TABLES OF THE IONIZATION IN THE UPPER ATMOSPHERE*

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Abstract

The present paper continues the work on the ionization of the upper atmosphere due to the ultraviolet light of the sun, developed in a series of papers in this Journal, and takes into account as far as possible the entire meteorology of the upper atmosphere and the effects of recombination, temperature diffusion and the motions of the ions and electrons in the gravitational, electric and magnetic fields of the earth. Tables are presented of the ion and electron densities over the earth. The ionization at night is worked out more completely than was done previously. After sunset as a result of temperature diffusion and electric-magnetic drift, the ionized region separates into two banks, one with a maximum of ionization at about 110 km and one with a maximum at about 140 km. Quantitative agreement is found with the skip distances and other phenomena of wireless waves and with various facts of terrestrial magnetism.

I. Introduction

The ionization of the upper atmosphere of the earth attributable to the ultraviolet light of the sun was worked out in a series of papers,[1] and was shown to be in agreement with that inferred from various facts of terrestrial magnetism and of the propagation of radio waves. In calculating the ionization the entire physical meteorology of the high atmosphere was taken into account as far as possible as well as the laws of gaseous diffusion, ionic recombination, electronic attachment, the motions of the ions and electrons under the actions of the gravitational, magnetic and electric fields, etc. The tables of the ionization which were obtained were not published although glimpses of them were given.

During the past year the tables have been worked over completely with more rigor as to certain details and are presented in the following pages. No new physical ideas have been added to those which have already been used.[1] In one instance an improvement has been effected (the necessity for this was outlined in reference 1, section 21), namely, the combined action of temperature diffusion and electric-magnetic drift of the ionization banks at night has been worked out more carefully with the result that for the first time a fairly satisfactory quantitative agreement is found with the night-time radio wave phenomena.

The notation, definitions, assumptions, etc., of this paper are the same as those of reference 1, and to avoid continual restatement of physical ideas and equations we shall refer repeatedly to reference 1.

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[1] Hulburt, Phys. Rev. **34,** 1167 (1929); **35,** 240 (1930); and references therein.

II. Temperatures and Molecular Densities in the High Atmosphere

It is assumed that the geographic and magnetic axes of the earth coincide and are perpendicular to the plane of the ecliptic. Thus our results hold approximately for equinoxial conditions but become untrustworthy for solstice conditions at high latitudes; the tables therefore cease at latitudes above 60°. It is assumed that the temperature T of the atmosphere in levels above 50 km is 400°K, at the subsolar point of the earth, is 220°K at night, and at any point of the daylight hemisphere is given by

$$T = 220 + 180 \cos c, \quad (1)$$

where c is the angular distance to the subsolar point. If θ and ψ denote, respectively, the latitude and longitude, or hour angle, measured from equinoxial high noon at the equator, then

$$\cos c = \cos \theta \cos \psi. \quad (2)$$

Values of T calculated from (1) and (2) are given in the third column of Table I. The assumption that T in the higher levels is constant with altitude is a simplifying approximation made in lack of more definite knowledge. It probably is not exactly true, for on general grounds one would expect T to vary to some extent with the altitude and the variance to be a function of θ and ψ. In the present paper we have taken T to be 400° at high noon, in reference 1 we used 500°. And, although Maris' calculations[2] show that T is considerably higher during the day than in the night, there is no way of knowing at the present time which value is more nearly correct. However, none of our conclusions depend critically on the exact value assumed for T at high noon.

Table 1. *Values of various quantities for the daylight latitudes and longitudes.*

	θ	T	z_c	D	i	v	E
noon	0°	400°K	150 km	1.88×10^{16}	43.8×10^{-4}	785	232
noon	10°	397	150	1.93	41.0	680	221
noon	20°	389	149	1.87	30.5	632	265
noon	30°	376	147	1.75	20.3	566	335
noon	40°	358	145	1.60	12.8	484	416
noon	50°	336	137	1.37	7.4	349	440
noon	60°	310	127	1.02	3.7	330	628
noon	70°	282	119	0.76	1.7	250	830
noon	80°	251	111	0.45	0.5	172	1150
noon	90°	220	104	—	—	—	—
10 A.M. or 2 P.M.	0°	376	150	1.52	36.5	841	249
10 A.M. or 2 P.M.	40°	340	136	1.32	10.5	510	438
10 A.M. or 2 P.M.	60°	299	124	0.87	3.1	402	777
8 A.M. or 4 P.M.	0°	310	132	0.68	15.8	1406	413
8 A.M. or 4 P.M.	40°	285	122	0.62	5.0	910	740
8 A.M. or 4 P.M.	60°	267	115	0.45	1.9	445	855

The molecular density n in the high atmosphere at a height z cm above sea level is given by

$$n = n_0 \epsilon^{-pz},$$

[2] Maris, Terr. Mag. and Atmos. Elec. **33**, 233 (1928); **34**, 45 (1929).

where

$$p = mg/kT, \tag{3}$$

where g is the acceleration of gravity, k is the Boltzmann constant and m is the mass of the average air molecule. With $T = 360°$, $250°$, $230°$ and $220°$, respectively, (3) gives to a close approximation Maris' molecular density curves[2] for summer day, winter day, summer night and winter night conditions in temperate latitudes. From (1) and (3) the n, z curves were drawn for all points on the earth; they are not reproduced here. The curves were used throughout the ionization calculations with two insignificant exceptions, namely: for all temperatures above 360° the n, z curve for 360° was used, this being Maris' summer day curve; for tropical nights the n, z curve for 230° was used, this being Maris' summer night curve.

3. The Ions During the Day

It is probable that the ions at the greater heights are mainly atomic ions and that the proportion of molecular ions increases with decreasing height. This possible change in the mass of the average ion with height was not recognized in the present calculations, but at all heights the average ion was assumed to have a mass of 3.83×10^{-23} grams which is the average of the mass of a nitrogen atomic ion and an oxygen molecular ion. The average ion was assumed to be singly charged. The critical level z_c was calculated as in reference 1, section 7. z_c is the level where the free path of the ion is about 1/5 of the radius of magnetic gyration and is the boundary between the long and the short free path ions. Above z_c the ionized regions are diamagnetic and the ions drift under the magnetic, electric and gravitational fields. Below z_c the ions give rise to the ordinary electrical conductivity of an ionized gas. z_c is a function of θ and ψ, values of z_c are given in the fourth column of Table I for the daylight hours. At night z_c decreases from about 115 km at the equator to 100 km at the poles.

It is assumed that the earth as a whole is approximately electrically neutral so that there are no strong electric fields in the high atmosphere except the relatively small fields which arise from the motions of the ions themselves. The method underlying the ionization calculations was exactly that of reference 1, section 8, namely, to assume an ion curve, caused by the ultraviolet light of the sun, at the subsolar point $\theta = \psi = 0$ which in the long free path region is that required by Gunn's diamagnetic theory of the diurnal magnetic variation[3] and which in the short free path region agrees with the facts of wireless waves as far as they are known, i.e., the absorption of and the apparent heights reached by the waves longer than 100 meters. The curve is given in curve 1, Fig. 4, and the values of y the density of ion pairs are tabulated in column 2, Table III, for noon at latitude 0°.

From this curve the y, z curves due to the solar ultraviolet radiations were calculated at all points of the earth. The calculation fell into two interdependent parts, one dealing with the short free path ions, the S ions, and

[3] Gunn, Terr. Mag. and Atmos. Elec. 34, 17 (1929).

one with the long free path ions, the D ions. The S ions were calculated as in reference 1, sections 11 and 34, and from these the electrical conductivity σ of a 1 cm² vertical column of the S region was calculated. The values of y for the S ions are given in Table III for z below z_c. The values of σ are in Table II, these are practically the same as those in Fig. 5, reference 1.

TABLE II. *Electrical conductivity of the upper atmosphere* $\sigma \times 10^7$ *c.g.s.e.m.u.*

Latitude	noon	10 A.M. 2 P.M.	8 A.M. 4 P.M.	6 P.M.	8 P.M.	10 P.M.	Mid-night	2 A.M.	4 A.M.	6 A.M.
0°	144	110	29	8.6	4.3	3.0	2.1	1.7	1.4	1.2
10°	139	90	26	8.3	4.1	2.8	2.0	1.6	1.3	1.1
20°	76	52	18	6.3	3.5	2.3	1.6	1.3	1.1	0.9
30°	46	34	10	4.0	2.5	1.6	1.2	1.0	0.86	0.7
40°	23	18	5.1	2.4	1.6	1.1	0.8	0.7	0.6	0.5
50°	13	8.5	2.5	1.6	1.0	0.72	0.54	0.46	0.43	0.36
60°	4.5	3.0	1.7	1.0	0.6	0.45	0.35	0.32	0.30	0.25
70°	1.5	1.0	0.8	0.6	0.3	0.27	0.25	0.21	0.18	0.15

The long free path ions in the daytime were calculated as in reference 1, sections 13 to 16, except that the present case is somewhat more complicated because a different n, z curve is used for each θ and ψ. The values of y for the D ions are given in Table III for z above z_c. There resulted from the calculation D, i, v, and E; these are given in Table I, which is similar to Table I, reference 1. D is the number of ion pairs in a 1 cm² vertical column of the D region, i and E are the current and difference of potential, respectively, in c.g.s. electromagnetic units in the horizontal direction of a 1 cm² vertical column of the D region, and v is the component in the vertical direction of the velocity of drift of the D ions due to the action of the electric field E and the earth's magnetic field. v is in the same direction for both positive and negative ions and is downward in the daylight hemisphere and upward in the night hemisphere.

The ions in the daylight hours of Table III are given a symmetrical distribution on either side of the noon meridian with the maximum at noon. Actually this cannot be exactly true, since the maximum is probably half an hour or so later than noon, as calculated in reference 1, section 34, and the distribution is only approximately symmetrical on either side of the maximum.

4. The Ions at Night

We shall first describe the case for latitude 40° and later take up the case of the equator. The y, z curves (or rather $\log_{10} y$) in the daytime for latitude 40° are plotted in Fig. 1. It is seen that during the afternoon y decreases because of the weakening sunlight and the curves sink to lower levels because of the cooling and contraction of the atmosphere. After sunset the electrical field E, which is westward during the day, reverses to become eastward and causes the D ions of both signs to rise with a velocity v. If there were no magnetic field H, or if H were vertical as at the poles, v would not exist and the ions would move downward by diffusion due to the random velocities of temperature agitation (this is the ordinary gaseous or temperature diffusion) to attain, if possible, their equilibrium distribution in the atmosphere; the equilibrium distribution obtains when y/n is constant with z. Such equilibrium can, however, never come to pass because the ions are produced mainly in levels above 100 km and their destruction by recombination increases with decreasing z; hence the ions continually diffuse downward. If the magnetic field H were horizontal, as at the equator, there would be no downward diffusion of the D ions at night. If H is inclined to the vertical, as in temperate latitudes, the ion banks are subjected to both influences, they rise with the magnetic-electric drift velocity v and slide down along the lines of magnetic force with the diffusion velocity v' resolved along the lines. In general v' increases with z, at the greater heights it is greater than v and at the lower heights is less than v. Thus the D ions in the lower levels move up and in the higher levels move down to form a bank with the maximum value of y approximately at the level z' where $v = v'$.

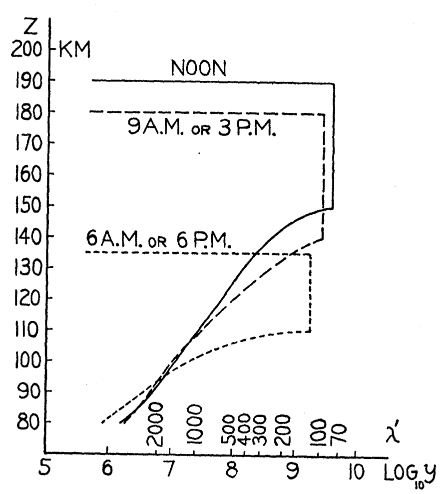

Fig. 1. Density y of ion pairs in the daytime, latitude 40°.

At a height z the number of ion pairs N which diffuse cm^{-2} sec^{-1} downward is

$$N = 0.41\gamma u(py + dy/dz)/n, \tag{4}$$

where γ is the mean free path of the ions and u their velocity of temperature agitation (reference 1, paragraph 9). Then

$$v' = (N \cos \zeta)/y, \tag{5}$$

where ζ is the angle between H and the vertical. At the level where y is a maximum $dy/dz = 0$. Putting $\gamma = 1/2^{1/2}\pi n\sigma^2$ where the molecular diameter σ is taken to be 3×10^{-8} cm and using $T = 220$°K, (4) becomes $N = 0.735 \times 10^{13}$ y/n. This value in (5) yields

$$v' = (0.735 \times 10^{13} \cos \zeta)/n. \tag{6}$$

To determine v during the night we turn to Table I and find that E averaged over the sunlit hemisphere is 335 e.m.u. cm^{-1}. As discussed in reference 1, section 27, the values in the table were obtained by calculating the equilibrium conditions for each slice of the atmosphere along parallels of latitude independently of the influence of adjacent slices. Such a procedure is not entirely correct because the adjacent slices influence each other, and, to make a long story short, the influence is such as to make the value 335 too large. We therefore take E to be 250 e.m.u. cm^{-1}. From this the sunset meridian of the earth is at a potential of about 5000 volts above that of the sunrise meridian. The voltage in the night D region is of course due to the eastward current flowing in the underlying S region. The values of the conductivity decrease all night, see Table II, and therefore, since the total current

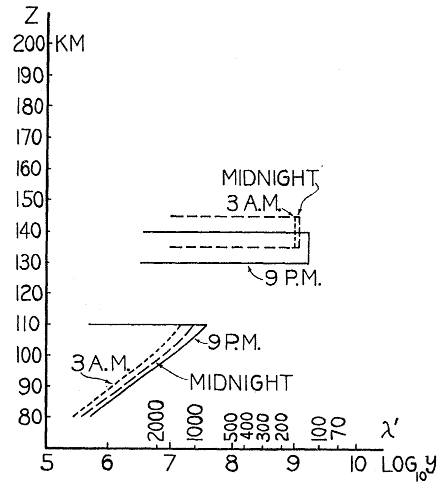

Fig. 2. Density y of ion pairs at night, latitude 40°.

in the S region is constant during the night, E increases all night, being about three times as great in the early morning hours as it is in the evening hours. Hence v, which is equal to $E \sin \zeta/H$, increases all night. As a simplifying approximation we take v to be 250 cm $sec.^{-1}$ from sunset to 10 P.M., to be 500 cm $sec.^{-1}$ from 10 P.M. until dawn, and to be constant along the meridians of longitude. Our results do not depend critically on the approximation.

To find the level where the maximum value of y of the D ion bank is between 6 and 10 P.M. we put $v' = 250$ cm $sec.^{-1}$ in (6) and get

$$n = 2.94 \times 10^{10} \cos \zeta. \tag{7}$$

At 40° latitude $\zeta = 30° \, 45'$ and $n = 2.51 \times 10^{10}$. Whence, from the night n, z curve, z' is 139 km. After 10 P.M. v' is 500 cm $sec.^{-1}$ and z' shifts up 5 km to 144 km. Therefore the D ions which at 6 P.M. are mainly in the levels from $z = 110$ to 135 km, see Fig. 1, move up to assemble in a bank around $z = 139$ km. $v = 250$ cm $sec.^{-1}$ amounts to 9 km $hour^{-1}$, so that the ions starting from 110 km reach the bank at about 9 P.M., although their numbers are considerably reduced by recombination during the journey (reference 1, Eq. (17)). Calculation shows that the supply into the bank from the ascending D ions is just about sufficient to maintain $y=1.7\times10^9$ in a bank 10 km thick from 130 to 140 km against the loss due to recombination of the ions. After about 9 P.M. the supply due to this source ceases. Since z_c is 110 km the ions below 110 km are S ions and are not subjected to the drift velocity v but remain where they are except for the relatively slow temperature diffusion upward and downward. Thus the ions separate into two banks. After about 10 P.M. the upper bank moves up 5 km. The degredation of y in each bank through the night due to the recombination of the ions was calculated from Sir J. J. Thomson's formula (reference 1, section 9). The night y, z curves for latitude 40° are given in Fig. 2.

The night-time ionization for latitude 60° was calculated in a manner similar to that for latitude 40°; the values of y are in Table III. A zero value in Table III merely means a small value.

TABLE III. *Ion density y in the upper atmosphere.*

	z	Geographic Latitude 0°	40°	60°
Noon	200 km	0	0	0
	190	4.7×10^9	4.0×10^9	0
	180	4.7×10^9	4.0×10^9	0
	170	4.7×10^9	4.0×10^9	3.1×10^9
	160	4.7×10^9	4.0×10^9	3.1×10^9
	150	4.7×10^9	4.0×10^9	3.1×10^9
	140	7.0×10^8	4.0×10^8	3.1×10^9
	130	2.3×10^8	1.4×10^8	3.1×10^9
	120	9.1×10^7	6.7×10^7	5.1×10^7
	110	3.6×10^7	2.7×10^7	2.3×10^7
	100	1.4×10^7	1.2×10^7	7.1×10^6
	90	5.8×10^6	5.4×10^6	2.7×10^6
	80	2.1×10^6	1.6×10^6	1.1×10^6
3 P.M. or 9 A.M.	200 km	0	0	0
	190	3.0×10^9	0	0
	180	3.0×10^9	2.8×10^9	0
	170	3.0×10^9	2.8×10^9	2.3×10^9
	160	3.0×10^9	2.8×10^9	2.3×10^9
	150	3.0×10^9	2.8×10^9	2.3×10^9
	140	2.8×10^8	2.8×10^9	2.3×10^9
	130	1.1×10^8	4.0×10^8	2.3×10^9
	120	4.6×10^7	1.0×10^8	1.1×10^8
	110	1.9×10^7	3.4×10^7	3.7×10^7
	100	9.1×10^6	1.2×10^7	1.1×10^7
	90	2.8×10^6	4.5×10^6	3.1×10^6
	80	1.2×10^6	1.8×10^6	1.0×10^6
6 P.M. or 6 A.M.	150 km	0	0	0
	140	1.7×10^9	0	0
	135	1.7×10^9	1.7×10^9	1.6×10^9
	130	1.7×10^9	1.7×10^9	1.6×10^9
	120	1.7×10^9	1.7×10^9	1.6×10^9
	115	1.7×10^9	1.7×10^9	1.6×10^9
	110	5.1×10^7	1.7×10^9	1.6×10^9
	100	1.3×10^7	1.9×10^7	1.4×10^7
	90	2.9×10^6	3.9×10^6	3.4×10^6
	80	8.0×10^5	8.4×10^5	8.1×10^5

TABLE III. (*Continued*).

	z	0°	40°	60°
9 P.M.	170 km	0	0	0
	160	1.7×10^9	0	0
	150	1.7×10^9	0	0
	140	0	1.7×10^9	1.6×10^9
	130	0	1.7×10^9	1.6×10^9
	120	0	0	0
	115	5.2×10^7	0	0
	110	2.4×10^7	3.8×10^7	3.3×10^7
	100	5.9×10^6	1.0×10^7	9.2×10^6
	90	1.3×10^6	2.1×10^6	1.9×10^6
	80	3.8×10^5	5.1×10^5	4.6×10^5
Midnight	170 km	0	0	0
	160	1.6×10^9	0	0
	150	1.6×10^9	0	0
	145	0	1.2×10^9	1.1×10^9
	135	0	1.2×10^9	1.1×10^9
	125	0	0	0
	115	3.4×10^7	0	0
	110	1.6×10^7	2.3×10^7	2.1×10^7
	100	3.6×10^6	2.3×10^6	6.5×10^6
	90	8.7×10^6	1.8×10^6	1.3×10^6
	80	4.6×10^5	3.7×10^5	3.1×10^5
3 A.M.	170 km	0	0	0
	160	1.5×10^9	0	0
	150	1.5×10^9	0	0
	145	0	1.0×10^9	9.0×10^8
	135	0	1.0×10^9	9.0×10^8
	125	0	0	0
	115	2.4×10^7	0	0
	110	1.0×10^7	1.5×10^7	1.4×10^7
	100	3.1×10^6	5.3×10^6	4.9×10^6
	90	5.5×10^5	1.1×10^6	9.4×10^5
	80	1.9×10^5	2.8×10^5	2.2×10^5

The calculation of the night-time ionization for latitude 0° presents an interesting feature. At the equator $\zeta=90°$ and from (7) $n=0$, and from the n, z curve for a summer night $z'=\infty$; which means that at the equator where H is horizontal the downward diffusion velocity v' is zero and therefore the D ions continue to rise all night with the velocity v. As a matter of fact, this does not occur because of the lateral diffusion of the ions. What happens is seen in the following way: z' was calculated for tropical latitudes from $\theta=0°$ to 20°. The z',θ curve is shown in Fig. 3, its rises sharply to ∞ at $\theta=0$, and gives the heights where the night D ion banks would be if there were no lateral diffusion to the north or south (and if the night were long enough). Such diffusion, however, occurs, for in a distance of only 0.5° or 55 km (or 30 nautical miles) from the equator z' drops from its equatorial value ∞ to 173 km. As a result the D ions rise up at night and diffuse to the north and sound along the lines of magnetic force in the tropical latitudes from, say, 20° north to 20° south to form a bank in the levels from 150 to 160 km. The tropical night values of y were then calculated just as in the case of latitude 40° and are given in the 0° latitude column of Table III.

5. Electrons

In general the electron density y_e increases with z to a maximum value y_m at a height z_m. For the daylight hours y_m was calculated from the equation

$$y_m = 3.15 \times 10^5 \cos\theta(0.126 \sin\psi + \cos\psi). \quad (8)$$

(8) is the same as (21) of reference 1 with a slight change in the numerical constants to conform with the slightly different atmospheric temperatures used in the present paper. The night values of y_m were worked out just as for the case of the ions at latitude 40° described in the preceding section; it seems unnecessary to give the details of the calculations. y_m and z_m are given in Table IV. y_e above and below y_m can not yet be calculated with certainty.

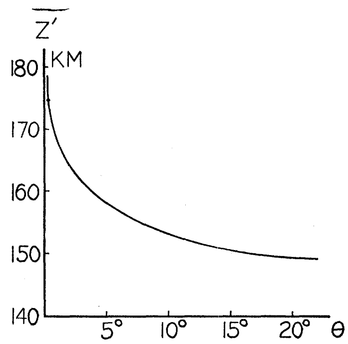

Fig. 3. z' as a function of θ.

Table IV. *Maximum electron density y_m in the upper atmosphere.*

Geographic Latitude	0°		40°		60°	
	z_m	y_m	z_m	y_m	z_m	y_m
Noon	195 km	3.2×10^5	195 km	2.4×10^5	175	1.6×10^5
3 P.M.	195	2.5×10^5	195	1.9×10^5	175	1.3×10^5
6 P.M.	144	1.2×10^5	140	0.9×10^5	140	0.6×10^5
9 P.M.	165	1.1×10^5	145	0.1×10^5	145	0
Midnight	165	1.0×10^5	145	0	145	0
3 A.M.	165	0.8×10^5	145	0	145	0
6 A.M.	144	1.0×10^5	140	0.8×10^5	140	0.5×10^5
9 A.M.	195	2.0×10^5	195	1.5×10^5	175	1.0×10^5

6. Ionization by the Ultraviolet Light of the Sun

Although explicit calculations of the ion and electron banks produced by the absorption of the ultraviolet light of the sun in the high atmosphere have been made for certain cases,[4] it is important to show that the y, z curve assumed at $\theta=\psi=0$ is the sort of curve which might be caused by the solar ultraviolet light. The curve is drawn in curve 1, Fig. 4; it is idealized of course, y being taken to be constant with z through the D region in order to simplify the problem. Without some such simplification the calculations of sections 3 and 4 could hardly even have been attempted.

[4] Hulburt, Phys. Rev. **31**, 1018 (1928).

The ions disappear only by recombination, therefore j the total rate of loss in a 1 cm^2 vertical column of the atmosphere is

$$j = \int_0^\infty \alpha n y^2 dz,$$

where α is the recombination coefficient. For curve 1, Fig. 4, (j) is 1.1×10^{13} pairs cm^{-2} $sec.^{-1}$ or 1.1×10^{13} W erg cm^{-2} $sec.^{-1}$, if W is the work of ionization. This is the energy necessary to maintain the ion bank and must be supplied by the sunlight. For $W=1.28\times10^{11}$ erg, corresponding to 8 volts or 1520A, the energy is 141 erg cm^{-2} $sec.^{-1}$. The energy in the solar spectrum region from 0 to 1520A is about 100 erg cm^{-2} $sec.^{-1}$ calculated on the assumption that the sun is a black body at a temperature of 6000°K and taking the solar constant to be 1.35×10^6 erg cm^{-2} $sec.^{-1}$. In this case the solar energy is able

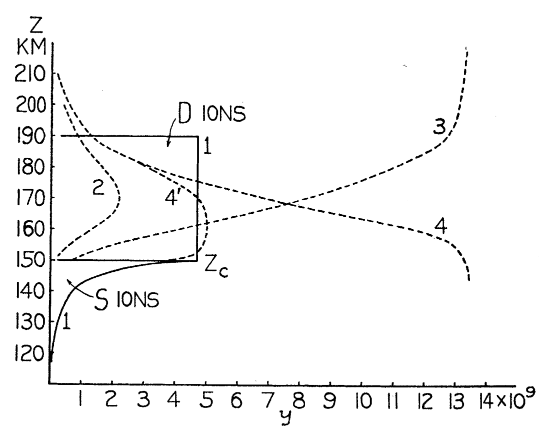

Fig. 4. Curve 1 is the assumed y, z curve for noon at the equator, the other curves are theoretical.

to maintain the ion bank. If the ionization potential is 10 volts, corresponding to 1234A the energy required to maintain the bank is 176 erg cm^{-2} $sec.^{-1}$, and the solar energy from 0 to 1234A is about 4 erg cm^{-2} $sec.^{-1}$, which is somewhat less than that required. On the whole, since the amount of the sun's energy in the far ultraviolet is entirely unknown as well as the exact processes of photo-ionization, the calculation supports the view that the solar ultraviolet energy is adequate to cause the ionization in the high atmosphere.

The form of the y, z curve will depend on the absorption coefficients of the ionizing wave-lengths of the solar ultraviolet light, on the recombination of the ions and on the manner in which the ions move about by temperature diffusion and electric-magnetic drift. If I is the energy carried by the ionizing wave-lengths outside of the atmosphere and if β is the average light absorption coefficient of these wave-lengths, the number of ion pairs v (i.e., positive and negative ions) produced per cm^3 in the atmosphere at a height z is, for the sunlight incident vertically,[4]

$$q = \frac{\beta I}{W} n\epsilon^{-\beta n/p}. \tag{9}$$

With $\beta = 2.63 \times 10^{-18}$ (a reasonable value, reference 4, page 1026) and with the summer day value of $p = 0.872 \times 10^{-6}$, q was calculated from (9); the q, z curve is plotted in curve 2, Fig. 4, q being in arbitrary units.

For a static atmosphere, i.e., no diffusion or drift of the ions, $q = \alpha n y^2$. Substituting in (9) gives

$$\alpha y^2 = \frac{\beta I}{W} \epsilon^{-\beta n/p}. \tag{10}$$

From (10) with $\alpha = 0.975 \times 10^{-25}$ and $I/W = 6.83 \times 10^{12}$ y was determined, the y, z curve is plotted in curve 3, Fig. 2. In the S region the electric-magnetic drift of the ions does not exist, and calculation shows that, because of the relatively high values of the molecular density n temperature diffusion is unimportant compared to q and the recombination loss. Hence, approximately, the S region may be regarded as a static atmosphere as far as the y, z curve is concerned, and therefore curve 3, Fig. 4, is of the type which the ultraviolet light would produce in the S region. The curve is similar to curve 1 below z_c. It is seen that by a suitable choice of several values of β and of I for each β, and combining the results, the y, z curve of the S ions may be accounted for.

In the D region we take into account the electric-magnetic drift of the ions and neglect the effect of temperature diffusion; this is a justifiable approximation in the lower portion of the D region, but is not so in the upper portion and must be allowed for later. Consider an element of the D region 1 cm square and of thickness dz. The number of ion pairs produced per second in the element is qdz; the number which disappear per second by recombination is $\alpha n y^2 dz$; the number which leave the lower face of the element is vy, the drift velocity v being downward; and the number which enter the upper face of the element is $v(y + (dy/dz)\, dz)$. Then for a steady state

$$q = \alpha n y^2 - v dy/dz.$$

Substituting (9) we get

$$\frac{\beta I}{W} n \epsilon^{-\beta n/p} = \alpha n y^2 - v dy/dz. \tag{11}$$

We solve (11) by approximate methods. From (3) $dy/dz = -pn\, dy/dn$. Above 180 km $\beta n/p$ is small, less than 0.4, and $\epsilon^{-\beta n/p}$ is approximately unity. (11) then becomes approximately

$$\beta I/W = \alpha y^2 + pv dy/dn. \tag{12}$$

Solving (12), and putting $y = 0$ at $n = 0$, gives

$$y = (\beta I/\alpha n)^{1/2}\left(\exp \frac{2n}{pv} (I\beta\alpha/W)^{1/2} - 1\right)\left(\exp \frac{2n}{pv} (I\beta\alpha/W)^{1/2} + 1\right)^{-1}. \tag{13}$$

With the numerical values used for curves 1, 2 and 3, Fig. 4, the y, z curve from (13) was calculated and is given in curve 4, Fig. 4. Curve 4 is the solution of (12) approximately valid down to about 180 km. In lower levels

$\epsilon^{-\beta n/p}$ is much less than unity and curve 4 is not an approximate solution of (11) but yields values of y which are too large. Therefore the curve for (11) lies below curve 4. Its maximum, obtained by putting $dy/dz=0$, lies somewhere on curve 3. Below the maximum the curve must satisfy the requirement that the total rate of supply of the ions due to the ultraviolet light and to the drift of the ions across the maximum equal the rate of loss from recombination. Above the maximum the total rate of supply from the ultraviolet light must equal the loss from recombination and the drift of ions across the maximum. At all points on the curve y, z and dy/dz must satisfy (11). These conditions are sufficient to enable one to draw the curve with an accuracy of at least 15 percent without much trouble. Curve 4′, Fig. 4, is the curve thus obtained and is seen to be not unlike curve 1. Furthermore, the effect of temperature diffusion is to bring down ions from the levels above 190 km; these

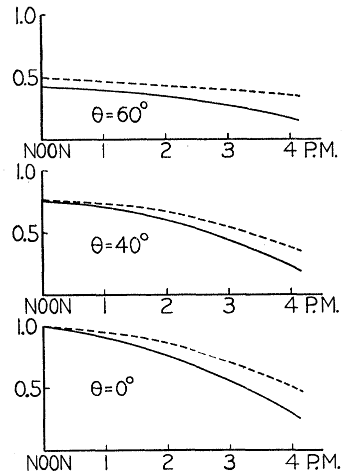

Fig. 5. The full line and dotted curves are the values of $T\ D$ and $\cos\theta\cos\psi$, respectively.

ions will bulge out curve 4′ in levels from 170 to 190 km, and make it even more like curve 1. On the whole it is concluded that curve 1 is much the sort of curve which might reasonably be expected to be produced by the ultraviolet light of the sun.

7. Agreement with the Solar Diurnal Variation of the Earth's Magnetic Field

The diamagnetic theory,[3] which accounted well for the prominent features of the solar diurnal variation of the earth's magnetism, assumed that the diamagnetism of the atmosphere due to the D ions was proportional to $\cos\theta\cos\psi$. The diamagnetism of the ions is proportional to DT. We have taken the y, z curve at $\theta=\psi=0$ such that DT is the value required by the diamagnetic theory and have calculated all the other y, z curves without reference to the theory. We must now see whether the values of Table III yield a diamagnetism in accord with the theory. The values of $T\ D$, calculated from Table

III, are plotted in the full line curves of Fig. 5; cos θ cos ψ is plotted in the dotted line curves. The two sets of curves agree within the tolerance of the theory; it should be remembered that they were made to fit at only one point, noon at the equator or $\theta=\psi=0$. Furthermore, the magnetic effects at the surface of the earth depend on the distance away of the diamagnetic ion bank, and the D ions are closer to the earth for the greater values of θ and ψ than they are for the lesser values. If this were taken into account the T D curves would be even closer to the dotted curves than they are in Fig. 5, and we conclude that the ionization of Table III is in close accord with the diamagnetic theory.

8. Agreement with Bauer's Analysis

From a harmonic analysis of the 1922 survey of the permanent magnetic

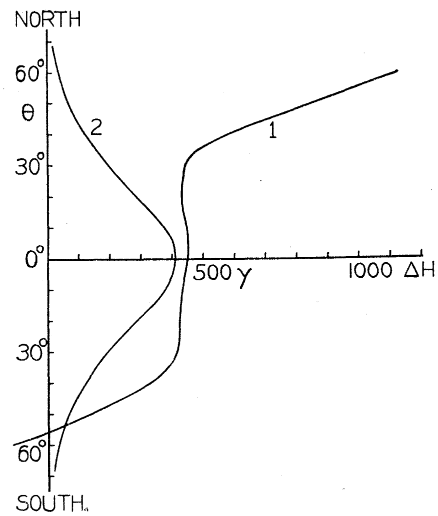

Fig. 6. Curve 1 gives the observed, curve 2 the theoretical, values of ΔH the horizontal component of the portion of the earth's magnetic field of external origin.

field of the earth Bauer[5] concluded that a portion is of external origin. His values of the horizontal component ΔH of this portion for various latitudes are plotted in curve 1, Fig. 6; the scale of abscissas is in γ or 10^{-5} gauss. From the values of i of Table I, from the values of σ of Table II and from the sunset to sunrise difference of potential of about 5000 volts (section 4) the currents from pole to pole in the D and S regions of the day and night hemispheres were calculated. They were, day D region, 21.2×10^6 amperes, day S region 15.9×10^6 amperes, night D region, 4.6×10^6 amperes and night S region, 0.7×10^6 amperes; all are eastward except the day S region current which is westward. These amount to a current of 5.3×10^6 amperes flowing eastward around the earth all the time. The distribution of the current density with latitude has the shape of curve 2, Fig. 6. The horizontal component

[5] Bauer, Terr. Mag. and Atmos. Elec. **28**, 1 (1923).

of the magnetic field due to this current sheet was calculated from the relation

$$\Delta H = 2\pi i, \tag{14}$$

and is plotted in curve 2, Fig. 6. The calculated and observed curves are most reliable in tropical latitudes and here they agree well. At high latitudes there is some disparity. However, at high latitudes (14) is a poor approximation and curve 2 is not trustworthy. The observed curve, too, is not to be trusted at high latitudes, for the data on which it was based were meagre. The program of the International Polar Year which begins this year contemplates obtaining more complete observations in polar regions which may permit a more exact magnetic analysis.

9. Agreement with Wireless Waves

From the ionization of Tables III and IV the skip distances of wireless waves were calculated taking into account the refraction of the waves in both the ion and electron banks and were found to agree well with the observed values.[4] The calculated skip distances for a 25 meter wave are given in Fig. 6.

The height z at which a wireless ray of wave-length λ is totally reflected (refracted) at normal incidence in the ionization bank is obtained by putting the refractive index μ equal to zero in the equation

$$\mu^2 = 1 - 2ye^2\lambda^2/\pi m, \tag{15}$$

solving for y and determining z from the y, z curve. For the ion bank y and m in (15) are the ionic density and mass, for the electron bank y and m are the electronic density and mass and in this case the factor 2 in (15) is omitted. In the case of electrons (15) is the dispersion formula for wave propagation polarized with the electric vector parallel to the magnetic field; for other states of polarization the appropriate dispersion formulas should be used, which are however not greatly different from (15) for wireless waves below 50 meters in length. For ions (15) is closely valid for all states of polarization of the wireless waves. The values of y for total reflection of waves of various wave-lengths are marked along the x-axis of Figs. 1 and 2. It is seen, for example, that 150 to 400 meter waves are turned down at the 100 km level during the day and that in the early part of the evening they pierce through the lower layer to be reflected (refracted) from the higher level at about 140 km. This agrees with the facts except that the observed retardation times indicated night-time heights of around 200 km or more.[6] The difference may reasonably be attributed to the group velocity retardation which is always present in the observations.

The longest wireless wave λ' which pierces vertically through the atmosphere is determined by setting $\mu=0$ in (15), substituting the maximum value of y or y_e from Table III or IV for the time of day and the latitude in question and solving for λ. The values of λ' are given in Table V, and agree roughly

[6] Kenrick and Pickard, Proc. Inst. Rad. Eng. **18**, 649 (1930) give a summary of references; Appleton and Green, Proc. Roy. Soc. **A128**, 159 (1930); Mars, Gilliland and Kenrick, Proc. Inst. Rad. Eng. **19**, 106 (1931).

TABLE V.

Latitude	0°	40°	60°
noon	60 m	68 m	84 m
midnight	103	140	144

with the values which have been observed.[6] The agreement is to be expected because the λ' relation is a special case of the skip distance relation, namely, λ' is the wave for which the skip distance is zero. And the agreement means nothing more than that the measurements of λ' checked the predictions from the skip distances.

The limiting wave, defined as the shortest wave which can be used for reliable long distance radio communication, was observed[4] by Taylor to be about 10.5, 17, 14 and 23 meters for summer noon, summer midnight, winter noon and winter midnight, respectively, for temperate latitudes in 1925

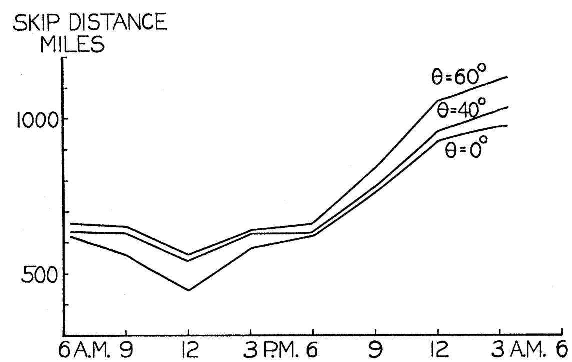

Fig. 7. Skip distances in miles for a 25 meter radio wave calculated as a function of the latitude and the time of day.

and 1926, an epoch midway between maximum and minimum solar activity. The values calculated from Tables III and IV agree within 15 percent with the observed values.

Keeping in mind that the ionization banks of Tables III and IV are necessarily somewhat idealized and are incomplete in that they do not give the electrons which exist below the maxima of the y_e, z curves, it is concluded that the ionization accounts for the main facts of the propagation of wireless waves over the earth. Further experimental facts may enable the tables to be made more complete and, if necessary, to be modified to some extent in various details. Certain modifications are possible without changing any of the present calculations. For example, the amount of ionization in the lower portion of the S region was estimated from rather uncertain experimental data, namely, the apparent heights reached by long radio waves. Future observations may indicate a greater ionization in these levels than we have put into the tables. Such changes would not change perceptibly the values of σ of Table II or any of the other calculations.

10. Summary

Each cm^3 of the upper atmosphere is assumed to be approximately electrically neutral. Above 60 km the ionization is assumed to be caused by the ultraviolet light of the sun, below, by cosmic radiation. At any height z km above sea-level, denote the numbers per cm^3 of singly-charged positive ions, negative ions and electrons by $y+$, $y-$ and y_e respectively. Then $y+=y-+y_e$, and since y_e is in general small compared to $y-$, the values of $y+$ and $y-$ are nearly equal and are each denoted by y; then y also denotes the density of ion pairs. Above 60 km the values of y are given in Table III. The electron density y_e increases with z to a maximum value y_m at a height z_m; y_m and z_m are given in Table IV. Above and below the maximum y_e can not yet be estimated with certainty. The values given for y and y_m may perhaps be correct within a factor of 2; a zero value merely means a small value. The tables are for average equinoxial conditions and solar quiescence at an epoch halfway between the periods of maximum and minimum solar activity. The values should be increased, and reduced, by roughly 25 percent to refer to epochs of maximum and minimum solar activity, respectively. During magnetic storms the ionization is increased, the values being perhaps double those in the tables for a severe storm. The ionization in polar regions is probably not greatly different from that given for latitude 60°. The seasonal changes in the ionization are small at the equator. In temperate latitudes for winter and summer use the values for latitudes about 20° higher and lower, respectively, in the tables. Below 60 km the ionization due to cosmic radiation is independent of the latitude, hour of the day, solar activity, etc., and y is 8×10^2, 1.6×10^3, 1.9×10^3, 2.2×10^3, 2.9×10^3, 3.4×10^3 and 3.5×10^3 at 0, 10, 20, 30, 40, 50 and 60 km, respectively.[7]

[7] Hulburt, Phys. Rev. **37, 1** (1931).