OPERATING CHARACTERISTICS OF THE ELECTRO-OPTICAL SHUTTER

By Harold W. Washburn
University of California, Berkeley

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Abstract

The electro-optical shutter is being employed at the University of California in the study of the electrical breakdown of gases and liquids. In these studies it is desirable to know the time it takes the shutter to close. A calculation of this time can be made from the electrical constants of the circuit and a knowledge of the rate at which the voltage drops across the spark gap. For some of the experimental conditions it is sufficiently accurate to base these calculations on an electrical circuit which replaces the actual distributed constants by lumped constants. In other cases however the error involved by this assumption is too great. It is the purpose of this paper to present an accurate solution of the electrical circuit taking into consideration that the constants are distributed, and by means of this solution to bring out the following important facts: (1) For relatively large distributed electrical capacities of the Kerr-cell leads the rate of closing of the shutter is greater than indicated by the lumped constant solution. (2) The rate of closing is materially increased by using leads separated only by a sheet of mica instead of spacing them farther apart in air. For completeness the results of a few experimental observations are also given and compared with results obtained by calculation.

Introduction

This paper is concerned with an accurate mathematical solution of the Abraham and Lemoine[1] type of electro-optical shutter. Until this solution was made we were using an approximate solution in which it was assumed that the electrical constants were lumped instead of distributed. The accurate solution which takes into consideration that the constants are distributed was originally made in order to determine the type and magnitude of errors involved in the approximate solution. However, it accomplished more than this. Calculations demonstrate the important fact that it is possible materially to speed up the rate of closing of the shutter by increasing the distributed capacity of the leads to the Kerr-cell. Moreover one is enabled to trace out the travelling waves of current and voltage and thereby obtain a true picture of the operation of the shutter.

Arrangement of Apparatus

A diagram of the arrangement of the apparatus is shown in Fig. 1. It will be noted that there is an optical system and an electrical circuit. In the optical system the two Nicol prisms are crossed and the direction of the electrical field in the Kerr-cell is placed at 45 degrees to the plane of polarization of the light passing through the first Nicol prism. This means that light can

[1] Abraham and Lemoine, C. R. **129**, 206 (1899).

pass from the spark gap through the prisms to the eye only when a voltage is on the Kerr-cell. The light from the spark which will be observed therefore is that light which passes through the Kerr-cell before the cell has discharged to an extent which effectively closes the shutter.

In the electrical system C_1 and R_1 are of such a size as to make the time it takes the voltage to reach its final value on the gap of the order of one second. The distributed resistance between C_2 and the spark gap is large enough to make the discharge of C_2 through the gap aperiodic. The distributed resistance between the spark gap and Kerr-cell is made large enough so that the second and all succeeding oscillations of voltage across the Kerr-cell are so small that the light transmitted is too feeble to be observed.

Operation of the Kerr-Cell Circuit

The solution of the electrical circuit shown in Fig. 1 is extremely complicated but fortunately the arrangement of the experimental set-up allows an

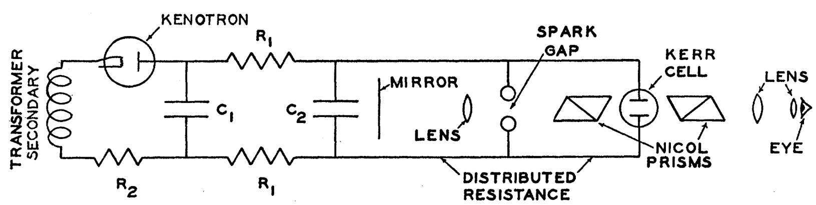


Fig. 1. Diagram of electrical and optical circuits.

assumption to be made which greatly simplifies the circuit. In order to avoid induced disturbances in the Kerr-cell circuit the coupling between this part of the circuit and the remainder was made as small as possible. It will therefore be assumed that this coupling is zero. This means that the circuit reduces to that shown in Fig 2. The solution given here will be for the voltage dropping instantaneously. If the solution is desired for any other variation of vol-

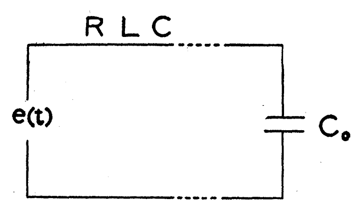


Fig. 2. Kerr cell circuit with spark-gap replaced by $e(t)$.

tage with time it can be obtained by applying the superposition theorem[2] to the solution for instantaneous voltage drop.

In solving this problem the following artifice is used.

$$e_{c_0}(t) = E_0 - e_{c_0}'(t) \tag{1}$$

where $e_{c_0}(t)$ is the voltage on the Kerr-cell, as a function of time, resulting from closing switch in the circuit shown in Fig. 3; E_0 is the initial voltage on

[2] See Operational Circuit Analysis by V. Bush, Page 125.

the Kerr-cell (Fig. 3); and $e_{c_0}'(t)$ is the voltage on the Kerr-cell as a function of time resulting from closing switch in the circuit shown in Fig. 4. It is only necessary therefore to obtain a solution for a voltage applied to the circuit as shown in Fig. 4.

In the solution which follows, the operational method will be used.[3] The plan is briefly: (1) Set up steady state alternating current solution. (2) Replace $j\omega$ in this expression by the operator p. For continuous functions

$$pf(t) = \frac{d}{dt}f(t); \quad \frac{1}{p}f(t) = \int_0^t f(t)dt, \text{ etc.}$$

(3) Carry out the indicated operations according to the rules of operational calculus.

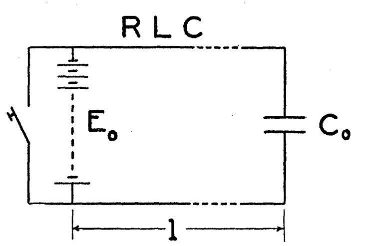


Fig. 3. Schematic Kerr cell circuit representing an instantaneous voltage-drop across spark-gap.

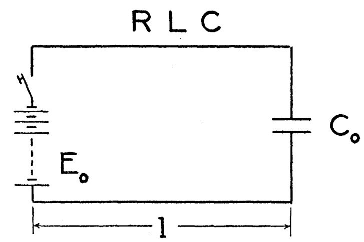


Fig. 4. Schematic Kerr cell circuit representing an instantaneous voltage rise across spark-gap.

Now proceeding in accordance with this plan: the relation between the voltage E_0' and E_{c_0}' for steady state alternating current is given by

$$E_0' = E_{c_0}' \cosh (ZY)^{1/2}l + I_{c_0}'\left(\frac{Z}{Y}\right)^{1/2} \sinh (ZY)^{1/2}l. \tag{2}$$

Where E_0' = applied a.c. voltage

I_0' and E_0' = alternating current and voltage at C_0.

R = resistance per unit length of line

L = inductance per unit length of line

C = capacitance per unit length of line

$\omega/2\pi$ = frequency of E_0'

The current through the condenser may be expressed in terms of the condenser voltage and its impedance:

$$I_{c_0}' = j\omega C_0 E_{c_0}'. \tag{3}$$

Substituting this in Eq. (1) and solving for E_{c_0}

$$E_{c_0}' = \frac{E_0'}{\cosh (ZY)^{1/2}l + j\omega C_0\left(\frac{Z}{Y}\right)^{1/2} \sinh (ZY)^{1/2}l}. \tag{4}$$

[3] See Operational Circuit Analysis by V. Bush, and Heaviside's Operational Calculus by E. J. Berg.

Replacing $j\omega$ in this expression by the operator p and expressing E_0' as a suddenly applied voltage:

$$e_{c_0}' = \frac{E_0}{\cosh\left((R+Lp)pCl^2\right)^{1/2} + pC_0\left(\frac{R+Lp}{pC}\right)^{1/2}\sinh\left((R+Lp)pCl^2\right)^{1/2}}\,1 \quad (5)$$

where 1 represents a function which is zero for all negative values of time and which is unity for all positive values of time.

Rearranging the expressions under the radicals and placing $R/2L=\sigma$ the following expression results.

$$e_{c_0}' = \frac{E_0 1}{\cosh\left([(p+\sigma)^2-\sigma^2]CLl^2\right)^{1/2}+\frac{C_0}{C}(LC)^{1/2}((p+\sigma)^2-\sigma^2)^{1/2}\sinh\left([(p+\sigma)^2-\sigma^2]CLl^2\right)^{1/2}} \quad (5')$$

This may be simplified by "shifting" by means of the operational expression

$$\frac{1}{Z(p)}1 = \epsilon^{-bt}\frac{p}{p-b}\,\frac{1}{Z(p-b)}1 \quad (6)$$

or

$$e_{c_0}' = \frac{\epsilon^{-\sigma t}\frac{p}{p-\sigma}E_0 1}{\cosh\left(CLl^2(p^2-\sigma^2)\right)^{1/2}+\frac{C_0}{C}(LC)^{1/2}(p^2-\sigma^2)^{1/2}\sinh\left(CLl^2(p^2-\sigma^2)\right)^{1/2}}.$$

Now the numerator and denominator of this expression will be treated separately and then combined by means of the superposition theorem.[2]

The result of the numerator operating on the unit function is known.

$$\frac{p}{p-\sigma}1 = \epsilon^{\sigma t}. \quad (7)$$

The result of the denominator operating on the unit function may be obtained with the aid of the expansion theorem:

$$\frac{1}{Z(p)}1 = \frac{1}{Z(0)} + \sum_{p=p_1,p_2\cdots}\frac{\epsilon^{pt}}{p\frac{dZ(p)}{dp}} \quad (8)$$

where $Z(0)=Z(p)$ with p placed equal to zero.

p_1, p_2 etc, are the roots of the equation $Z(p)=0$.

$$Z(p) = \cosh\left(CLl^2(p^2-\sigma^2)\right)^{1/2}+\frac{C_0}{C}(LC)^{1/2}(p^2-\sigma^2)^{1/2}\sinh\left(CLl^2(p^2-\sigma^2)\right)^{1/2} \quad (9)$$

$$Z(0) = \cos\left(CLl^2\sigma^2\right)^{1/2}-\frac{C_0}{C}(LC\sigma^2)^{1/2}\sin\left(CLl^2\sigma^2\right)^{1/2}. \quad (10)$$

In order to make the roots of $Z(p)=0$ easier to calculate place $p=jq$.

$$Z(p) = Z(jq) = \cos\,(CLl^2(q^2+\sigma^2))^{1/2} - \frac{C_0}{C}(LC)^{1/2}(q^2+\sigma^2)^{1/2}\sin\,(CLl^2(q^2+\sigma^2))^{1/2} = 0 \quad (11)$$

or

$$(CLl^2(q^2+\sigma^2))^{1/2}\tan\,(CLl^2(q^2+\sigma^2))^{1/2} = \frac{Cl}{C_0}\cdot \quad (12)$$

The values of q which are the roots of this equation are to be substituted in the final expression for the voltage e_{c_0}. It will be noted that there are an infinite number of roots and that they are in pairs of plus and minus an imaginary quantity. This means that Eq. (8) may be written as

$$\frac{1}{Z(p)}1 = \frac{1}{Z(0)} + \sum_{q=q_1,q_2\cdots}\frac{2\cos qt}{p\dfrac{dZ(p)}{dp}}\cdot \quad (8')$$

The expression $d\,Z(p)/dp$ will now be obtained

$$\frac{dZ(p)}{dp} = \frac{CLl^2\sinh\,(CLl^2(p^2-\sigma^2))^{1/2}}{(CLl^2(p^2-\sigma^2))^{1/2}} + \frac{\dfrac{C_0}{C}(LC)^{1/2}(p^2-\sigma^2)^{1/2}(CLl^2p)}{(CLl^2(p^2-\sigma^2))^{1/2}}\cosh\,(CLl^2(p^2-\sigma^2)) + \frac{\dfrac{C_0}{C}(LC)^{1/2}p}{(p^2-\sigma^2)^{1/2}}\sinh\,(CLl^2(p^2-\sigma^2))^{1/2}. \quad (13)$$

This may be simplified by placing $p=jq$ and then substituting Eq. (11) in Eq. (13). The result is

$$p\frac{dZ(p)}{dp} = -\frac{q^2\left(\dfrac{Cl}{C_0}+1+C_0Ll(q^2+\sigma^2)\right)}{q^2+\sigma^2}\cos\,(CLl^2(q^2+\sigma^2))^{1/2}. \quad (14)$$

Now all the expressions to substitute in Eq. (8′) have been found and it remains to apply the superposition theorem to Eqs. (7) and (8′). The form of superposition theorem most convenient in this case is

$$e(t) = \frac{d}{dt}\int_0^t A_1(t-\lambda)A_2(\lambda)d\lambda \quad (15)$$

where

$$A_1(t) = \frac{1}{Z_1(p)} 1$$

$$A_2(t) = \frac{1}{Z_2(p)} 1$$

$$e(t) = \frac{1}{Z_1(p)} \cdot \frac{1}{Z_2(p)} 1$$

$$e(t) = e_{c_0}' = E_0 \epsilon^{-\sigma t} \frac{d}{dt} \int_0^t \epsilon^{\sigma(t-\lambda)} \left[\frac{1}{Z(0)} - \sum \frac{2\cos q\lambda}{p \dfrac{dZ(p)}{dp}} \right] d\lambda. \tag{16}$$

Performing the integration and differentiation this becomes:

$$e_{c_0}' = \frac{E_0}{Z(0)} - E_0 \sum \frac{2}{p \dfrac{dZ(p)}{dp}} \frac{\sigma^2}{\sigma^2 + q^2} - \epsilon^{-\sigma t} E_0 \sum \frac{2q \cos\left(qt - \tan^{-1} \dfrac{\sigma}{q}\right)}{(\sigma^2 + q^2)^{1/2} p \dfrac{dZ(p)}{dp}} \cdot \tag{17}$$

From physical considerations it is known that for $t = \infty$, $e_{c_0}' = E_0$. The constant term in Eq. (17) is therefore E_0 and Eq. (17) becomes:

$$e_{c_0}' = E_0 \left\{ 1 - \epsilon^{-\sigma t} \sum \frac{2q \cos\left(qt - \tan^{-1} \dfrac{\sigma}{q}\right)}{(\sigma^2 + q^2)^{1/2} p \dfrac{dZ(p)}{dp}} \right\}. \tag{18}$$

Now substituting Eq. (14) in Eq. (18) and substituting the result in Eq. (1) the expression desired is obtained.

$$e_{c_0} = \epsilon^{-\sigma t} \sum \frac{2(\sigma^2 + q^2)^{1/2} \cos\left(qt - \tan^{-1} \dfrac{\sigma}{q}\right)}{q\left(\dfrac{Cl}{C_0} + 1 + C_0 Ll(q^2 + \sigma^2)\right) \cos\left(CLl^2(q^2 + \sigma^2)\right)^{1/2}} \tag{19}$$

where q is determined from Eq. (12).

It is rather difficult to determine by inspection from this expression just what is the effect of varying the different factors and so a few examples will be given. Before this is done, however, it will be instructive to note the similarity between this expression and the expression derived when the constants are considered as lumped. The solution of the circuit shown in Fig. 5 is

$$e_{c_0} = \frac{E}{(1 - \sigma^2 LC_0)^{1/2}} \epsilon^{-\sigma t} \cos\left[\left(\frac{1}{LC_0} - \sigma^2 \right)^{1/2} t - \tan^{-1} \frac{\sigma}{\left(\dfrac{1}{LC_0} - \sigma^2 \right)^{1/2}} \right]. \tag{20}$$

As the ratio of the total distributed capacity of the leads to the capacity of the Kerr cell approaches zero the first term of Eq. (19) approaches Eq. (20) and all other terms approach zero. This means that when the ratio Cl/C_0 is small enough so that $A \tan A = Cl/C_0$ (see Eq. (12)) may be closely enough approximated by $A^2 = Cl/C_0$ the lumped constant solution is a good approximation. The errors involved in this lumped constant solution are; (1) An error in phase, frequency, and magnitude; of the same order of magnitude as the error involved in assuming $A \tan A = A^2$; and (2) An error, due to neglecting higher harmonics, of an amplitude roughly $Cl/5C_0$ times the amplitude of the fundamental.

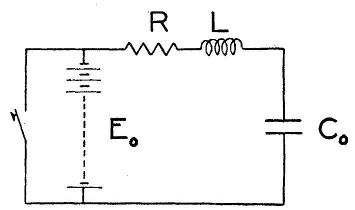


Fig. 5. Schematic Kerr cell circuit with the distributed constants replaced by lumped constants.

To bring out more clearly the characteristics of this circuit and the errors involved in the approximate solution three examples have been selected:

(1) Leads from spark gap to Kerr cell 90 cm long, spaced 15 cm in air, having a total resistance of 150 ohms (both leads).

(2) Leads 90 cm long, spaced 1.3 mm, insulated from each other with mica, a resistance of 150 ohms.

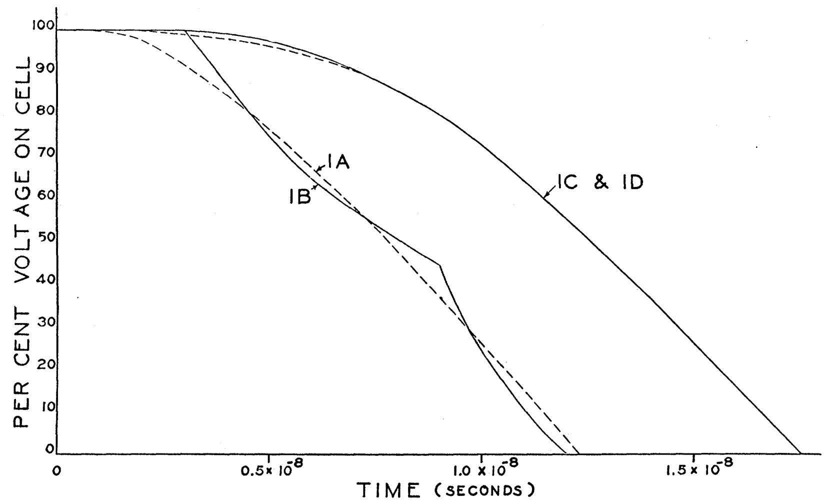
(3) Leads 37 cm long (this length is such that the first wave of voltage arrives at the Kerr cell at the same time that it would if the leads were 90 cm long and had air between them instead of mica), spaced 1.3 mm, insulated from each other with mica, a resistance of 61 ohms (same resistance per cm of leads as in cases (1) and (2).

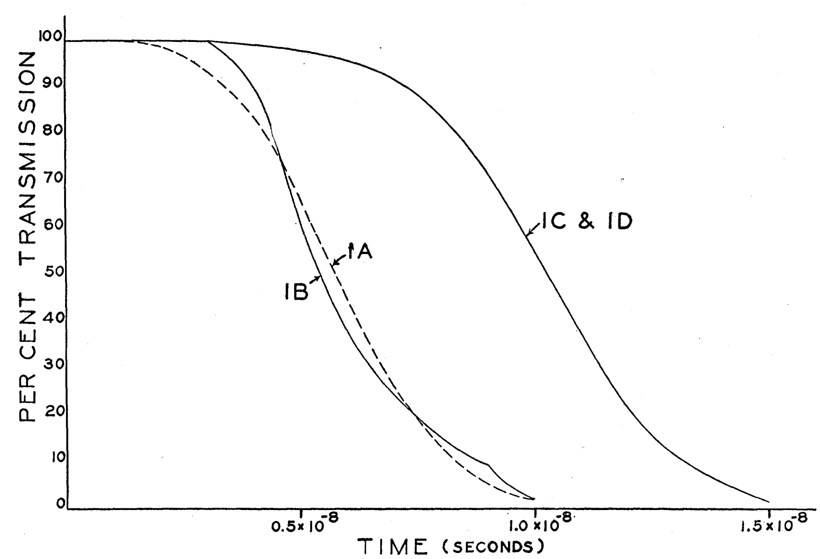
Calculations were made of the voltage on the cell (as a percent, of the break-down voltage of the gap) and the percent transmission[4] of the cell as a function of time for two rates of voltage drop on the spark gap (1). Instantaneous drop of voltage to zero and (2) voltage dropping to zero linearly with time in 10^{-8} seconds.[5] This latter case corresponds roughly to that which we have observed in air with gaps of from 3 to 6 mm. The results of these calculations are given by the curves in Figs. 6 to 10.

[4] The percent, transmission was calculated by the relations $I = \sin^2 \delta/2$, $\delta = 2\pi b l E^2$, (where 100 percent transmission was taken as the transmission obtained when $\delta = \pi$ radians, I = transmission, b and l are constants of the Kerr cell, and E = voltage gradient in the cell) and making the assumption that the Kerr cell employed has its maximum opening at the breakdown voltage. If this later condition is not obtained the curves would be somewhat changed but would have in general the same appearance.

[5] The curves for the voltage dropping in 10^{-8} seconds were obtained by applying the superposition theorem to the curves for instantaneous voltage drop. As accuracy was not desired much labor was saved by employing a step by step method which assumed the voltage to fall in five equal instantaneous drops spaced 0.2×10^{-8} seconds apart.

In Figs. 6 and 7 curves 1*a* and 1*b* are for the conditions (1) as stated above and instantaneous voltage drop. Curve 1*a* was computed by the lumped constant equation and 1*b* by the distributed constant equation. For these conditions the difference between these two solutions is sufficiently

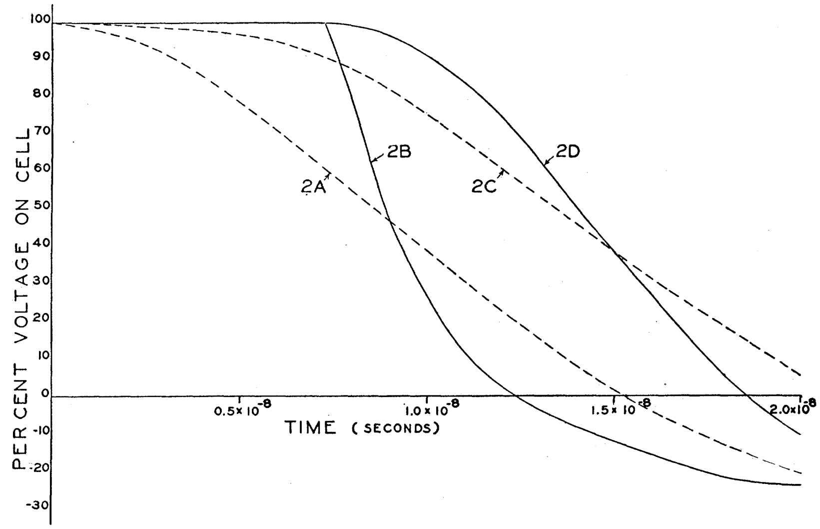


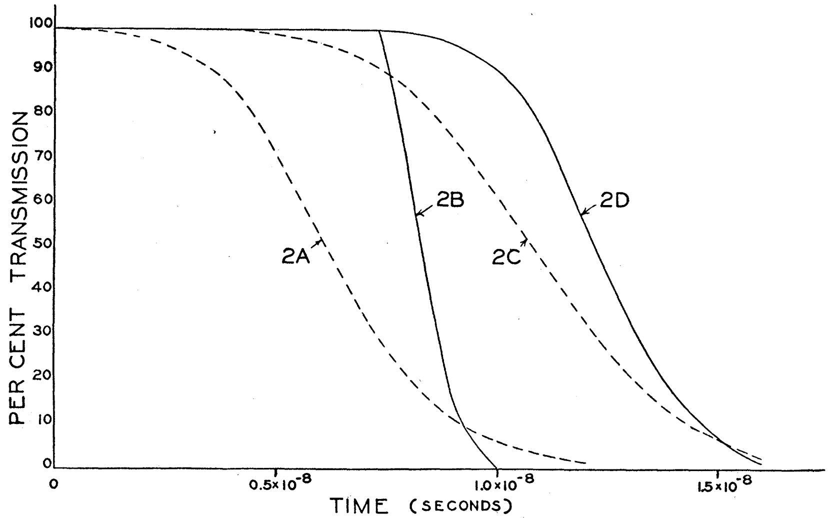


Figs. 6 and 7. Calculated electro-optical shutter performance curves. For 90 cm lead-spaced 15 cm apart in air. 1A, for instantaneous voltage drop across gap—lumped constant solution. 1B, for instantanious voltage drop across gap—distributed constant solution. 1C, for voltage across gap dropping in 10^{-8} seconds—lumped constant solution. 1D, for voltage across gap dropping in 10^{-8} seconds—distributed constant solution.

small to be neglected for our purposes. The breaks in the distributed constant curve at 0.3×10^{-8} and 0.9×10^{-8} seconds are due to the arrival of the first and second waves of current at the Kerr cell.

Curve 1C is for conditions (1) and the voltage dropping in 10^{-8} seconds. The lumped constant and distributed solutions differ so little in this case that they cannot be distinguished on the graph. This is due to the smoothing out of the small irregularities resulting from the less steep wave front associated





Figs. 8 and 9. Calculated electro-optical shutter performance curves. For 90 cm leads spaced 1.3 mm apart on mica. 2A, for instantaneous voltage drop across gap—lumped constant solution. 2B, for instantaneous voltage drop across gap—distributed constant solution. 2C, for voltage across gap dropping in 10^{-8} seconds—lumped constant solution. 2D, for voltage across gap dropping in 10^{-8} seconds—distributed constant solutions.

with the slower rate of voltage drop across the gap. It may be concluded that for leads in air up to one meter in length the lumped constant solution gives a very close approximation of the operation of the circuit.

When the current in the wave front is sufficiently large to make the breaks in the voltage-time curves more pronounced the lumped constant solution no longer gives a good approximation. This is brought out in Figs. 8 and 9 which are for conditions (2). The larger current in the wave front here is due to the closer spacing and to the use of mica both of which increase the distributed capacity. The error which would be involved if the time of operation of the shutter were calculated from the lumped constant equation is shown in Fig. 9 where the percent transmission is plotted vs. time. For instantaneous voltage drop these curves give for time of operation (90 to 10 percent transmission) 5.5×10^{-9} seconds for the lumped and 1.6×10^{-9} seconds for the distributed calulations. For the voltage dropping in 10^{-8} seconds these values are 6.7×10^{-9} seconds for the lumped and 4.6×10^{-9} seconds for the distributed calcu-

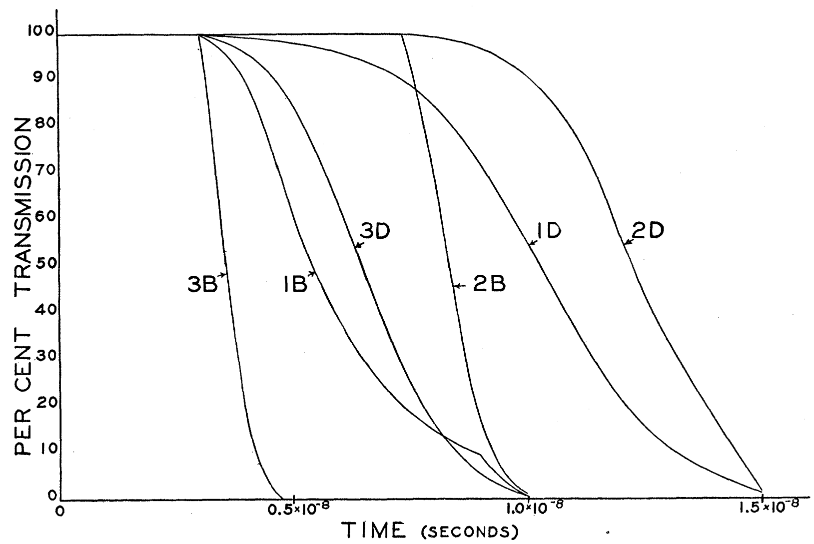


Fig. 10. Calculated electo-optical shutter performance curves. 3B, for 37 cm leads spaced 1.3 mm apart on mica-instantaneous voltage across gap—distributed constants. 3D, for 37 cm leads spaces 1.3 mm apart on mica—voltage across gap dropping in 10^{-8} seconds—distributed constants. 1B, 1D, 2B, and 2D are the same as in Figs. 6, 7, 8, and 9.

lations. It will be seen that the error involved is less for the slower change in voltage but still too large to be neglected. It is interesting to note the extremely short time of operation which is possible even when the voltage drop takes as much as 10^{-8} seconds. That is, if the voltage drops to zero in 10^{-8} seconds the shutter closes in less than half this time. This is due to the fact that the transmission of the cell drops to two percent, when the voltage has dropped to thirty percent, of its initial value.

In order better to compare the operation with the leads in air and the leads on mica, curves were calculated for leads on mica of such a length that the shutter starts to close at the same instant as for leads in air 90 cm long (conditions given under 3 above). This length is given by $90/k^{1/2}$ where k is the dielectric constant of mica. The results of this calculation are given by

Fig. 10. For instantaneous voltage drop on the gap the time of closing of the shutter (90 to 10 percent transmission) for leads in air is 5×10^{-9} seconds and for leads on mica is 1×10^{-9} seconds. If the gap voltage takes 10^{-8} seconds to drop this value is 5.9×10^{-9} for leads in air and 3.8×10^{-9} seconds for the leads on mica. *It is seen that the placing of leads on mica materially shortens the time of closing of the shutter especially if the time of voltage drop across the gap is very small.*

The curves for the 90 cm leads on mica are also drawn in Fig. 10 to bring out the important fact that *the time that the shutter remains open may be increased without appreciably lengthening the time required for the shutter to close.* That is, for the voltage on the gap dropping in 10^{-8} seconds the 37 cm leads give an open time (90 percent transmission or above) of 4.7×10^{-9} seconds and a closing time (90 percent transmission to 10 percent transmission) of 3.8×10^{-9} seconds. The 90 cm leads give an open time of 10.1×10^{-9} seconds and a closing time of 4.3×10^{-9} seconds. The open time is more than doubled while the closing time is increased only 13 percent. This characteristic of the shutter makes possible the observation of various stages of the breakdown with approximately the same sharpness of cut-off.

Experimental Work

It is a simple matter experimentally to measure differences of the open time of the shutter.[6] Referring to Fig. 1., it is seen that two images of the spark will be observed. One image is formed by the light which passes directly from the spark through the Nicol prisms and Kerr cell to the observer. The other is formed by light which first travels to the mirror and is then reflected back through the prisms and Kerr cell to the observer. Different stages of the progression of the spark may be observed in the reflected image by merely moving the mirror nearer to or farther from the spark gap. The difference in time that it takes the spark to progress to these different observed stages is then merely twice the difference of the distance of the mirror from the spark gap divided by the velocity of light. If now it is desired to determine the difference of open time of the shutter for different electrical constants of the Kerr cell spark gap circuit, it is merely necessary to adjust the mirror for each condition of the electrical circuit so that the same state of progression of the spark is observed. The difference of position of the mirror will then give the difference in open time of the shutter for the different conditions of the electrical circuit. This of course assumes that the change in electrical constants does not alter the rate of progression of the spark. The extent to which this is true must be determined experimentally as will presently be shown.

It was desired to determine experimentally the effect of the Kerr cell capacity on the speed of operation of the shutter. To do this a Kerr cell was made which had a constant Kerr effect with a variable capacity. That is, it

[6] The open time of the shutter can also be experimentally determined by a method which makes use of the effect on the brightness of the spark upon receiving the reflected wave from the supply condenser. For an explanation of this method see a paper by Mr. Frank Dunnington in the Phys. Rev. **38**, 1506 (1931).

had two fixed plates between which the light passed. In the same cell and connected in parallel (the connecting leads being very short 1 cm) were a fixed and a movable plate by means of which the capacity of the cell was varied.

In order to determine to a fair degree of accuracy the state of progression of the spark the electrodes were chosen so that the spark formed with a narrow bright pencil of light starting from the cathode and moving nearly all the way across the gap before meeting the anode pencil. The state of progression could then be determined by merely measuring with the aid of cross-hairs the fraction of the gap which this pencil had travelled. The arrangement of electrodes which gives this condition is a sphere cathode and a plane anode. The cathode used was a sphere 0.5 cm in diameter, the anode a disk 2 cm in diameter, and the spacing was 0.4 cm.

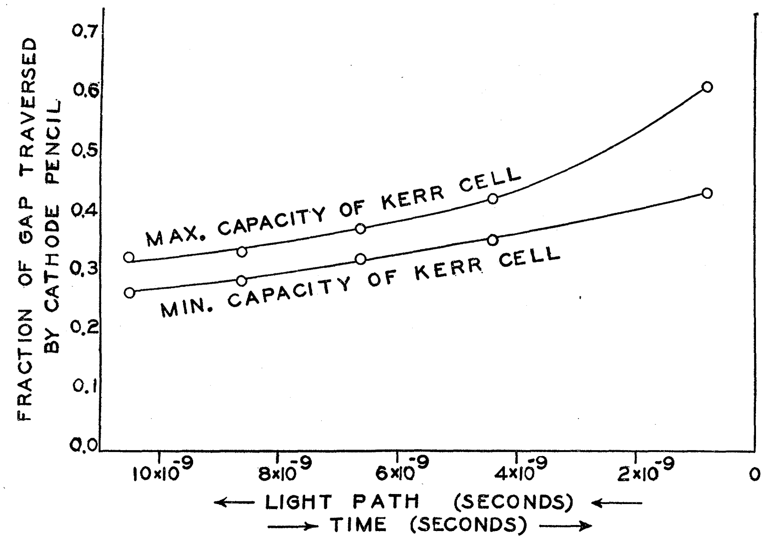


Fig. 11. Experimental curves of progression of spark vs. time.

Three sets of leads were used between the Kerr cell and gap:

(a) 39 cm long 37 ohms resistance (each wire) spaced 15 cm in air.

(b) 39 cm long 84 ohms resistance (each wire) spaced 15 cm in air.

(c) 74 cm long 64 ohms resistance (each wire) spaced 15 cm in air.

Curves of percent, of the gap traversed by the cathode pencil vs. length of light path from gap to Kerr cell (which was varied by means of the mirror) were taken for the minimum and for the maximum capacity of the Kerr cell. The curves thus obtained for the leads *b* are shown in Fig. 11. The abscissa for these curves is plotted from right to left in seconds instead of cm, i.e., the length of light path divided by the velocity of light. Since the longer light paths correspond to earlier times in the formation of the spark the curves from left to right give the progression of the cathode pencil vs. time. If the change in capacity in the Kerr cell affected the rate of progression of the cathode pencil across the gap the slope of these two curves for a given ordinate would be different. But it is found that if the curves are shifted horizontally they coincide throughout, that is, within the limits of accuracy of the measurements. The change in cell capacity therefore did not measureably affect the rate of formation of the spark in the range for which these curves were taken. Theoretical considerations show us that if the spark were to be effected by the cell capacity it would be effected in the range here shown. These considerations are as follows: The position of the cathode pencil at the instant the shutter closes is given by zero light path or by the zero of the abscissa in Fig. 9. Calculations show that at about 10^{-8} seconds previous to this the voltage across the Kerr cell is 95 percent, of its initial value. Now the only way in which the Kerr cell can affect the spark is by the height of the reflected waves which is in turn determined by the voltage on the cell. Since for times earlier than are shown on the curves this voltage is between 95 and 100 percent, for both minimum and maximum capacity adjustments the spark could not be appreciably affected by a change of cell capacity at these earlier times. This is still more certain when it is realized that the energy fed into the spark by the leads from the supply condenser in these earlier times is about three times as great as that fed in by the Kerr cell circuit.

Since it may now be concluded that the rate of formation of the cathode pencil is the same for both curves in Fig. 9. the difference on the abscissa of the two curves for any given ordinate gives the difference in open time of the shutter resulting from the change in cell capacity.

In order to ascertain whether the Kerr cell and circuit were operating in agreement with the equations developed, the above three differences of time were calculated. The variable Kerr cell had a minimum capacity of 47×10^{-12} farads. This large minimum capacity was made necessary by the variable feature of the cell. For the three sets of leads used in the above measurements the ratio of the total distributed capacity of the leads to the Kerr cell capacity was so small that the lumped constant solution was sufficiently accurate. The next question which was confronted was the time it takes the voltage across the gap to drop. It was known from measurements by other experimenters with the cathode-ray oscillograph that this time was about 10^{-8} seconds.[7] Curves of voltage vs. time were therefore calculated for both instantaneous voltage drop across the gap and for a linear decrease of voltage with time in 10^{-8} seconds. From these curves the difference in open time of the shutter due to the change in cell capacity was determined for: (1) assuming instantaneous voltage drop across the gap, (2) assuming linear voltage drop in 10^{-8} seconds. The result showed that this difference of time for the two cases was the same, or stated better, it changed an amount less than could be distinguished by the experimental measurements. This constancy of the time difference was due to the large capacity of the variable Kerr cell which made the shutter close more slowly than with the regular cell. This situation was very

[7] For a determination of this time with the Kerr cell see article by Mr. Frank Dunnington in Phys. Rev. **38**, 1506 (1931).

fortunate as it made the calculated results entirely indepandant of the way in which the voltage dropped across the gap. This makes possible a mathematical check upon the operation of the Kerr cell and leads alone. Once this agreement is established the calculations may be extended to a case where the rate of voltage drop across the gap has considerable influence and thus determine by trial what the rate of voltage drop is.[7]

The results of the experiments and calculations are given in Table I. It is seen that the results agree within the limits of experimental accuracy. It is unfortunate that the accuracy of the experiment could not have been better.

TABLE I.

Leads	Experimental time difference	Calculated time difference
a	$2.9\times10^{-9}\pm0.5\times10^{-9}$ secs.	2.5×10^{-9} secs.
b	$3.2\times10^{-9}\pm0.5\times10^{-9}$ secs.	2.8×10^{-9} secs.
c	$2.9\times10^{-9}\pm0.5\times10^{-9}$ secs.	3.2×10^{-9} secs.

The percentage error however is not as great as it first appears. The total time open of the cell is about 10^{-8} seconds so that the errors of observation are only about five percent of this value. The remarkable thing is that measurements of such small time intervals in the operation of an optical shutter can even be approximately measured.

At present no quantitative checks have been made on the Kerr cell circuit where the distributed constant solution differs greatly from the lumped constant solution (i.e., when there is a relatively large distributed capacity in the leads). It has however been observed that the cut-off is much sharper when the leads are placed close together and separated by mica. That is, when mica is used the cathode pencil is much more sharply defined at its tip. This is in qualitative agreement with the predictions of the distributed constant solution.

To sum up, it is seen that the accurate solution of the problem of the discharge of the Kerr cell shows that the initial discharge wave causes the cell to close more rapidly than is indicated by the approximate solution involving lumped constants and moreover that the effect of the discharge wave can be enhanced by increasing the distributed capacity along the leads, thereby materially increasing the rate of closing of the shutter.

In conclusion I wish to express my appreciation to Mr. Frank Dunnington for his valuable suggestions and to Professor E. O. Lawrence under whose direction this experimentation is being carried out.