DETERMINATION OF e/m FROM MEASUREMENTS OF THERMIONIC CURRENTS.[1]

By Saul Dushman.

In a recent paper on "The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum,"[2] Dr. I. Langmuir has shown that the thermionic current from a heated metallic surface in high vacuum is limited by (*a*) temperature, (*b*) space charge.

With a given potential difference between the electrodes, the current from the heated cathode increases with the temperature in accordance with the equation of Richardson,

$$i = a\sqrt{T}\epsilon^{-\frac{b}{T}}. \tag{1}$$

However, above a certain temperature, this current becomes constant; further increase in temperature does not cause any corresponding increase in thermionic current. The temperature at which this limitation occurs increases with increase in anode potential. It was shown in the above paper that this effect is due to the existence of a space charge produced by the emitted electrons, and it was furthermore deduced that at a fixed cathode temperature, the electron current ought to increase with the three-halves power of the voltage (until the saturation current as defined by the Richardson equation is attained), that is, for electrodes of any shape, the space charge current

$$i = k \cdot V^{3/2}, \tag{2}$$

where V denotes the potential difference, and k is a constant depending on the shape of the electrodes, their area and the distance between them. For the case of the heated filament in a concentric cylindrical anode (infinite length),

$$i = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}} \cdot \frac{V^{3/2}}{r}, \tag{3}$$

where i is the thermionic current emitted per unit length of filament, and r the radius of the anode.

While in the above investigation, a number of results had been ob-

[1] An abstract of this paper was read at the Chicago Meeting, November 29, 1913. See Phys. Rev., *3*, 65, 1914.

[2] Phys. Rev., *2*, 450, 1913.

tained which were in accord with equation (2), it was considered, in view of the skepticism exhibited by many investigators with regard to the actual existence of a pure electron emission that a separate experimental test of the validity of equation (3) would be of interest, especially as it leads to a new and probably very accurate method for the determination of the ratio e/m.

Description of Apparatus.

In order to be able to apply equation (3) to the case of a finite length of anode, the latter was inserted between two other anodes on the guard-ring principle. A tungsten filament 15 cm. long and 0.25 mm. diameter was supported by means of a glass tube about 7 cm. in diameter and 36 cm. long (see Fig. 1). The current required to heat the filament was carried by a separate wire at the top, thus avoiding any weakening in the tension exerted by the spring, owing to heating. The anode consisted of three molybdenum cylinders 2.54 cm. in diameter, made from sheet metal about .37 mm. thick. The two edges of the sheet were brought together to form a cylinder and heavy molybdenum strap riveted to the outside of the cylinders, held these edges close together. At the same time, these straps were drilled and tapped to fit 2.5 mm. molybdenum rods. The latter were held rigidly in the side arms by means of molybdenum springs. The outside anodes were each 2.54 cm. long, while the center one was 7.62 cm. long. The connections through the glass were made by means of platinum leads.

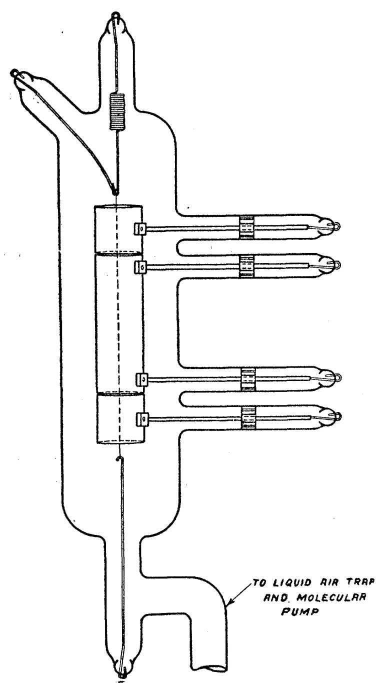

Fig. 1.

The tube shown in Fig. 1 was connected through a large-bore liquid air trap to a Gaede molecular pump. The latter was connected in series with a Gaede "box" oil pump and the rough vacuum line (pressure of about 1 cm.).

A McLeod-gage was inserted between the molecular pump and the oil pump, as well as a tap connected to the atmosphere through a phosphorus pentoxide tube for letting in dry air into the vacuum tube before stopping the molecular pump.

The tube could be exhausted at a temperature of 350° C. by means of an electrically heated oven which was lowered over it.

Vacuum Obtainable.

As stated by Dr. Langmuir in his paper, the most essential conditions for obtaining pure thermionic currents are an extremely high vacuum and absolutely gas-free electrodes.

A special investigation, the results of which will be published shortly in a separate paper, has shown that in order to obtain a very high vacuum even with the molecular pump, it is necessary to insert a liquid air trap between the tube to be exhausted and the pump, and also to heat the tube to over 300° C. for a period of at least one hour. The water-vapor absorbed in the glass walls is thus removed much more quickly and the liquid air trap prevents the diffusion backwards of stopcock grease vapor and other condensible gases. After this preliminary heating, the vacuum is found to be improved considerably. With a pressure of about 0.01 mm. on the rough side of the pump, it is possible in this manner to obtain a vacuum in the tube, which is certainly less than 2×10^{-7} mm.

Preparation of Anodes.

However, under the action of the electrons emitted from the heated cathode, there is a continuous liberation of gas from the anodes, which not only leads to an actual decrease in the thermionic currents themselves, but also prevents the space charge limitation from occurring, owing to the production of positive ionization. It is, therefore, very important to free the anodes thoroughly from occluded gases and volatile oxides which may dissociate gradually when the anodes are heated. This may be accomplished either by heating the anodes to a white heat in a tungsten vacuum furnace,[1] or by powerful electron bombardment in the exhausted tube itself.

The latter method was adopted in this case. A potential varying from 1,000 to 5,000 volts was applied to the electrodes by means of a transformer, the filament being maintained at such a temperature that anywhere from 50 to 200 milliamperes thermionic current was obtained. Under the influence of the electron bombardment at high voltages, the temperature of the anodes was raised to 1000° C. or more and the gases contained in the metal were gradually eliminated. This progressive clean-up of the gases was accompanied by a disappearance of the blue glow, which always indicates the presence of positive ions and by a corresponding increase in the magnitude of the thermionic currents actually

[1] Coolidge,

emitted at any given temperature of the filament. Whenever the blue glow ceased, the voltage was raised, thus heating the anodes to a still higher temperature and liberating more gas from them. By this procedure, the anodes were finally raised nearly to a white heat, and made gas-free.

METHOD OF DETERMINING THE TEMPERATURE OF THE FILAMENT.

The temperatures of the filament were determined from the relation,

$$T = \frac{11{,}230}{7.029 - \log_{10} H},[1]$$

where H is the intrinsic brilliancy of the filament in international candles per square cm. of projected area, and T denotes the temperatures in degrees Kelvin.

The variation in H with the current carried by the filament was determined by photometric measurements carried out on a special lamp, made up from the same coil of tungsten wire as that used in making up the filament of the discharge tube described above. From these data, a curve was obtained giving the relation between the current carried by the filament and the resulting temperature.

EXPERIMENTS.

In Table I. is given a typical record of observations made on the magnitude of the thermionic currents during the process of evacuation. It will be observed that at the very beginning, when the vacuum was comparatively bad, the electron emission was very low. Replacing the ice and salt mixture around the trap by liquid air caused only a small increase in the thermionic currents obtained. But after the anodes were heated to a high temperature and made fairly gas-free, the increase in the electron emission was considerable. Thus, at 2130° K. the currents observed increased from 0.08 to 2.30 milliamperes, while at 2210° the increase was from 0.1 to 18 milliamperes, and similarly for other temperatures.

These observations are most easily interpreted in terms of Dr. Langmuir's surface film theory. According to this theory, "the effect of gases in changing the electron emission, is due to the formation of unstable compounds on the surface of the wire. In the cases observed (so far), the presence of the compound decreases the electron emission. The extent to which the surface is covered by the compound depends on the *rate of formation* of the compound and on its *rate of removal* from the

[1] I. Langmuir, PHYS. REV., 2, 452, 1913.

TABLE I.

Effect of Gradual Clean-up of Gas in Electrodes.

T = temperature of filament; V = voltage between electrodes; i_1 = thermionic current to center anode only; i_3 = current to all three anodes. Currents measured in milliamperes.

Time.	T	V	i_3	Remarks.
8:00 A.M.				Tube heated in oven to 330° C. for 1 hour.
9:00	2000°	20	0.065	
	2030	20	.14	
	2080	20	.04	*Ice and salt mixture on trap.*
	2130	20	.08	Pressure on rough pump side at about 50 μ. Estimated pressure in tube about 0.001 μ.
	2175	20	.08	Current gradually decreased to 0.04
9:15	2175	50	.04	
	2210	20	.10	
	2210	50	.10	
	2250	50	.22	
	2300	50	.44	
		20	.40	
	2250	50	.26	
	2300	20	.48	
9:30	2315	20	.72	
		50	1.00	Had to stop pump. Pressure on rough pump side still high (50 microns). Cleaned out pump. Put on *liquid air* on trap. Baked out tube in oven for 1 hour.
		95	1.00	
12:00	2180	21	.07	Pressure on rough pump side, 25 μ. Estimated pressure in tube about 0.0005 μ.
	2230	21	.25	
	2330	21	1.1	
12:30 P.M.	2230	21	.38	
	2190	21	.30	
	2230	21	.75	
	2330	21	2.2	Note the gradual increase in thermionic current.
12:45	2250	1,600	1.6	Applied high voltage to bombard anodes.
	2250	2,000	1.2	
	2290	2,000	2.6	Current decreased rapidly to 1.6.
	2330	1,500	8	Current decreased rapidly to 3.
1:00	2330	3,000	40	Current decreased rapidly to 15.
	2365	3,000	90	Current decreased rapidly to 30.
2:00	2400	3,500	130	Current decreased rapidly to 30.
	2360	3,000	70	Current decreased rapidly to 40.
3:00	2400	4,000	140	Current does not decrease so much. No blue glow apparent. Anodes red hot. Changed to low voltage battery.
4:00	2230°	21	12	
	2360	21	14	
	2170	21	9	
	2130	21	3.5	
	2230	21	12	
	2250	28	19	

Time.	T	V	i_3	Remarks.
	2290	28.5	21	
	2160	28	7	
	2160	50	7	Temperature saturation.
	2210	50	18	
	2240	50	27	
	2290	50	48	Space charge effect.
	2330	50.6	49	
			i_1	Measured current to centre anode only.
4:45	2240	50	15.5	See Curve 1, Fig. 2.
	2260	50.2	20	
	2290	50.2	27	
	2310	50.4	28.5	
	2320	50.6	29.0	
	2260	50.2	21	
	2310	50.4	28.5	
		43.4	24.5	
		73.4	32.5	Temperature saturation.
		94.9	32.0	
	2330	95.0	40	
	2345	95.2	55	
	2360	95.3	65	
	2440	96.5	75	
	2460	97.0	77	See Curve 1, Fig. 2.
6:00	2480	97.0	77	Let in dry air.
6:30				Re-exhausted. Fresh liquid air on trap.
7:00	2250	50	10	
	2300	50.4	20	
	2340	50.6	31	
	2390	50.6	34.5	
	2270	50.6	8	
	2270	73	8	
	2310	96	15	
	2340	96	32	
	2390	96	53	
7:30	2440	96	110	See Curve 3, Fig. 2.
			i_3	Applied high voltage to bombard anodes again.
	2340	4,800	66	Current decreased to 40.
	2390	5,000	110	Current decreased to 75.
	2410	5,000	125	Current decreased to 115.
8:00	2450°	5,000	250	Anodes at bright red heat.
	2450	5,000	300	
	2320	5,000	110	
	2380	5,000	225	
	2400	5,000	250	Current does not decrease so much.
			i_1	Measured current to centre anode only at low voltages.
9:00	2440	88	71	

Time.	T	V	i_3	Remarks.
	2400	87	70	See Curve 4, Fig. 2.
	2360	87	68	
	2344	87	60	
	2320	87	50	
	2300	87	38	
	2275	87	28	
	2238	87	17.5	
	2216	87	12.0	
9:30	2350	87	65	
	2380	87	69	
	2450	88	70	Space charge current calculated = 73.
	2200	106	9.5	
	2240	106	20.5	
	2320	106	49	
	2360	107	72	
	2370	107	79	
	2420	108	94	
	2440	108	94	
	2460	108	95	Space charge current calculated = 99.
	2540	108	95	See Curve 4, Fig. 2.

surface." Now even low velocity electrons striking the anode liberate gas from it. If the pump is unable to remove the gas as fast as liberated, the rate of formation of the film on the surface of the cathode exceeds its rate of destruction and consequently the electron emission decreases. This was exhibited strikingly in the actual experiments during the treatment with high voltages. The volume of gas liberated during the interval of only a few minutes was so great that the electron emission would decrease very rapidly.[1] It was only towards the end of the high voltage treatment that the thermionic currents remained fairly steady, thus indicating that little or no gas was being liberated.

The subsequent observations on the thermionic currents at lower voltages show that the vacuum was now considerably better than before the high voltage treatment. Otherwise no space charge effects could have been observed; for as long as gases are given off by the electrodes or the glass walls, there is no indication of space charge effects. In Curve 1, Fig. 2, are plotted the observations obtained between 4:45 and 6:00 P.M. The space charge currents calculated for 50.6 and 97 volts are 32 and 84.5 milliamperes respectively; the values actually obtained were 29 and 77.

The effect of bad vacuum conditions on the thermionic currents is

[1] The high voltage was applied for only a few minutes at a time, until blue glow occurred; the filament was then allowed to cool for a couple of minutes to permit the pump to remove the liberated gases.

shown by the observations made between 7:00 and 7:30 P.M., and plotted in Curves 2 and 3, Fig. 2. After letting in dry air and re-exhausting, it was evidently not possible to obtain a good enough vacuum until the anodes were heated again by high voltage bombardment.

It will be seen from these curves that the effect of the presence of gas is to remove the limitation set by space charge effect, while decreasing the actual electron emission. Curve 3 is interesting because it shows the complete absence of any space charge effects. Evidently the amount of

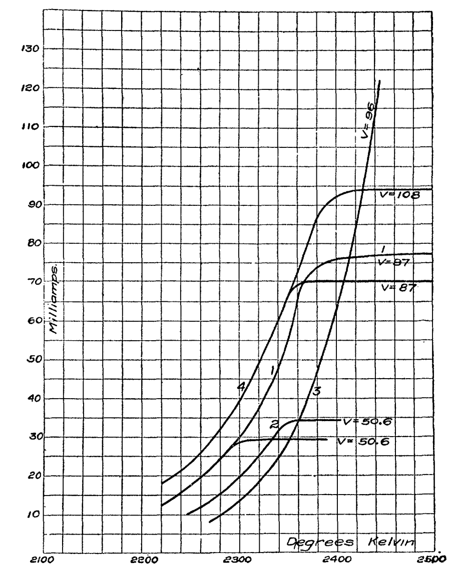

Fig. 2.

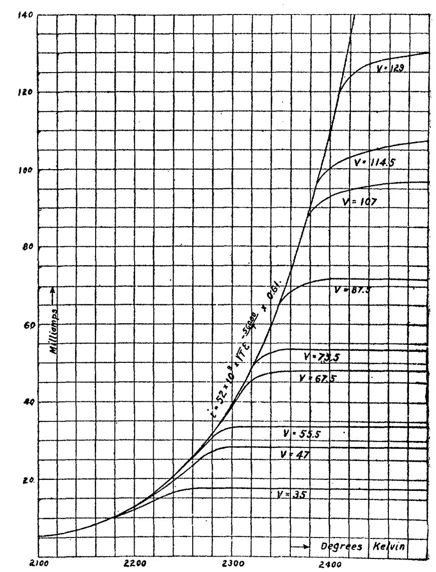

Fig. 3.[1]

gas present in the tube had kept on increasing during these measurements, so that the space charge effect disappeared completely.

After bombarding the anodes again, the results plotted in Curve 4, Fig. 2, were obtained. Owing to the improved vacuum conditions, the electron emission at any temperature was increased, while the space charge currents observed were not very different from those calculated in accordance with equation (3), as shown below.

Table II gives the actually observed space charge currents at different voltages, together with a number of the observations (made under the same conditions) on the total electron emission at different temperatures. The results are plotted in Fig. 3. It is seen that heating the anodes with high voltage again led to no further changes in the values of the thermionic currents observed.

The values under i_1 (calc.) were calculated by means of equation (3)

[1] The ordinates give the current in milliamperes *not* microamperes.

Denoting the constant, $2\sqrt{2/9}\sqrt{e/m}$ by k_0, the values of this constant for different values of e/m are as follows:

$10^{-7} \times e/m$.	$10^3 \times k$.
1.775	14.71
1.766	14.68
1.755	14.62

The last seems to be the most accurate value of e/m, according to the

TABLE II.

Space Charge Currents Obtained at Different Voltages under Best Vacuum Conditions.

Tube exhausted as before. Anodes bombarded with electrons of high velocity by using high voltage, Measured current to center anode only. Pressure on rough pump side, 18 microns. Estimated pressures in tube, less than 0.0004 micron.

T	V	i_1	i_1 (calc).
2125°	47.0	3	
2185	47.0	9	
2236	47.0	18	
2280	47.0	27	
2320	47.0	28	28.5
2340	35	17	17
2300	35	17.3	
2240	35	16.2	
2208	35	12	
2180	35	8.5	
2240	68	19.5	
2262	68	27	
2318	68	45	
2340	68	47	
2400	68	48	49
2340	55.5	34	36
2340	76.0	54	
2400	75.2	54	57.5
2400	88.5	72	
2450	87.5	71	72.5
2480	89	72	74
2480	107.0	96.5	98
2430	108.5	97.7	100
2440	107.5	95.0	
2396	107.5	92.0	
2300	111	49	
2390	114.2	99	
2440	114.5	104.8	
2476	114.8	106.4	
2500	115	107.3	
2522	115	108.0	109
2440	128.5	126	
2480	128.8	128	
2500	129	130	130
2520	129	130	

Treated anodes with high voltage again, and measured thermionic currents to centre, anode only. Pressure on rough pump side down to 10 microns; estimated pressure in tube less than 0.0002 micron.

T	V	i_1	i_1 calc.
2350°	50.8	31.8	32
2180	49	15.5	
2180	70.5	15.0	
2240	71.0	37.5	
2315	71.5	52.2	54
2400	93.5	81.0	80
2490	114.0	111.0	109
2500	136.0	144.0	140

observations of Bucherer and others and is in good accord with the value calculated from the Zeeman effect. It was, therefore, used to calculate the space charge currents for different voltages for the length of filament, corresponding to that of the center anode (7.62 cm.). The results are given in Fig. 4. From this curve, the values given in Table II., under i_1

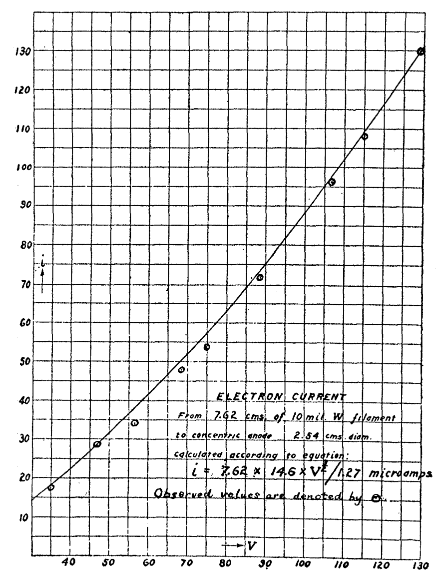

Fig. 4.

(calc.), were taken. The actually observed thermionic currents are also indicated in Fig. 4.

SOURCES OF ERROR.

No attempt was made in this investigation to attain great accuracy; the object being merely to show the validity of the space charge equations.

It may, however, be well to indicate briefly the sources of error involved in the above measurements.

The largest errors undoubtedly occurred in the measurement of V, the potential difference between the electrodes. The voltmeter used was of the ordinary Weston type. As direct current was used to heat the filament, a correction had to be made for the voltage drop through the filament. This drop differed, of course, with the temperature, and was determined by means of a separate set of measurements. To correct for the possibility that the center anode might not be situated symmetrically with respect to the voltage drop through the filament, the thermionic currents were observed for both directions of current through the filament, and the means of pairs of observations are recorded in Table II.

Other sources of error besides that involved in measuring V occurred also in the determination of i_1, the thermionic current to the center anode, and of r, the radius of the cylinder. In neither case was an accuracy greater than 1 to 2 per cent. obtained.

Besides these sources of error involved in the actual experimental manipulation there is another factor which must be taken into account when one attempts to use the method for an accurate determination of e/m.

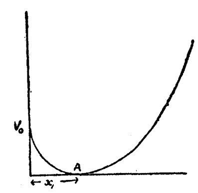

Fig. 5.

It will be observed that in deriving his space charge formula, Dr. Langmuir intentionally assumed that the electrons emitted from the hot cathode have no initial velocity. This is, however, manifestly not true. Thus the average velocity of electrons emitted from a hot cathode at 3000° K. is about 0.4 volt. In consequence of this initial velocity, the curve giving the potential distribution between anode and cathode will have a form such as that indicated in Fig. 5, where V_0 denotes the potential due to the initial velocity of the electrons, and V, the potential of the anode, both potentials being measured with respect to the minimum point A.

It is therefore evident that instead of using for V in equation (2) the potential difference as measured, we ought to add to the observed value of V another term V_0.[1] Since for small wires, the diameter of the wire does not affect the value of i calculated according to equation (3) we may consider the small distance x_1 which the electrons have to travel before reaching the minimum point as merely an increase in the diameter of the wire and we may therefore neglect it.

[1] It must also be observed that in all the calculations we have assumed an average velocity for the electrons, whereas in the more rigorous calculation Maxwell's distribution law ought to be taken into account.

A more accurate form of equation (3) is therefore:

$$i = \frac{2\sqrt{2}}{6}\sqrt{\frac{e}{m}}\frac{(V_{\text{obs}} + V_0)^{3/2}}{r}.^{2}$$

As in all the experiments recorded above, V_0 was less than 1 per cent. of the observed potential difference it was neglected.

There is no doubt, however, that by carrying out these measurements under more exact conditions the method described in this paper could be used to determine e/m with a very high degree of accuracy.

Effect of Gases.

As has been pointed out by Dr. Langmuir the effect of gases is two-fold. Firstly, it tends to eliminate space charge effects owing to the presence of positive ionization; secondly, the actual thermionic emission is decreased below the value obtained in a good vacuum.

It has already been pointed out that such effects were observed during the process of evacuation. In Table III. are given some data obtained at

Table III.

Effect of Nitrogen on Thermionic Currents.

P Rough Side of Pump.	T	V	i_R	i_S	i_0	Remarks.
200 microns	2300°	15	39	9	28	P in tube estimated at 0.01 micron.
		80	39	63	28	
1,000 microns	2325	100	51	86	30	
		80		63	25	
		60		41	18	
		40		22	18	
	2400	79	109	61	85	
		58		39	70	
		38		21	57	
		15		9	20	
	2450	22.5	180	9	75	
		30		14	68	
		52.5		32	75	Blue glow.
		72.5		54	87	Blue glow.
		93.5		79	105	Blue glow.

the end of an experiment with gas-free electrodes in presence of nitrogen. Nitrogen of the same degree of purity used in the manufacture of the nitrogen-filled tungsten lamps was allowed to enter the discharge tube

[2] The effect of taking into account the initial velocity of the electrons and Maxwell's distribution law has been discussed very fully by W. Schottky in a recent paper (Physik. Zeitsch. *15*, 624) who has also pointed out that in making use of the space charge equation for an accurate determination of e/m, failure to consider the above factors might involve errors of over 2 per cent. (Physik. Zeitsch. *15*, 528, footnote.)

through a narrow-bore stopcock connection sealed between the liquid air trap and the pump. The rate of flow of the gas was measured approximately by noting the pressure on the rough pump side. From this reading it was possible to obtain an estimate of the pressure in the tube.

Under i_r is given the value of the electron emission at the corresponding temperature as obtained from the curve $i = a\sqrt{T}\epsilon^{-\frac{b}{T}}$ given in Fig. 3. The values under i_s are the space charge currents calculated for the corresponding voltages by means of equation (3) and plotted in Fig. 4. The actually observed thermionic currents are given under i_0. It will be observed that *even with blue glow* the thermionic currents obtained were much lower than those calculated from the Richardson equation.

This decrease in thermionic currents was certainly not due to space charge effects, as is seen from the data at $T = 2325$. In some other experiments in which air was allowed to enter the tube, the decrease in thermionic current was so pronounced that even with 3,000 volts on the anode the sudden appearance of gas in the tube (whether originating in the anode or allowed to flow in through a side connection) immediately caused the electron emission to fall to almost zero.

General Conclusions.

The above experiments are thus in uniform agreement with the observations and explanation given by Dr. Langmuir in his paper on space charge effects, and in the light of all our subsequent experiments his conclusions on the nature of electron emission from heated metals appear to be well justified. The *perfect definiteness* of the results obtained which are independent of vacuum conditions after a sufficiently high vacuum has once been attained, the *reproducibility* of the observations even after allowing gas to enter the tube and then re-exhausting, and the quantitative agreement obtained not only in the above experiments but in all the experiments so far carried out in this laboratory point to the existence of a *pure electron emission per ipse*, which is not a secondary effect due to chemical reactions, as assumed by a number of other investigators, and which is a *function of the temperature only*.

In conclusion the writer wishes to express his indebtedness to Dr. I. Langmuir for kindly suggestions and interest during the progress of the above investigations.

Summary.

The space charge formula developed by Dr. I. Langmuir for the thermionic current from a heated filament to a coaxial cylindrical anode has been tested experimentally over a range of voltages from 35 to 140, for

the case of a tungsten filament in a concentric molybdenum cylinder.

The results obtained are in good agreement with this formula and indicate that the method ought to prove a very accurate one for the determination of e/m. The observations obtained on the effect of gases on the thermionic currents are also found to be in accord with Dr. Langmuir's surface film theory and justify the conclusion that there exists a pure electron emission from heated metals which is a function of the temperature only and is not a secondary effect due to presence of gases as assumed by a large number of investigators.

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