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VELOCITY OF SOUND IN TUBES AT AUDIBLE AND ULTRASONIC FREQUENCIES

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ABSTRACT

The velocity of sound in air contained in glass tubes ranging in diameter from 0.1 cm to 3.0 cm was measured at ultrasonic frequencies over a frequency range from 30 to 200 kilocycles per second by using quartz crystals as sound oscillators, and the Kundt's dust tube method. The results do not agree with those calculated by use of Helmholtz-Kirchhoff equation

$$V^1 = Vo(1 - C/2R(2\pi N)^{1/2}).$$

An equation of the form

$$V^1 = Vo(1 - C/D^2 - K/D(N)^{1/2})$$

is fairly satisfactory for both audible and ultrasonic frequencies. In this equation $V_0 = 331.77$ m/sec., $C = 0.001512$, $K = 0.174$, N = frequency and D = diameter of the tube. The agreement between experimental and calculated values is better for the larger tubes than for the smaller.

INTRODUCTION

EXPERIMENTAL observations of the velocity of sound in pipes may be divided into three classes, in which direct measurements were made of the time required (1) for the sound to pass from one end of the pipe to the other end; (2) to return to the source after being reflected from the closed farther end; (3) to traverse a U-tube and return to the source.

From experiments Regnault[1] found that the velocity of sound increased with the diameter of the pipes, reaching the ordinary value in large pipes. Rink's[2] analysis of Regnault's observations gives a mean value of the velocity of 330.5 m/sec. at 0°C. Voille and Vautier,[3] by observations similar to those of Regnault, obtained 331.1 m/sec. at 0°C.

Helmholtz[4] and Kirchhoff[5] considered theoretically the propagation of sound waves in small tubes and arrived at the following expression for the speed:

$$V^1 = V_0(1 - C/2R(2\pi N)^{1/2}) \tag{1}$$

where V^1 is the velocity of sound for frequency N, in a pipe of radius R, V_0 being the velocity in free air at 0°C and C a constant with no definite agreement as to its exact value. Schneebeli[6] and Seebeck[7] found from their experiments that the decrease of velocity varies as the radius of the tube but that

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[1] V. Regnault, Comptes Rendus **66**, 209 (1868).
[2] Rink, Pogg. Ann. **149**, 533 (1873).
[3] Voille and Vautier, Phil. Mag. **26**, 77 (1888).
[4] Helmholtz, Crelles Jour. **57**, 1 (1859).
[5] Kirchhoff, Pogg. Ann. **134**, 177 (1868).
[6] Schneebeli, Pogg. Ann. **136**, 296 (1869).
[7] Seebeck, Pogg. Ann. **139**, 104 (1870).

when the frequency varies it is inversely proportional to the squareroot of the cube of the frequency instead of the squareroot as given in Eq. (1). Müller[8] found that the equation is not valid but that the velocity of sound in pipes depends on the material of the pipes. Stevens[9] found that C is valid for pipes of 2.0 cm to 4.0 cm diameters, while Schulze[10] claimed that C depends on the diameter and nature of the pipes. The results obtained by Wertheim[11] and Blaikley[12] are in support of Eq. (1).

The above results were obtained with frequencies (256 to 5550) within the audible region. When the frequency of the sound is above the audible limit, or too high to be estimated by the ear, the velocity is more conveniently measured by Rayleigh's[13] "stationary wave" method or Kundt's[14] dust tube. W. Altberg[15] measured the wave-length of the sound emitted by an electric spark, frequency $N=340{,}000$, and found that the product $N\lambda$ was in moderately good agreement with the accepted values for the velocity of sound in air. A. Campbell and D. W. Dye[16] obtained a similar result at $N=900{,}000$ using a high frequency spark as a source and a Kundt's dust tube to indicate the wave-length. A. L. Foley[17] found by the photographic method that a very intense sound, such as produced by an intense electric spark, has an abnormally high velocity at points near the source where the intensity is great. If such a source be placed in one end of a pipe so that the wave does not have any chance to expand and thus decrease in intensity, such a wave travels through a tube with a higher velocity than normal. If the spark were placed at a distance of two to five centimeters, depending on the intensity of the spark, from the end of the tube, the velocity in the tube would be less than it is in free air.

In recent years the piezoelectic property of crystals has been used as a source of high frequency vibrations. Cady[18] made a thorough investigation of crystal oscillations and crystal resonators, and adapted them to use as constants of electrical frequency. Harrison[19] and others made an investigation as to the possible modes of vibration of quartz crystals. Pierce,[20] Reid,[21] Wood and Loomis,[22] Hubbard and Loomis[23] and others have used piezoelec-

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[8] Müller, Ann. d. Physik **11,** 331 (1903).

[9] Stevens, Ann. d. Physik **7,** 285, (1902).

[10] Schulze, Ann. d. Physik **13,** 1060 (1904).

[11] Wertheim, Pogg. Ann. **77,** 427 (1844).

[12] Blaikley, Phil. Mag. **16,** 477 (1883).

[13] Rayleigh, Sound **2,** 403.

[14] Kundt, Pogg. Ann. **135,** 337 (1868).

[15] W. Altberg, Ann. d. Physik **23,** (1907).

[16] A. Campbell and D. W. Dye, Electrician **66,** 862 (1911).

[17] A. L. Foley, Phys. Rev. **14,** 2 (1919).

[18] W. G. Cady, Proc. Inst. Rad. Eng. **10,** 83 (1922); Phys. Rev. **19,** 1 (1922); Jour. Opt. Soc. Am. **10,** No. 4, April (1925).

[19] J. R. Harrison, Proc. Inst. Rad. Eng. 1040, Dec. 1927.

[20] G. W. Pierce, Proc. Am. Acad. of Arts and Science **60,** 271 (1925).

[21] C. D. Reid, Phys. Rev. **35,** 814 (1930).

[22] R. W. Wood and A. L. Loomis, Phil. Mag. **4,** 417 (1927).

[23] J. C. Hubbard and A. L. Loomis, Phil. Mag. **5,** 1177 (1928).

tric crystals as the source of ultrasonic vibrations, of accurately known frequency, for the determination of the velocity of sound in liquids or gases.

EXPERIMENTAL PROCEDURE

The velocity of sound waves in tubes of small diameters has been found in most cases by the Kundt's dust tube or some resonance tube method. It was suggested that quartz resonators be used as a source of high frequency sound for the investigation of sound waves in tubes of small diameter to find whether or not the Helmholtz-Kirchhoff equation is valid for ultrasonic waves. This work was carried on by the writer in the ultrasonic region at frequencies from 30 kc to 200 kc, using glass tubes varying in diameter from 0.1 cm to 3.0 cm. A quartz crystal was mounted as shown in Fig. 1 which is a modified form of Pierce's circuit.

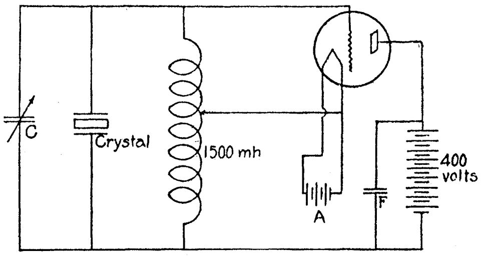

Fig. 1. Diagram of apparatus.

A type UX210 Radiotron tube was used with 400 volts on the plate. The capacity and inductance used depended on the frequency of the various crystals (7 crystals were used). When the oscillating circuit was tuned to crystal frequency, the crystal was set in vigorous vibration, producing ultrasonic waves of constant frequency. Round glass tubes were fitted with movable pistons and lycopodium powder was dusted in them. Each tube in turn was brought close to the face of the vibrating crystal and the position of the piston adjusted so as to produce nodes of lycopodium dust. The distance between nodes was then measured and this value taken as the half wavelength. Knowing this value and the frequency of the crystal, the velocity of the sound wave in the tube was calculated from $V=N\lambda$, where V is the speed, N the frequency, and λ the wave-length. The crystals which were calibrated by the Bureau of Standards had the following frequencies: 30.3 kc, 49.58 kc, 51.5 kc, 69.4 kc, 100 kc, 145 kc, and 200 kc. In order to obtain the best possible results, care was taken to avoid change in the frequency of the crystals. This was done (1) by having the crystal perfectly clean, (2) by proper adjustment of the electrodes to the crystal, thus preventing sparking and heating, (3) by operating the crystal for a comparatively short time only, to avoid heating. The lycopodium powder and tubes were kept dry, as any dampness in either of these made results difficult to obtain.

DISCUSSION

For measuring the distance between nodes a micrometer microscope, which could be read accurately to 0.005 cm, was used. The maximum point on the nodes could not be located accurately but after a number of trials, the probable error of any one setting of the micrometer was found to be less than 0.5 mm. Then in order to increase the accuracy of the readings the microscope was set on every tenth node and the reading taken. The difference between any two consecutive readings was divided by ten, thus reducing the error to one-tenth of that for one reading alone. The value of the wavelength for each tube and crystal was obtained by taking the average of fifty or more measurements, as described above, and then multiplying this value by two. Due to the difficulty in obtaining distinct nodes beyond the thirty-fifth in the smaller tubes, the measurements were limited to the first thirty-five nodes in all tubes. The first two or three nodes were not measured as they were more or less distorted. The temperature at which the observations were made was 21°C, but the velocity of sound in every case was reduced to 0°C for comparison. In reducing the velocity to 0°C it was assumed that the velocity of ultrasonic waves varies with the temperature the same as audible waves, i.e., 0.6 m/sec. per degree centigrade change in temperature.

No correction was made for the humidity. The relative humidity was less than 30 percent and was practically constant. Wood[24] states that the calculated velocity in air saturated with moisture at 10° C is from 2 to 3 feet/sec. greater than in dry air. Stewart-Lindsay[25] states that the velocity of sound in saturated air at 20° C and 760 mm Hg is about 0.33 percent greater than that in perfectly dry air at the same temperature and pressure.

The velocity of sound in tubes of different diameters was calculated by use of the Helmholtz-Kirchhoff equation in which V_0 is given the value of 331.7 7m/sec., and C that of 0.132, which is the ratio of the coefficient of viscosity of air to its density at 0°C, or what Richardson[26] calls the kinematic coefficient of viscosity.

Table I shows my experimental and calculated results.

The experimental results are shown graphically in Fig. 2, in which the average value of the velocities for all frequencies for a given tube diameter is plotted against the tube diameter.

It will be seen that the values calculated by use of Eq. (1) do not agree with the experimental values (see Table I). A greater decrease would be expected for the smaller tubes due to the increased effect of viscosity and heat conduction. When the diameter of the tube is small the conduction of heat from the center of the air column to the walls becomes more and more rapid.[27] The velocity of sound in such a tube might be expected to be more nearly that of Newton's isothermal value $K/\rho^{1/2}$. Viscosity may be expected to exert a greater influence when sound waves pass through a relatively narrow tube

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[24] A. B. Wood, A Textbook of Sound, p .231.

[25] Stewart-Lindsay, Acoustics, p. 308.

[26] F. G. Richardson, Sound, p. 166.

TABLE I. *Velocity of sound in glass tubes, of different diameters, and at different frequencies.*

Frequency	Diameter of tube	Velocity experimental at 0°C	Velocity from Eq. (1) at 0°C	Velocity from Eq. (3) at 0°C	Difference exp−Eq. (3)
30.3 kc	3.0 cm	332.30m/sec.	331.74 m/sec.	331.47 m/sec.	+1.83
	1.0	332.05	331.67	329.76	+2.89
	0.7	328.55	331.60	327.85	+0.70
	0.5	325.05	331.56	324.37	+0.68
	0.4	319.90	331.51	320.46	−0.56
	0.3	310.68	331.42	311.86	−1.18
	0.2	288.49	331.28	287.64	+0.85
	0.1	161.75	330.75	158.59	+3.16
49.58 kc	3.0	331.88	331.74	331.50	+0.38
	1.0	331.70	331.68	329.78	+1.92
	0.7	328.29	331.62	327.92	+0.37
	0.5	324.70	331.59	324.47	+0.23
	0.4	319.55	331.55	320.66	−1.11
	0.3	310.35	331.48	312.20	−1.85
	0.2	288.28	331.33	288.14	+0.14
	0.1	161.53	330.91	159.25	+2.28
51.5 kc	3.0	331.82	331.74	331.50	+0.32
	1.0	331.68	331.69	329.81	+1.87
	0.7	328.26	331.65	327.99	+0.27
	0.5	324.66	331.62	324.47	+0.19
	0.4	319.50	331.58	320.57	−1.09
	0.3	310.29	331.52	312.20	−1.91
	0.2	288.24	331.39	288.14	+0.10
	0.1	161.51	331.01	159.58	+1.93
69.4 kc	3.0	331.66	331.75	331.50	+0.16
	1.0	331.46	331.70	329.85	+1.61
	0.7	328.07	331.66	328.05	+0.02
	0.5	324.48	331.64	324.54	−0.06
	0.4	319.30	331.61	320.69	−1.39
	0.3	310.08	331.55	312.26	−2.18
	0.2	288.11	331.44	288.31	−0.21
	0.1	161.40	331.11	159.71	+1.69
100 kc	3.0	331.40	331.75	331.50	−0.10
	1.0	331.20	331.71	329.88	+1.32
	0.7	327.90	331.69	328.12	−0.32
	0.5	324.32	331.67	324.64	−0.32
	0.4	319.17	331.63	320.79	−1.62
	0.3	309.90	331.57	312.36	−2.46
	0.2	288.02	331.50	288.47	−0.45
	0.1	161.30	331.21	160.08	+1.22
145 kc	3.0	331.35	331.75	331.54	−0.19
	1.0	331.10	331.72	329.91	+1.19
	0.7	327.80	331.70	328.12	−0.42
	0.5	324.18	331.68	324.70	−0.52
	0.4	319.02	331.65	320.85	−1.83
	0.3	309.78	331.62	312.49	−2.71
	0.2	288.01	331.54	288.64	−0.63
	0.1	161.28	331.31	160.38	+0.90
200 kc	3.0	331.20	331.76	331.54	−0.34
	1.0	331.00	331.73	329.95	+1.05
	0.7	327.65	331.71	328.15	−0.50
	0.5	324.13	331.69	324.74	−0.61
	0.4	318.97	331.67	320.92	−1.95
	0.3	309.73	331.64	312.56	−2.83
	0.2	287.98	331.57	288.97	−0.99
	0.1	161.25	331.38	160.58	+0.67

in which the wall offers greater resistance to the motion of the air in contact with it.[28]

When the relation between the wave-length at the frequencies used, 30 kc to 200 kc, and the diameter of the tubes (smallest 0.1 cm diameter) is taken into consideration as compared with the frequency and diameter of tubes at audible frequency, it is found that the diameters of the tubes used are relatively large. The above experimental results, Table I, show that in the ultrasonic region (30 kc to 200 kc) the frequency has but little effect on the velocity of sound. The percent of difference between the velocity of sound at any frequency and the average velocity of sound for all frequencies for a given tube diameter is less than 0.2 of one percent.

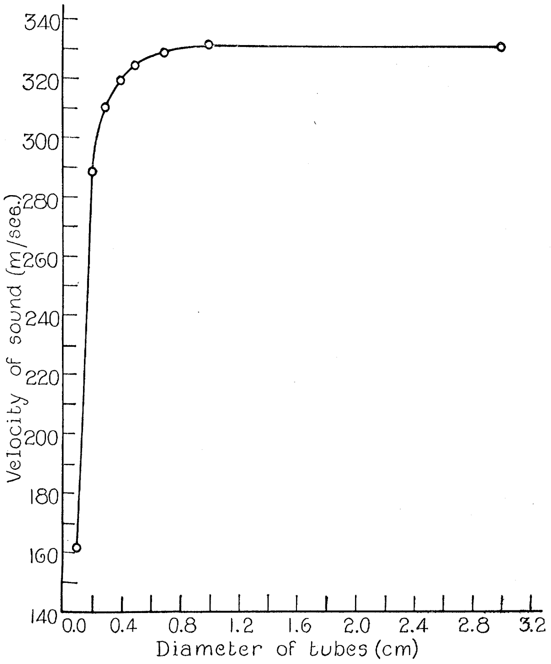

Fig. 2. Relation between the velocity of sound in tubes and the diameter of tubes.

The effect of viscosity and heat conduction in reducing the sound energy is greatly increased when a gaseous medium is brought into contact with a large surface area of a solid or a liquid. Lord Rayleigh[29] shows that the extent to which the 'viscous' dragging effect penetrates into the gas is proportional to $\nu/D(N)^{1/2}$. The effect of viscosity in modifying the motion is dependent on the ratio of $\mu/\rho=\nu$ rather than on μ, the viscosity, alone. Kirchhoff pointed out that the loss of energy due to heat conduction in a medium cannot be neglected. If the diameter if the tube is large compared with the wave-

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[27] A. B. Wood, A Textbook of Sound, p. 323.

[28] Stewart-Lindsay, Acoustics, p. 68.

[29] Lord Rayleigh, Sound **2**, p. 319.

length the effect of viscosity will be small while if the tube diameter is small compared with the wave-length the effect will be much greater.

Foley[17] has shown that the velocity of sound in a pipe depends on the intensity of the wave as it enters the pipe rather than upon the intensity of the sound source. The source of sound being outside the tubes the amount of energy entering the tubes, placed the same distance from the sound source, would be directly proportional to the square of the diameter of the tube.

TABLE II.

Observer	Frequency	Diameter of tube	Velocity experimental at 0°C	Velocity from Eq. (3) at 0°C	Difference exp−Eq. (3)
Low[30]	256	0.93 cm	320.60 m/sec.	325.80 m/sec.	−5.20
		1.71	325.24	327.19	−1.95
		2.80	327.29	327.56	+0.27
Seebeck[7]	320	0.34	317.26	309.38	+8.90
		0.90	328.02	326.03	+1.99
		1.75	329.24	327.46	+1.78
Schulze[10]	384	0.101	258.00	133.44	+124.56
		0.151	282.00	235.56	+46.44
Schulze[10]	512	0.101	265.00	139.34	+125.66
Seebeck[7]	512	0.34	322.98	310.39	+12.79
Seebeck[7]	512	0.90	328.44	326.76	+1.68
Seebeck[7]	512	1.75	330.92	328.29	+2.63
Low[30]	512	0.93	323.60	326.86	−3.26
Low[30]	512	1.71	326.70	328.25	−1.55
Low[30]	512	2.80	328.33	328.62	−0.29
Muller[8]	903	1.55	327.30	328.88	−1.58
Low[30]	1023	0.93	325.29	327.69	−2.40
		1.71	327.80	329.04	−1.24
		2.80	328.68	329.45	−0.77
Muller[8]	2482	1.55	330.20	329.75	+0.45
Kundt[14]	3700	0.35	318.88	317.07	+1.81
		0.65	327.14	326.69	+0.45
		1.30	329.88	329.65	+0.23
Kundt[14]	5550	0.65	328.14	326.89	+1.25
		1.30	330.87	329.85	+1.02

In order to include the effect of viscosity, heat conduction and intensity for all relations of wave-length to tube diameters, an equation of the form

$$V^1 = V_0(1 - C/D^n - K/D(N)^{1/2}) \tag{2}$$

was used with V_0 as the velocity of sound at 0°C, V^1 as the velocity of sound in a tube of diameter D and at a frequency N, and C, K and n were constants. The value of C, K and n were determined and found to be $C = 0.00512$, $K = 0.174$, and $n = 2$. It was found that the best value for the velocity of sound at audible frequencies and 0°C is 331.45 m/sec.,[31] while for ultrasonic frequen-

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[30] Low, Ann. d. Physik **52,** 841 (1894).

[31] A. L. Foley, International Critical Tables **6.**

cies and 0°C it appears to be 332.08 m/sec.[32] The value for V_0 was taken as the average of these two values or 331.77 m/sec. Eq. (2) now becomes

$$V^1 = V_0(1 - 0.00512/D^2 - 0.174/D(N)^{1/2}). \tag{3}$$

At the various ultrasonic frequencies the velocity of sound in tubes of different diameters was calculated by use of Eq. (3) and the results thus obtained were compared with my experimental values as shown in Table I. At various audible frequencies the velocity of sound in tubes of different diameters was calculated by use of Eq. (3). The results were compared with the experimental values obtained by other experimenters using glass tubes. Table II shows the comparison. From Table I we see that for ultrasonic frequencies, the experimental and calculated values for the velocity of sound agree very well, while from Table II for audible frequencies, the experimental and calculated values for the velocity of sound compare fairly well except for the smallest tubes, 0.101 and 0.151 cm diameters. As the experimental and calculated values for the velocity of sound for these tubes differ widely, I concluded to redetermine them at the same frequencies and in tubes of approximately the same diameter as were used by Schulze.

My method was as follows: the sound produced by a constant frequency phonograph record was amplified and then transmitted to a telephone receiver held close to the end of the tube. In making these measurements the same general plan was followed as that used for the measurements at ultrasonic frequency. The results (the average of 20 measurements for each tube and frequency) thus obtained are shown in Table III.

TABLE III.

Frequency	Diameter of tube	Velocity experimental at 0°C	Velocity from Eq. (3) at 0°C	Difference exp−Eq. (3)
384	0.105 cm	151.04 m/sec.	149.30 m/sec.	+1.74
	0.160	250.72	247.17	+3.55
512	0.105	153.60	152.28	+1.32
	0.160	251.80	249.48	+2.31

It will be seen by Table III that my values are less than those obtained by Schulze but that they agree fairly well with those calculated by use of Eq. (3).

I wish to express my appreciation to Dr. A. L. Foley for his very valuable suggestions, and also to Dr. R. R. Ramsey for his interest in the work.

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[32] C. D. Reid, Phys. Rev. **37**, 1147 (1931).