one to adjust it to any desired position.

The observing telescope, which was made by Wm. Gaertner and Company especially for this experiment, is mounted on a cathetometer. In addition to the two tangent screws ordinarily found on this instrument, namely one for changing the heighth and the other for changing the azimuth, the telescope is mounted on a ratchet and pinion which enables one to move it back and forward without changing the magnifying power, *i. e.*, without changing the distance between the objective and eye piece. The objective is corrected for the distance for which it is used, namely about ten inches. In the eye piece is mounted a scale having ten large divisions each divided into five equal parts. After each set of observations this scale is calibrated by comparison with a standard centimeter.

The time is taken by means of a kymograph. The paper which is to receive the record is mounted on two slowly revolving drums which are placed about three feet apart, and it is then covered with lamp black. By means of a signal key the time when the small oil droplet crosses each division in the eye-piece is recorded on the kymograph. Fig. 1

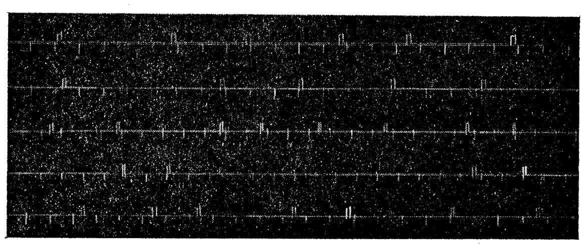

Fig. 1.

shows a photograph of a very small part of the record after it has been fixed with shellac.

The complete record measures $90 \times 8$ inches. The time marker was compared with a standard clock and found to agree to one fortieth per cent.

Fig. 2 shows a vertical cross section of the vessel in which the small

[1] PHYS. REV., Aug., 1911.

oil drops are suspended. The chamber $B$ is exhausted to the desired pressure and the stopcock $C_2$ is closed. A fine spray of liquid vaseline is produced in the bottle $A$ by means of an ordinary atomizer. The cock is then opened, permitting part of the spray to pass along the tube into the chamber $B$, and to finally fall through the small hole in the top plate of the condenser into the position indicated in the figure. Here the

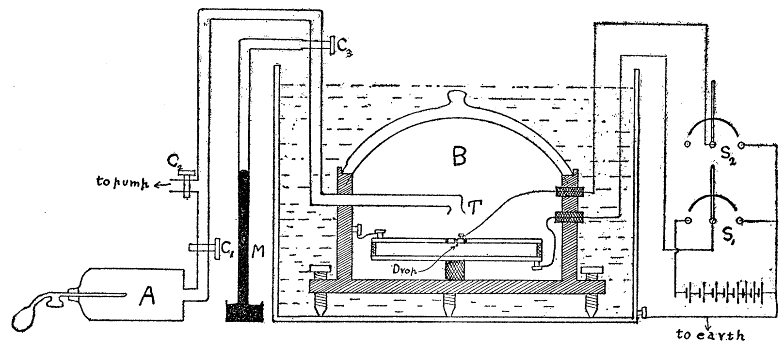


Fig. 2.

spray is illuminated and observed as described above. During the observation the three-way stopcock $C_3$ is turned so that the vessel and pressure gauge $M$ are in communication. The entire vessel containing the condenser plates is closed at the top with a bell jar and immersed in a bath of transformer oil. This serves two purposes; it stops small leaks which might exist at the different joints, and keeps the air in the vessel at a constant temperature—thus preventing convection currents.

In order to control the drop with electric forces, it is necessary that it be charged. To prevent it from remaining uncharged in case it should catch an ion from the air an X-ray bulb is arranged so as to produce intense ionization between the plates making it possible for the drop to become quickly recharged. However it was found that for pressures under 20 cm. of mercury that the drop very seldom caught ions from the air. In fact only two of the drops recorded in this paper changed their charge, although some of them were under observation six hours.

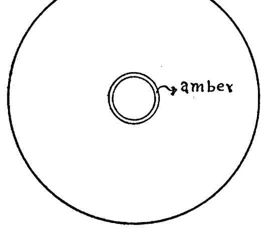


Fig. 3.

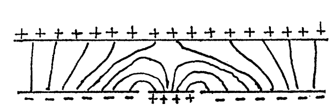

Fig. 4.

To enable one to raise the drop after it had fallen, a condenser of dimensions like that used by Professor Millikan in all the oil drop work was employed. It was found that the small droplets, due to Brownian movements, would sometimes drift to the right or left and finally go out of the beam of light and be lost. To prevent this the top plate was cut into two concentric circular plates which were insulated from each other by a thin strip of amber as indicated in Fig. 3. If a negatively charged drop should drift in any direction whatever from the center of the condenser, the switch $S_1$, is thrown to the right, and the switch $S_2$ to the left, producing an electrostatic field similar to that shown in Fig. 4. The drop would be forced back again into the center of the beam of light. The switches would just be reversed for a positively charged drop. This precaution was justified for the drop was shifted from three to ten times during the progress of a set of observations.

With these precautions it was found that a drop could be held under observation for almost an unlimited time. One drop was under observation six hours and if we had so desired we could have held it twice that long as the seeing was just as good at the end as it was at the beginning.

## § 4. THE DATA.

The density of the liquid vaseline was found to be .845 gm./cm.$^3$ at 20° C.; the viscosity coefficient of air was taken as .0001826 at 23° C.; and the gas constant $R$, as $82.9\times10^6$. Because of the large amount of space that would be taken the complete data is not given but only the final values. Table I. shows a sample of the original data as it was read from the kymograph record. It is the first 100 observation on drop 1. In the first column is recorded the time at which the drop crossed each scale division. It was found impracticable to permit the drop to fall the full fifty divisions, as it would drift slightly out of focus and in the process of refocusing the scale would be shifted. So that the breaks in the column represents periods of refocusing or some other difficulty, while from one break to the next represents a continuous falling of the drop under the influence of gravity. For example, the first time it fell 24 divisions in 71.0 seconds, and was then brought back by the electric field. It then fell 23 divisions in 72.1 seconds; 45 divisions in 131.3 seconds; and 25 divisions in 75.6 seconds; etc. The time of fall $t_g$ for one division due to gravity was obtained by dividing the total distance of fall by the number of divisions crossed, *i. e.*, in this case

$$t_g = \frac{71.0 + 72.1 + 131.3 + 75.6 + \text{etc.}}{24 + 23 + 45 + 25 + \text{etc.}} = 3.092.$$

In the second, third, and fourth columns are recorded the times necessary to travel each scale division, each two scale divisions, and each three scale divisions respectively. These times are derived from the first column in a very obvious manner. The average of all of the values in the first column above 3.09 is found to be 3.587, while the average of those below 3.09 is 2.661. These values are designated $\overset{+}{t}_a$ and $\overset{-}{t}_a$ and are tabulated in Table II. Then

$$\tau = \frac{3.587 - 2.661}{2} = .463 \text{ second.}$$

$z$ can now be computed from equation 9 and is found to be 7.58. If this value is substituted in (10) there results for $N$, $60.9 \times 10^{22}$. The velocity $V$ is simply the fall distance .00916 divided by 3.09 or .002962 cm./sec. From this value $a$ and $k$ can be computed as explained in Section 2. The final values for this drop are tabulated in Table II. It is interesting to note that $z$ for the three different fall distances is proportional to 1, 1.408, and 1.697 *i. e.*, to 1, $\sqrt{2}$, and $\sqrt{3}$ as it should be according to equation (10).

The amount of labor that was necessary to obtain a value of $N$ according to the original method of calculation can be partly realized when it is pointed out that Table I. contains only one seventh of the observations on this drop (Drop 12 has five times as many observations) and that each observation required a separate calculation[1] to obtain a value of $u$, and finally an average of all of these values of $u$ had to be found. It still requires considerable time to work up the data, but by using the adding machine one can add 1,000 numbers in a comparatively few minutes.

Tables II. to XIII. contain the final values obtained from observations taken during the last three months. This represents all of the data taken during this time with the exception of two drops, the first being discarded because it was lost when only a few observations were taken; the second being discarded because the seeing was poor and consequently inaccurate timing resulted. At the top of each table are given the constants $a$ and $k$ obtained from the velocity $V$. In the different columns are given the average values obtained from Brownian movements.

Particular attention is called to Drop 12, the last drop which was observed. As is seen from the table we obtained 5,950 observations on the time of fall through ten different fixed distances. In order to bring out more clearly the fact that $z$ is proportional to $\sqrt{t_g}$ the value of $z/\sqrt{t_g}$ is recorded in the 7th column. Although there is some little

[1] $u = \frac{V(t_g - t)}{\sqrt{t}}$ equation 28 of the art. already cited.

TABLE I.

| Time. | 1 Division. | 2 Division. | 3 Division. | Time. | 1 Division. | 2 Division. | 3 Division. |
|---|---|---|---|---|---|---|---|
| | | | | 36.10 | 2.30 | | 8.90 |
| 0.20 | | | | 38.40 | 2.80 | 4.90 | |
| 3.20 | 3.00 | 5.40 | 7.70 | 41.20 | 2.10 | | |
| 5.60 | 2.40 | | | 43.30 | 3.90 | 6.80 | |
| 7.90 | 2.30 | 5.70 | 9.7 | 47.20 | 2.90 | 5.60 | 9.70 |
| 11.20 | 3.40 | | | 50.10 | 2.10 | | |
| 14.80 | 3.50 | 6.30 | | 52.20 | 3.20 | | |
| 17.60 | 2.80 | | 8.40 | 55.70 | 3.30 | 6.20 | |
| 20.70 | 3.10 | 5.60 | | 59.00 | 2.90 | | 10.00 |
| 23.20 | 2.50 | | | 61.90 | 3.00 | | |
| 26.00 | 2.80 | 5.50 | 9.60 | 64.90 | 2.90 | 7.00 | |
| 28.70 | 2.70 | | | 67.80 | 4.10 | 6.70 | 9.20 |
| 32.30 | 3.60 | 6.90 | | 71.90 | 3.60 | 6.70 | |
| 35.60 | 3.30 | | 9.70 | 75.50 | 3.10 | | |
| 38.70 | 3.60 | 7.40 | | 78.60 | 2.50 | 5.30 | 9.10 |
| 43.00 | 4.30 | | | 81.10 | 2.80 | | |
| 45.30 | 2.30 | 4.80 | 7.50 | 83.90 | 3.00 | 6.30 | |
| 47.80 | 2.50 | | | 86.90 | 3.30 | | 6.70 |
| 49.90 | 2.10 | 5.00 | | 90.20 | 2.70 | 4.70 | |
| 52.80 | 2.90 | | 8.20 | 92.90 | 2.00 | | |
| 56.10 | 3.30 | 5.90 | | 94.90 | 2.00 | 4.30 | 8.60 |
| 58.40 | 2.30 | | | 96.90 | 2.30 | | |
| 61.00 | 2.60 | 6.40 | 10.20 | 99.20 | 3.80 | 6.30 | |
| 64.80 | 3.80 | | | 103.00 | 2.50 | | 9.00 |
| 67.70 | 2.90 | 6.40 | | 105.50 | 3.20 | 6.30 | |
| 71.20 | 3.50 | | 10.90 | 108.70 | 3.10 | | |
| | | | | 111.80 | 2.70 | 4.90 | 8.10 |
| 20 | 3.20 | | | 114.50 | 2.20 | | |
| 3.40 | 3.50 | 6.70 | | 116.70 | 2.60 | 5.90 | |
| 6.90 | 3.50 | | 9.50 | 119.30 | 3.30 | | 9.00 |
| 10.40 | 3.90 | 7.40 | | 122.60 | 2.90 | 6.20 | |
| 14.30 | 3.70 | | | 125.50 | 3.30 | | |
| 18.00 | 2.20 | 5.90 | 9.50 | 128.80 | 2.80 | 4.70 | 6.90 |
| 20.20 | 3.60 | | | 131.60 | | | |
| 23.80 | 2.60 | 6.20 | | | | | |
| 26.40 | 3.30 | | 8.00 | 70 | 1.90 | 5.00 | |
| 29.70 | 3.60 | 6.90 | | 2.60 | 2.90 | | |
| 33.30 | 2.30 | | | 5.50 | 2.10 | 6.00 | 8.20 |
| 35.60 | 2.80 | 5.10 | 9.80 | 7.60 | 2.70 | | |
| 38.40 | 2.60 | | | 10.30 | 3.30 | 5.00 | |
| 41.00 | 2.60 | 5.20 | | 13.60 | 2.20 | | 9.20 |
| 43.60 | 3.90 | | 10.20 | 15.80 | 2.80 | 6.40 | |
| 47.50 | 3.30 | 7.20 | | 18.60 | 3.00 | | |
| 50.80 | 2.60 | | | 21.60 | 3.40 | 6.70 | 9.00 |
| 53.40 | 3.20 | 5.80 | 8.70 | 25.00 | 2.80 | | |
| 56.60 | 3.10 | | | 27.80 | 3.90 | | |
| 59.70 | 3.90 | 7.00 | | 31.70 | 3.30 | 6.30 | 10.30 |
| 63.60 | 3.10 | | 8.90 | 35.00 | 3.00 | | |

Table I.—*Continued.*

| Time. | 1 Division. | 2 Division. | 3 Division. | Time. | 1 Division. | 2 Division. | 3 Division. |
|---|---|---|---|---|---|---|---|
| 66.70 | 3.50 | 5.60 | | 38.00 | 3.80 | 6.10 | |
| 70.20 | 2.10 | | | 41.80 | 2.30 | 6.90 | 9.60 |
| 72.30 | | 6.00 | | 44.10 | 4.20 | | |
| | | | 9.90 | 48.30 | 2.70 | | |
| 30 | 3.90 | | | 51.00 | 2.80 | 6.90 | |
| 4.20 | 2.40 | | | 53.80 | 4.10 | | 10.80 |
| 6.60 | 2.60 | 5.00 | 8.40 | 57.90 | 4.50 | 71.0 | |
| 9.20 | 2.80 | 6.00 | | 62.40 | 2.60 | | |
| 12.00 | 3.20 | | | 65.00 | 3.70 | 6.40 | 7.60 |
| 15.20 | 3.90 | 7.00 | 8.60 | 68.70 | 2.70 | | |
| 19.10 | 3.60 | | | 71.40 | 2.20 | 4.90 | |
| 22.20 | 2.20 | 5.30 | | 73.60 | 2.70 | | |
| 24.40 | 3.10 | | 7.20 | 76.30 | | | |
| 27.50 | 2.80 | 6.20 | | | | | |
| 30.30 | 3.40 | | | | | | |
| 33.70 | 2.40 | 4.70 | | | | | |

variation from a constant it seems quite remarkable to think that an apparently haphazard set of numbers unite on the average according to such a definite law.

Table II.

*Drop No. 1.*

Pressure = 24.8 cm. Temperature = 18.8° C.

$V$ = .00296 cm./sec. $a = 4.04 \times 10^{-5}$ cm. $k$ = .611.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00916 | 3.92 | 3.587 | 2.661 | .463 | 7.58 | 60.9 | 707 |
| 2 × .00916 | 6.694 | 6.811 | 5.515 | .648 | 10.81 | 61.9 | 355 |
| 3 × .00916 | 9.276 | 10.052 | 8.444 | .804 | 13.03 | 60.2 | 239 |

Weighted mean $N = 61.0 \times 10^{22}$.

Table III.

*Drop. No. 2.*

Pressure = 24.8 cm. Temperature = 18.8° C.

$V$ = .002770 cm./sec. $a = 3.93 \times 10^{-5}$ cm. $k$ = .604.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00916 | 3.306 | 3.945 | 2.893 | .526 | 7.16 | 65.0 | 312 |
| 2 × .00916 | 6.612 | 7.589 | 5.901 | .844 | 9.00 | 51.6 | 150 |
| 3 × .00916 | 10.926 | 9.057 | 9.057 | .934 | 12.02 | 61.2 | 95 |

Weighted mean $N = 60.7 \times 10^{22}$.

### TABLE IV.

*Drop. No. 3.*

Pressure = 31.7 cm. Temperature = 19.5° C.

$V$ = .001465 cm./sec. $a = 2.79 \times 10^{-5}$ cm. $k$ = .577.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 6.27 | 8.11 | 4.85 | 1.63 | 4.44 | 64.7 | 208 |
| 2 × .00918 | 12.54 | 14.80 | 10.48 | 2.16 | 5.62 | 72.1 | 102 |
| 3 × .00918 | 18.81 | 22.44 | 16.38 | 3.03 | 7.08 | 54.8 | 70 |

Weighted mean $N = 64.8 \times 10^{22}$.

### TABLE V.

*Drop. No. 4.*

Pressure = 12.25 cm. Temperature = 20.0° C.

$V$ = .00303 cm./sec. $a = 3.12 \times 10^{-5}$ cm. $k$ = .348.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 3.033 | 3.834 | 2.401 | .716 | 4.87 | 55.8 | 338 |
| 2 × .00918 | 6.06 | 7.108 | 5.237 | .936 | 7.47 | 65.8 | 168 |
| 3 × .00918 | 9.09 | 10.41 | 7.97 | 1.220 | 8.47 | 56.5 | 111 |

Weighted mean $N = 58.8 \times 10^{22}$.

### TABLE VI.

*Drop. No. 5.*

Pressure = 13.52 cm. Temperature = 19.3° C.

$V$ = .00393 cm./sec. $a = 3.99 \times 10$ cm. $k$ = .442.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00914 | 2.325 | 2.735 | 1.997 | .369 | 7.19 | 58.2 | 1010 |
| 2 × .00914 | 4.650 | 5.172 | 4.146 | .516 | 10.22 | 58.9 | 508 |
| 3 × .00914 | 6.975 | 7.57 | 6.32 | .635 | 12.65 | 60.2 | 336 |

Weighted mean $N = 58.8 \times 10^{22}$.

### TABLE VII.

*Drop No. 6.*

Pressure = 15.65 cm. Temperature = 18.6° C.

$V$ = .00321 cm./sec. $a = 3.69 \times 10^{-5}$ cm. $k$ = .460.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00914 | 2.844 | 3.410 | 3.410 | .500 | 6.51 | 60.7 | 968 |
| 2 × .00914 | 5.688 | 6.404 | 4.962 | .721 | 8.97 | 57.8 | 481 |
| 3 × .00914 | 8.532 | 9.450 | 7.700 | .875 | 11.05 | 58.4 | 319 |

Weighted mean $N = 59.0 \times 10^{22}$.

## Table VIII.

*Drop No. 7.*

Pressure = 19.4 cm. Temperature = 19.1° C.

$V$ = .00332 cm./sec. $a = 4.11 \times 10^{-5}$ cm. $k$ = .551.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\bar{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 2.77 | 3.19 | 2.40 | .395 | 8.00 | 65.8 | 561 |
| 2 × .00918 | 5.54 | 6.15 | 5.00 | .575 | 10.88 | 60.6 | 283 |
| 3 × .00918 | 8.31 | 9.14 | 7.67 | .735 | 12.93 | 57.2 | 188 |
| 4 × .00918 | 11.08 | 11.80 | 10.36 | .780 | 16.05 | 66.1 | 142 |
| 5 × .00918 | 13.85 | 14.87 | 13.00 | .935 | 16.75 | 57.7 | 109 |
| 6 × .00918 | 16.62 | 17.77 | 15.73 | 1.020 | 18.40 | 58.0 | 94 |

Weighted mean $N = 62.2 \times 10^{22}$.

## Table IX.

*Drop No. 8.*

Pressure = 12.25 cm. Temperature = 20.0° C.

$V$ = .00313 cm./sec. $a = 3.21 \times 10^{-5}$ cm. $k$ = .357

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\bar{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 2.932 | 3.683 | 2.384 | .650 | 5.24 | 59.5 | 696 |
| 2 × .00918 | 5.864 | 6.792 | 5.030 | .881 | 7.66 | 63.4 | 364 |
| 3 × .00918 | 8.796 | 9.860 | 7.774 | 1.043 | 9.68 | 67.5 | 229 |
| 4 × .00918 | 11.728 | 13.04 | 10.39 | 1.326 | 10.04 | 54.5 | 180 |
| 5 × .00918 | 14.660 | 16.129 | 12.972 | 1.578 | 10.54 | 49.1 | 136 |
| 6 × .00918 | 17.592 | 19.15 | 16.05 | 1.55 | 12.98 | 60.8 | 108 |

Weighted mean $N = 60.1 \times 10^{22}$.

## Table X.

*Drop No. 9.*

Pressure = 19.5 cm. Temperature = 20.2° C.

$V$ = .002420 cm./sec. $a = 3.31 \times 10^{-5}$ cm. $k$ = .482.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\bar{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 3.795 | 4.708 | 3.109 | .800 | 5.44 | 59.6 | 733 |
| 2 × .00918 | 7.590 | 8.700 | 6.500 | 1.100 | 7.84 | 61.8 | 378 |
| 3 × .00918 | 11.385 | 12.637 | 10.093 | 1.272 | 10.15 | 68.9 | 243 |

Weighted mean $N = 61.8 \times 10^{22}$.

### TABLE XI.

*Drop No. 10.*

Pressure = 13.34 cm. Temperature = 20.5° C.
$V$ = .003302 cm./sec. $a$ = 3.39 cm. $k$ = .400.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 2.78 | 3.347 | 2.282 | .532 | 5.97 | 58.8 | 586 |
| 2 × .00918 | 5.56 | 6.308 | 4.770 | .769 | 8.20 | 55.4 | 301 |
| 3 × .00918 | 8.34 | 9.303 | 7.507 | .898 | 10.60 | 61.7 | 192 |

Weighted mean $N = 58.4 \times 10^{22}$.

### TABLE XII.

*Drop No. 11.*

Pressure Temperature
$V$ = .003207 cm./sec. $a = 3.41 \times 10^{-5}$ cm. $k$ = .395.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00918 | 2.862 | 3.496 | 2.336 | .580 | 5.67 | 58.2 | 503 |
| 2 × .00918 | 5.724 | 6.604 | 4.962 | .821 | 7.95 | 57.0 | 245 |
| 3 × .00918 | 8.586 | 9.596 | 7.614 | .991 | 9.83 | 58.2 | 159 |

Weighted mean $N = 57.9 \times 10^{22}$.

### TABLE XIII.

*Drop No. 12.*

Pressure = 24.29 cm. Temperature = 21.1° C.
$V$ = .00251 cm./sec. $a = 3.63 \times 10^{-5}$ cm. $k$ = .572.

| $b$ | $t_g$ | $\overset{+}{t}_a$ | $\overset{-}{t}_a$ | $\tau$ | $z$ | $\frac{z}{\sqrt{t_g}}$ | No. Obs. |
|---|---|---|---|---|---|---|---|
| 1 × .00922 | 3.672 | 4.365 | 3.078 | .6435 | 6.50 | 3.39 | 2056 |
| 2 × .00922 | 7.344 | 8.349 | 6.480 | .9345 | 8.93 | 3.31 | 1028 |
| 3 × .00922 | 11.016 | 12.237 | 9.926 | 1.156 | 10.83 | 3.27 | 681 |
| 4 × .00922 | 14.688 | 15.999 | 13.333 | 1.333 | 12.48 | 3.24 | 514 |
| 5 × .00922 | 18.360 | 19.916 | 16.915 | 1.501 | 13.85 | 3.23 | 393 |
| 6 × .00922 | 22.032 | 23.423 | 20.387 | 1.513 | 16.44 | 3.50 | 342 |
| 7 × .00922 | 25.704 | 27.630 | 23.918 | 1.856 | 15.67 | 3.10 | 268 |
| 8 × .00922 | 29.376 | 30.981 | 27.309 | 1.831 | 18.12 | 3.34 | 252 |
| 9 × .00922 | 33.048 | 35.143 | 30.941 | 2.10 | 17.85 | 3.11 | 226 |
| 10 × .00922 | 36.720 | 38.92 | 34.73 | 2.09 | 19.86 | 3.28 | 190 |

Weighted mean $N = 60.0 \times 10^{22}$. Total No. Obs. = 5,950.

Table XIV. contains a summary of the 12 drops with the final weighted mean values of $N$ which were obtained by assigning weights according to the number of observations that were taken. They are arranged in ascending values of $l/a$ in order to show that $N$ has no tendency to gradually increase or decrease, although $k$ changes from .357 to .611. This is a verification that Millikan's expression for $k$ is correct, at least for this region and for air.

TABLE XIV.

| Drop No. | $a \times 10^5$ | $z \times 10^5$ | $l/a$ | $k$ | $N \div 10^{22}$ | No. Obs. |
|---|---|---|---|---|---|---|
| 4 | 3.12 | 5.75 | 1.840 | .348 | 58.8 | 617 |
| 8 | 3.21 | 5.75 | 1.790 | .357 | 60.1 | 1,713 |
| 10 | 3.39 | 5.30 | 1.562 | .400 | 58.4 | 1,079 |
| 11 | 3.41 | 5.30 | 1.553 | .395 | 57.9 | 907 |
| 5 | 3.99 | 5.19 | 1.300 | .442 | 58.8 | 1,854 |
| 6 | 3.69 | 4.48 | 1.215 | .460 | 59.5 | 1,768 |
| 9 | 3.31 | 3.61 | .917 | .482 | 61.8 | 1,354 |
| 7 | 4.11 | 3.62 | .880 | .551 | 62.2 | 1,377 |
| 12 | 3.64 | 2.91 | .800 | .572 | 60.0 | 5,950 |
| 3 | 2.79 | 2.22 | .795 | .577 | 63.8 | 380 |
| 2 | 3.93 | 2.83 | .710 | .604 | 59.3 | 557 |
| 1 | 4.04 | 2.83 | .700 | .611 | 61.0 | 1,281 |

Weighted mean = $60.0 \times 10^{22}$. Total No. Obs. = 18,837.

As indicated heretofore all of the above computations have been made on the assumption that $R = 82.9 \times 10^6$ and $\mu_{23} = .0001826$. Professor Millikan calls my attention to the fact that the correct value of $R$ is $83.15 \times 10^6$ and that the most probable value of $\mu_{23}$ is .0001824.[1] The introduction of these changes increases the above value of $N$ by .41 per cent. The probable error computed on the ordinary way from column 6 of Table XIV. is $.3 \times 10^{22}$ or .5 per cent. This represents the "probable error" in the $z^2$ term of equation 10. The *uncertainty* in this term, the mean being obtained from but 12 numbers, is of course considerably larger. Professor Millikan estimates the uncertainty in his determination of the $k$ term of (10) [more accurately the $ak$ term, since this is the quantity actually given by his experiments] for these values of $l/a$, at about 1 per cent. So that we may consider that the uncertainty in the above determination of $N$ by the Brownian movement method does not exceed 2 per cent. The value actually obtained, viz., $60.3 \times 10^{22}$ differs by but .5 per cent from the value obtained by Professor Millikan, namely, $60.62 \times 10^{22}$.

The results of this investigation show then that the number of molecules in a gram molecule of air is

$$N = 60.3 \times 10^{22} \pm 1.2 \times 10^{22}.$$

In conclusion I wish to express my indebtedness to Mr. Carl F. Eyring, who has assisted me in taking a large portion of the observations and in computing the average values.

PHYSICAL LABORATORY,
BRIGHAM YOUNG UNIVERSITY,
July 17, 1914.

[1] Ann. d. Phys., 41, p. 759, 1913.

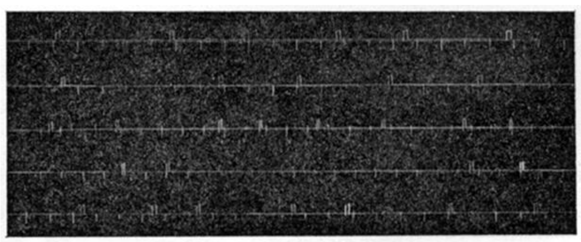

Fig. 1.