INVESTIGATIONS IN THE FIELD OF THE ULTRA-SHORT ELECTROMAGNETIC WAVES

II. THE NORMAL WAVES AND THE DWARF WAVES

By G. Potapenko*

Norman Bridge Laboratory of Physics, California Institute of Technology

(Received November 23, 1931)

Abstract

The results are presented of an investigation of the production of ultra-short undamped electromagnetic waves by using the method of H. Barkhausen and K. Kurz.

Method of working diagrams. Normal waves and dwarf waves. A method is developed for the graphic representation of the work of generators of ultra-short waves. This method is based on the construction of special "working diagrams." These diagrams define the location of "regions of oscillations," which show the values of the natural periods of the oscillating circuits and the values of the grid potentials at which oscillations are generated. Vacuum tubes can generate two kinds of ultra-short waves. The first kind have a wave-length approximating that computed by Barkhausen's formula $\lambda^2 E_g = d_a^2 10^6$. Their period is nearly equal to the time required for the electrons to move from the filament to the plate and back (normal waves). The second kind of waves are considerably shorter (dwarf waves). Both kinds of waves satisfy the equation $\lambda^2 E_g =$ const. for points on the working diagram where the plate current (the amplitude of the oscillations) has its maximum value.

Complex working diagrams. Dwarf waves of higher orders. Vacuum tubes can have complex working diagrams with a large number of regions of oscillations. In such a case the tube generates different dwarf waves. Their length is two, three and four times shorter than that of the normal waves. Dwarf waves are accordingly divided into waves of the 1st, 2nd, 3rd, etc. orders. The shortest dwarf waves of the 4th order, generated by tubes of the type *R*5, had a wave-length $\lambda = 9.4$ cm. The presence of dwarf waves of higher orders shows that *vacuum tubes can generate oscillations of a frequency considerably greater than the frequency of the electronic oscillations.* Both the normal and dwarf waves belong to the same type of GM-oscillations. Limits were determined within which Barkhausen's formula is applicable. It is shown that the difference in the number of regions of oscillations on the working diagrams depends on the difference in the time required for the electrons to pass in different directions within the tube. The latter depends on the asymmetry in the arrangement of the electrodes.

The nature of dwarf waves. Dwarf waves are oscillations of the circuits within the tube or coupled with the tube which are excited in such a manner that during the time τ it takes for the electrons to pass from the filament to the plate and back, the circuits perform two complete oscillations (dwarf waves of the 1st order), three complete oscillations (dwarf waves of the 2nd order) etc. Thus the wave-lengths are equal to: $\lambda_0 = c_0\tau$ (normal waves), $\lambda_1 = c_0\tau/2$ (dwarf waves of the 1st order), $\lambda_2 = c_0\tau/3$ (dwarf waves of the 2nd order), $\lambda_3 = c_0\tau/4$ (dwarf waves of the 3rt order), etc. Dwarf waves $9.5-18.5$ cm long originate in oscillating circuits, which are inside the tube. The advantages of dwarf waves of higher orders are shown, owing to the possibility of using lower grid potentials, which leads to a greater steadiness in the operation of the tube.

* International Research Fellow.

§1. Introduction

THE present article gives some of the results of an investigation on the production of ultra-short undamped electromagnetic waves. These waves were obtained using the method of H. Barkhausen and K. Kurz by means of a generator which has been described earlier.[1]

§2. The Method of Working Diagrams. Normal Waves and Dwarf Waves

There are two schemes of graphical representation of the work of vacuum tubes generating ultra-short waves by the method of H. Barkhausen and K. Kurz. Both schemes can be used regardless of what oscillating circuit the tube is connected with or of what generator it makes a part. The first method is to represent the work of a tube (or generator) by means of curves showing the relation between the energy of oscillations (or the direct current I_a in the plate circuit of the tube) and the grid potential E_g. The dimensions of the oscillating circuits connected with the tube, their natural periods and the heating current are kept constant. We have already used such curves (I, §8), and called them the *(I_a, E_g)-characteristics* of the generator. The second scheme consists in representing the energy of oscillations or the plate current as a function of the dimensions or (what is the same thing) of the natural periods of the oscillating circuits. The grid voltage and the heating current are kept constant. We can call such curves the *(I_a, L)-characteristics*, where L stands for the dimensions or for the natural periods of the oscillating circuits.

As a matter of fact, the energy of the generated oscillations depends both on the electrode potentials and on the natural periods of the oscillating circuits. Thus, neither of the characteristics gives a complete picture of the work of the tube.

This drawback can be obviated by systematically investigating the work of a tube, varying simultaneously the potentials and the periods of the oscillating circuits. This is easily accomplished. The results can be conveniently and clearly presented by means of special "working diagrams," which we shall presently discuss. This "*method of working diagrams*," as it may be called, has a number of advantages and permits us to give complete characteristics of short wave generators. This method does not preclude the possibility of using the (I_a, E_g)-and (I_a, L)-characteristics. The latter are useful in representing separate details of the working diagrams.

§3.

We shall now investigate the work of the generator described above, using tubes of the type $R5$. This is a tube commonly used in Russia for radio reception. It shows considerable steadiness in operation, not only at normal heating ($I_h=0.68$A; $I_s=5$–7 mA) but even at considerable overheating ($I_s=$ 20–30 mA and higher).

Table I gives the results of measurements of the dependence of the plate

[1] G. Potapenko, Phys. Rev. 39, 625 (1932); this article will be referred to as I.

TABLE I. E_g (volt). R5 (No. 2+No. 4) $I_h=1.36A$.

300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
260	0	0.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0.5	1.2	0	0	0	0	0	0	0	1.8	1.3	0	0	0	0	0	0	0.2	0.1	1.2	0	0	0
220	0	0.3	2.0	2.6	0.9	0	0	0	0	0	0.4	2.6	2.2	1.2	0	0	0	0	0.2	0.2	2.6	2.3	1.7	0.8
200	0	0	0.4	3.5	2.6	1.7	0.6	0	0	0	0.2	3.2	3.1	2.8	2.1	1.1	0	0	0	0.1	2.9	3.2	3.0	2.8
180	0	0	0.1	4.0	3.7	3.2	2.4	1.1	0.1	0	0	0.5	3.2	3.4	3.2	2.8	2.0	1.6	0	0	0.2	0.5	3.5	3.7
160	0	0	0	2.3	4.1	3.9	3.6	2.8	1.7	0.1	0	0	0.4	2.1	3.7	3.7	3.2	3.1	2.1	0.4	0	0.1	0.5	3.9
140	0	0.2	0	0.3	2.3	3.6	4.1	4.0	3.5	2.3	0.7	0	0	0.1	0.5	2.4	3.9	3.9	3.6	3.3	2.3	1.7	0.6	0.5
120	0	0	0.9	0.1	0	0.4	2.1	4.1	4.1	3.8	2.8	1.5	0.2	0	0	0	0.5	2.0	3.8	4.0	3.7	3.4	2.8	2.2
100	0	0	0	1.4	1.6	0.1	0	1.8	3.6	4.2	4.0	3.4	2.3	1.4	0.4	0	0	0	1.9	2.0	3.8	3.9	3.9	3.8
80	0.7	0.2	0.3	0	0	0	0	0	0	2.8	4.0	4.0	3.7	3.2	2.3	1.6	0.9	0.5	0	0	0	0.1	3.3	3.8
60	1.8	1.2	0.7	0.3	0.2	0.1	0.1	0	0	0	0.1	0.1	0.4	3.4	3.2	3.0	2.5	2.1	1.5	1.3	0.9	0.5	0.3	0.2
40	1.6	1.0	0.6	0.6	0.6	0.5	0.4	0.3	0.1	0	0	0	0	0.1	0.2	0.2	2.0	1.8	1.6	1.4	1.3	1.2	1.1	1.0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74 L(cm)

current I_a on the grid voltage E_g and on the lengths of the plate and the grid circuits $L = L_a = L_g$ (see I, Fig. 1). These measurements were made at the beginning of our investigation with two tubes Nos. 2 and 4 working simultaneously.[2]

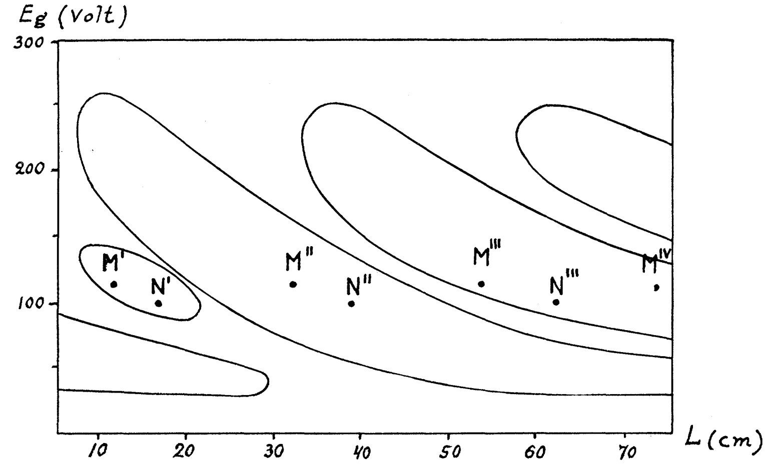

Fig. 1. A working diagram for vacuum tubes of the type *R*5. The regions of oscillation show the length of the oscillating circuits and the grid potentials at which oscillations are generated.

In our previous paper it was shown (I, §6) that the presence of the plate current indicates the presence of oscillations. The current is approximately proportional to the amplitude of the oscillations. Table I shows that the plate current and therefore the oscillations exist at definite values of E_g and L, ly-

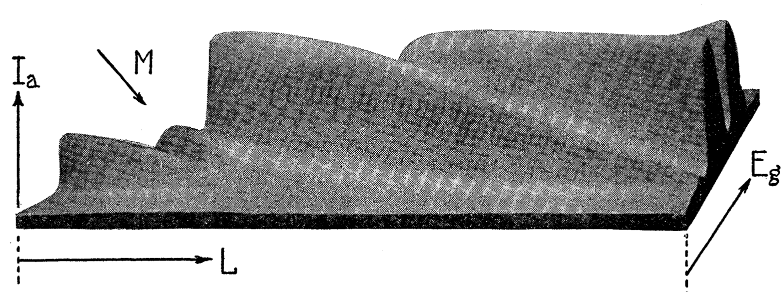

Fig. 2. A 3-dimensional working diagram for vacuum tubes of the type *R*5. The ridge *M* corresponds to dwarf waves.

ing within certain regions, which can be called *"regions of oscillations."* By plotting the locations of regions of oscillations, we obtain Fig. 1. Such diagrams will be called *working diagrams*.

To get an idea of the general appearance of the regions of oscillations, we can construct a 3-*dimensional working diagram* shown in Fig. 2. It is seen from

[2] The numbers are used merely to indicate the order in which the tubes were tested.

the diagram that all regions of oscillations have the same general appearance of continuous ridges. Their height corresponds to the value of the plate current in a given region, or as we have shown, to the amplitude of the oscillations. As all the tubes investigated gave working diagrams of the same type, there is no need to construct 3-dimensional working diagrams in each case. A simple working diagram, similar to the one of Fig. 1 will be sufficient. It should be mentioned that a simple working diagram gives also an approximate idea of the relative magnitude of the amplitude of the oscillations: the size of the area of a region of oscillations usually corresponds to the magnitude of the amplitude of the oscillations. A simple comparison of diagrams Figs. 1 and 2 shows the correctness of this statement. It is evident that a working diagram will give a more complete idea of the work of a generator than separate (I_a, E_g)-or (I_a, L)-characteristics, because it is a combination of all

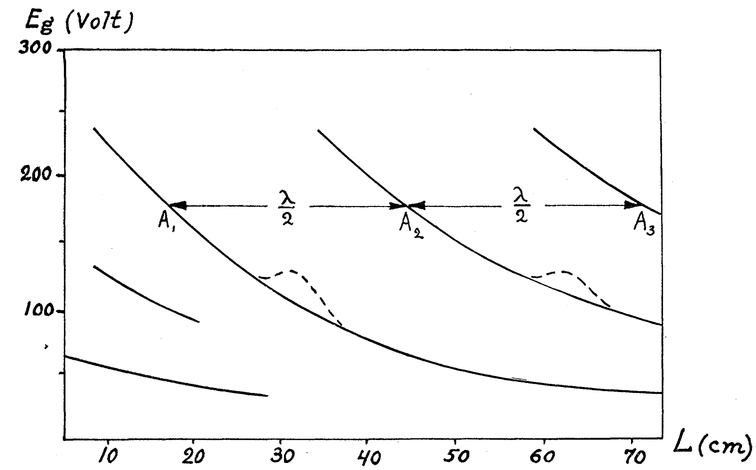

Fig. 3. Simplified working diagram for vacuum tubes of the type *R*5. —lines of maxima corresponding to the maximum values of the plate current, i.e., to the maximum values of the amplitude of the oscillations. ---bends on the lines of maxima indicating a leak of energy through the chokes.

such characteristics. It is seen that the (I_a, E_g)-characteristic represents a section of a 3-dimensional working diagram cut by a plane parallel to the E_g axis and the (I_a, L) characteristic is a section cut by a plane parallel to the L axis.

The work of measuring and of plotting the diagrams is greatly simplified if the position of only those points is determined at which the amplitude of the oscillation has a maximum value. If such points are plotted on the diagram and connected by smooth curves, a simple diagram is obtained which we shall call a *simplified working diagram*. Such a diagram is shown on Fig. 3. It is evident that the curves of such simplified diagrams or *lines of maxima* as they may be called correspond to the peaks of the ridges of the 3-dimensional diagrams. Unfortunately, the simplified diagrams do not give any idea of the relative energy of the oscillations. When such figures are of particular interest they can be given directly on the simplified diagram or the simplified diagram can be supplemented by one or two (I_a, E_g)-or (I_a, L)-characteristics.

The working diagrams have the following important property: from their general appearance it is frequently possible to make an approximate determination of the wave-length of the generated oscillations. Consideer, for example, the diagram of Fig. 3; we draw a line $E_g=180$ volts, intersecting all the three lines of maxima. The points of intersection A_1, A_2 and A_3 will correspond to circuit lengths $L_1=17$ cm, $L_2=44$ cm, $L_3=71$ cm. The distance between two consecutive points will be 27 cm. If we determine the wave-length of the oscillations at these three points, they will be found to be nearly equal and to have an average value $\lambda=54$ cm. Hence it follows that the distance (expressed in centimeters) between two corresponding neighboring points on the diagram equals approximately half the wave-length of the oscillations at these points.[3] By *corresponding points* we mean points lying in two similar regions of oscillation and corresponding to the same value of E_g.

§4.

What is the wave-length of oscillations in different regions on the diagram?

We determine the wave-lengths in such points of the regions of oscillations where the plate current has a maximum value. These points will evidently lie on the peaks of the ridges of the 3-dimensional working diagram or on the lines of maxima of the simplified working diagram. Table II gives the

TABLE II. *R5 (No. 2+No. 4).* $I_h=1.36A$.

E_g (Volt)	L (cm)	λ (cm)	λ^2E_g	E_g (Volt)	L (cm)	λ (cm)	λ^2E_g
40	65	114.0	5.20×10^5	82	21	48.9	1.96×10^5
50	5	109.0	5.94 "	95	19	46.2	2.09 "
80	40	80.0	5.12 "	100	17	44.9	2.01 "
100	71	74.4	5.53 "	110	11	42.1	1.95 "
120	30	66.0	5.23 "	120	9	40.9	2.01 "
120	62	68.8	5.68 "				
140	53	61.5	5.29 "			$(\lambda^2E_g)_{\text{aver.}}=1.99\times10^5$	
150	50	59.7	5.34 "				
160	20	60.4	5.84 "				
160	74	56.9	5.18 "				
180	41	54.2	5.29 "				
200	65	52.5	5.51 "				
210	11	51.6	5.59 "				
		$(\lambda^2E_g)_{\text{aver.}}=5.44\times10^5$					

results of such measurements. It is seen from the table that there are two groups of points. Within the limits of experimental error, Barkhausen's equation

[3] This rule is evidently analogous to the well known rule concerning the distance between two regions of oscillation on the (I_a, L)—characteristics, which was first formulated by Zilitinkewitch, Drahtl. Tel. u. Tel. (russ) **18,** 2–22 (1923); Arch. f. Electr. **15,** 470–484 (1926).

$$\lambda^2 E_g = \text{Const.} \quad (1)$$

holds true for each of the groups. However, the absolute value of the product $\lambda^2 E_g$ is several times greater for one group than for the other. We have seen (I, §2) that in our case, when the plate of the tube is directly connected to the filament, the following approximate expression can be derived from theoretical considerations:

$$\lambda^2 E_g = d_a^2 10^6. \quad (2)$$

The diameter d_a of the internal surface of the plate of tube R5 is equal to 0.87 cm. Substituting this value of d_a in (2) we get $\lambda^2 E_g = 7.6 \times 10^5$. This computation is not very accurate in view of the simplifying assumptions made in deriving Eq. (2). Still, the calculated value of the product $\lambda^2 E_g$ is not very far from the value observed for the first group of points.[4] Waves corresponding to this first group of points may be called normal Barkhausen's waves or simply *normal waves*. The second group of waves having a considerably shorter wave-length will be called *dwarf waves*.

Two types of Barkhausen's oscillations are known at the present time: (1) the *BK-oscillations*, when the length of the generated waves depends on the electrode potentials only, and is independent of the lengths or of the natural periods of the oscillating circuits connected with the tube, and (2) the *GM-oscillations*, when the length of the generated waves depends on the length (or the natural period) of the oscillating circuits. Also, as it was first shown by E. Gill and J. Morrell,[5] Eq. (1) is satisfied only at voltages corresponding to a maximum of the energy of oscillation.[6] Thus, in the case of BK-oscillations Eq. (1) is satisfied always,[7] i.e., for all grid voltages and for all lengths of circuits. In the case of GM-oscillations Eq. (1) is satisfied only for points corresponding to the maximum values of the plate current.

The observations presented in Table II refer to points where the plate current was at a maximum and therefore give no information as to which of the two types the observed observations belong. To obtain such information, observations were made at points not corresponding to the maximum of the plate current. The results are given in Table III. They cover two cases: the lengths of the oscillating circuits were varied at constant grid voltgae and the opposite case when the grid voltage was varied and the length of the oscillating circuits was kept constant. In both cases observations were made at points which did not correspond to the maximum values of the plate current.

[4] The difference between 7.6×10^5 and 5.4×10^5 corresponds to a difference of about 18 percent in the wave-length, which is permissible in view of the approximate nature of our computations. Later we shall attempt to explain the causes of the difference between the theoretical and observed wave-lengths.

[5] E. Gill and T. Morrell, Phil. Mag. **44**, 161–178 (1922).

[6] For further details on different types of Barkhausen oscillations see H. E. Hollmann, Proc. I.R.E. **17**, 229–251 (1929); Zeits. f. Hochfr. **33**, 66–74, 101–105 (1929); K. Kohl, Erg. d. ex. Wiss. **9**, ch. II, §4 (1930).

[7] If one disregards the small variations in the wave-length produced by changes in the heating current.

TABLE III. *R5* (*No. 2+No. 4*). $I_h=1.36A$.

L (cm)	E_g (volt)	I_a (mA)	λ (cm)	λ^2E_g	L (cm)	E_g (volt)	I_a (mA)	λ (cm)	λ^2E_g
				normal waves					
45	150	1.3	57.6	4.98×10^5	50	130	0.2	61.4	4.90×10^5
50	"	3.9	59.7	5.34 "	"	150	3.9	59.7	5.34 "
55	"	3.7	62.0	5.77 "	"	165	3.5	59.2	5.77 "
60	"	2.5	64.0	6.14 "	"	185	2.3	58.8	6.39 "
65	"	0.3	66.8	6.69 "	"	205	0.6	57.6	6.80 "
				dwarf waves					
12	100	0.2	42.8	1.83×10^5	17	90	0.1	45.6	1.87×10^5
17	"	1.6	44.9	2.01 "	"	100	1.6	44.9	2.01 "
19	"	0.3	45.8	2.10 "	"	110	0.2	44.4	2.17 "

The data of Table III clearly show that in either case the product λ^2E_g does not remain constant. Both for normal waves and for dwarf waves λ^2E_g varies in the same manner. The results this show unmistakably that both the normal waves and dwarf waves are GM-oscillations.

It must be mentioned, that for points which do not correspond to maximum values of the plate current the values of the products λ^2E_g differ from the average values found previously, namely, 5.44×10^5 for normal waves and 1.99×10^5 for dwarf waves. The difference, however, is not so great as when we pass from normal to dwarf waves. This shows that there is a fundamental difference between normal and dwarf waves. Before considering the causes of this difference, we shall discuss the results obtained with other tubes.

§5.

With the data of Table II and one of the working diagrams, it is easily found that dwarf waves are observed only in a very small region of oscillations, denoted with an arrow on Fig. 2. All the other regions of oscillations give normal waves. It is also seen from Fig. 2 and from the data of Table I that the region where dwarf waves are observed is of secondary importance as far as the place it occupies on the working diagrams and the energy of its waves are concerned.

In taking separate (I_a, E_g)-and (I_a, L)-characteristics one could easily fail to observe the dwarf waves. A complete picture of the work of a generator can be obtained only by constructing and thoroughly investigating working diagrams. The completeness of this method constitutes a very great advantage over all the other methods. The working diagrams have an additional advantage of being clear and convenient to use. The labor spent in constructing these diagrams is amply paid for.

It is of interest to know why the dwarf waves are observed in one small region only and not in several analogous regions as is the case for normal waves. Let us consider the diagram, Fig. 1, and let us take some point in the region of dwarf waves, point M^{I}, for example, which lies at $L=11$ cm and $E_g=110$ v. As seen from Table II, the waves generated at this point have a length $\lambda\sim42$ cm. According to what we said before (see §3), we should expect dwarf waves in points M^{II}, M^{III}, M^{IV}, lying at the same value of E_g and at

$L=11+\lambda/2$, $L=11+\lambda$, $L=11+3/2\lambda$. It is seen from the diagram that all these points lie in the regions of normal waves which are longer and therefore more easily generated. If we take some other point in the region of dwarf waves, point N^{I}, for example, which lies at $L=17$ cm and $E_g=100$ v and for which $\lambda \sim 45$ cm, then similarly we might expect dwarf waves in points N^{II} and N^{III}. These points are also situated in the regions of normal waves. Hence it follows that if it were possible to inhibit the generation of normal waves in points M^{II}, M^{III}, M^{IV}, or N^{II}, N^{III} we could obtain dwarf waves at these points. This is easily verified experimentally.

So far, our plate and grid circuits were of equal length $L=L_a=L_g$ (see I, Fig. 1), i.e., they were in resonance. We shall now destroy this resonance. Let us take $L_a=32$ cm and $E_g=110$ v, corresponding to point M^{II}, and let us

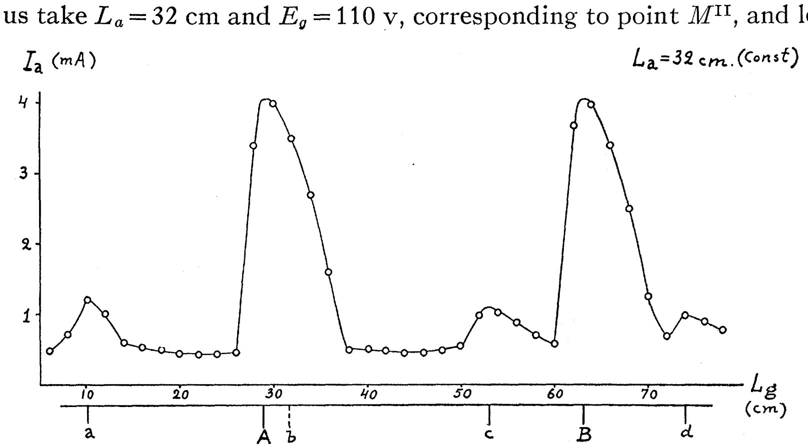

Fig. 4. (I_a, L)-characteristics, the lengths of the plate circuit and the grid circuit not being equal. The humps A and B correspond to normal waves, humps a, c and d, to dwarf waves.

vary the length L_g of the grid circuit, making simultaneous measurements of the plate current. The results of these observations are presented in the curve of Fig. 4. It is easily seen that the two main humps of this curve—A and B correspond to the normal waves. This is because at $L_g=32$ cm we should have normal waves[8] (at this value of L_g we have $L_a=L_g$) and also because the distance between the peaks of these two humps is approximately 33 cm, which is very nearly half the wave-length of the normal waves, which we should have at $E_g=110$ v (see Table II). It is easily shown that the three smaller humps, a, c, d, correspond to dwarf waves, since measurements show that at this value of L_g the average wave-length $\lambda=44.0$ cm. This is very near to the wave-length of dwarf waves, found previously at $E_g=110$ v. The distance between the humps c and d is 22 cm, i.e., is equal to a half wave-length. The distance between humps a and c is approximately 43 cm. It is easily guessed that between humps a and c there should be another hump b, which is not seen merely because it is concealed under the hump A.

[8] The peak of the hump A lies at $L_g \sim 29$ cm, that is, at $L_g \neq L_a$. This is due to the fact that the point $L_a=L_g=32$ cm and $E_g=110$ v does not correspond to the maximum of the plate current.

Hence, it becomes clear that it is possible to inhibit the generation of normal waves if we put out of tune the plate and grid circuits. If we take $L_a=32$ cm and $L_g=53$ cm, or $L_a=32$ cm and $L_g=74$ cm and keep $E_g=110$ v, we shall obtain dwarf waves, although, as we have seen, these dwarf waves are not observable either for $L_a=L_g=32$ cm, or for $L_a=L_g=53$ cm, or for $I_a=L_g=74$ cm.

§6.

The above described method of *putting the circuits out of tune* is very useful when several kinds of waves are present at the same time and it is necessary to separate them. For instance, if at a certain circuit length $L_0=L_a=L_g$

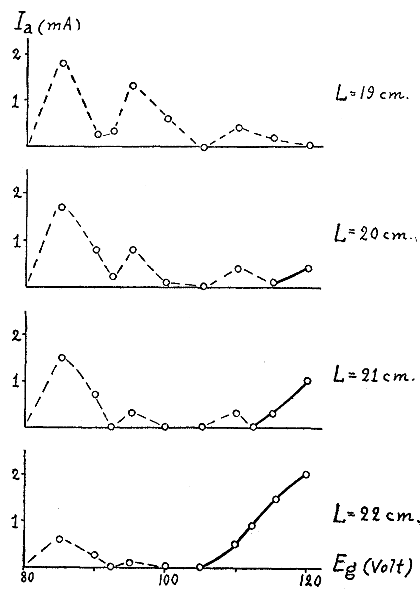

Fig. 5. (I_a, E_g)-characteristics showing the regions of dwarf waves at different circuit lengths;—normal waves; ---dwarf waves.

several kinds of oscillations are simultaneously generated with wave-lengths λ_1, λ_2, etc., we can evidently separate the first kind of waves by putting the circuits out of tune, making, for example, $L_a=L_0$ and $L_g=L_0+n\lambda_1/2$. The second kind of waves can be separated at $L_a=L_0$ and $L_g=L_0+n\lambda_2/2$ etc. Naturally, we can as well change the length of the plate circuit and keep the length of the grid circuit constant.

§7.

The regions of dwarf waves are not always of as simple an appearance as those seen on the preceding diagrams. Very frequently these regions consist of several parallel ridges merging gradually with a neighboring region of normal waves.

To discuss this case we shall consider several (I_a, E_g)-characteristics obtained with two other tubes of the same type R5. They are shown on Fig. 5.

Within the limits $L=19-22$ cm, $E_g=80-120$ v, i.e., where previously we had a region of dwarf waves, we see several regions of oscillations. As L increases these regions are gradually absorbed by the region of normal waves. The length of waves generated in different regions was determined at points corresponding to a maximum value of the plate crurent. Table IV gives the results of these observations for $L=19$ cm. It is seen from the table that for all the three regions $\lambda^2 E_g$ has approximately the same value, closely approaching that which was formerly obtained for dwarf waves. Measurements made at $L=20$, 21, and 22 cm gave for $\lambda^2 E_g$ values close to 2×10^5. Thus, there is no doubt that all the three regions belong to the same dwarf waves.

TABLE IV. *R5 (No. 3+No. 4).* $I_h=1.36A$.

E_g (volt)	L (cm)	I_a (mA)	λ (cm)	$\lambda^2 E_g$
85	19	1.8	48.3	1.98×10^5
95	"	1.3	46.2	2.03 "
110	"	0.4	42.2	1.96 "

The question as to why the region of dwarf waves is in the shape of a single ridge in some cases and in the shape of several ridges in other cases depending on the tube used, can be answered only after discussing the nature of oscillating circuits where dwarf waves originate. We shall not consider this problem now.

§8.

On the diagrams, shown above, all regions of oscillations appear very uniform and the lines of maxima (see Fig. 3) have neither gaps, nor sharp bends. This is not always the case. Sometimes the lines of maxima are sharply bent (see dotted lines on Fig. 3). In such a case typical depressions appear in the corresponding places of the 3-dimensional diagram. This is an indication that the oscillations generated at the corresponding values of E_g and L have a wave-length closely approaching one of the natural periods of the chokes and that the chokes no longer stop them. The energy of the oscillations is evidently dissipated in the connecting leads of the generator. The irregularities disappear when the choke is replaced by another one, having a different natural period. Thus, a regularly shaped diagram and the absence of bends in the lines of maxima is an indication that the generator is working correctly.

§9. Complex Working Diagrams. Dwarf Waves of Higher Orders.

We have seen before that in the case of simultaneous operation of two tubes the secondary (with respect to the energy) regions of oscillations are concealed by the neighboring regions of more powerful oscillations (I, §8). As the preceding results show, the dwarf waves are generated exactly in these secondary regions. Therefore, in all the subsequent experiments only one tube was used for the production of waves. The other tube, "ballast" tube, was used as a condenser and for each tube investigated a special ballast tube was selected, as described earlier (I, §9).

For some of the tubes investigated diagrams were obtained that were considerably more complicated than those shown on Figs. 1–3. There appear a larger number of regions of oscillations. Also, for relatively small overheating there appear regions of oscillations at considerably higher grid potentials—potentials at which tubes heated normally do not generate any oscillations, as a rule.

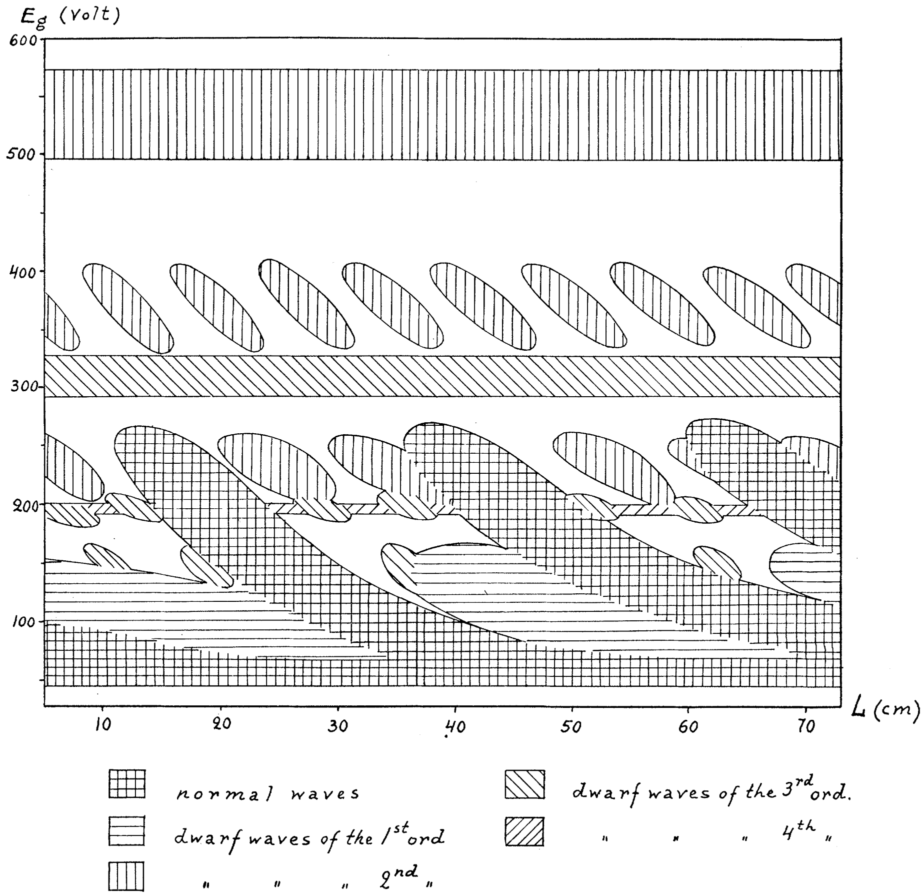

Fig. 6. Working diagram for vacuum tubes of the type *R*5, generating dwarf waves of higher order.

Fig. 6 shows a working diagram particularly rich in regions of oscillations. It was obtained with tube No. 20, type R5 and ballast tube No. 18 of the same type. The heating current was a little above normal so that the emission current reached as a maximum about 19 mA (normal $I_e = 5 - 7$ mA).

A comparison of this diagram with the diagram Fig. 1 shows the abundance of regions of oscillations. Still there is a similarity between the two diagrams. In the lower part of the new diagram, between $E_g \sim 50$ v and $E_g \sim 260$ v we observe the same principal regions of oscillations as before and which, as we know, belong to normal waves. These regions are considerably narrower than before, which is as we should expect, as in this case only one tube was working and not two as before. Between the regions of normal weves we see a great number of smaller regions. Although the regions of normal waves are narrower and their energy is smaller, as the second tube is no longer working, still these regions reach to higher values of E_g than before. This is due both to the greater heating current (I, §2) and to the greater degree of

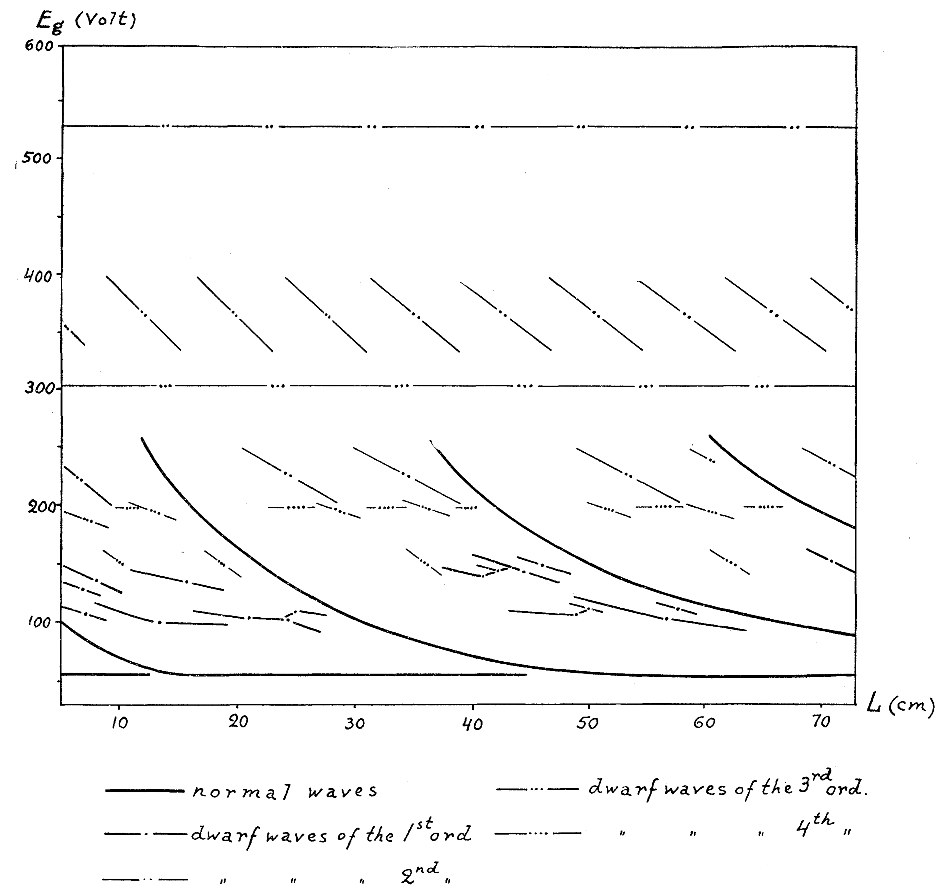

Fig. 7. Simplified working diagram for vacuum tubes of the type $R5$ generating dwarf waves of higher orders.

correspondence between the internal capacities of the two tubes, as it is found experimentally that the smaller the difference between the internal capacities of the working and the ballast tubes, the greater will be the length of the regions of normal oscillations. The lower parts of the regions of oscillations converge into a common asymptote. As we shall see later, this asymptote corresponds to the upper bend of the usual static characteristic of the tube.

We can also identify on Fig. 6 the region of dwarf waves. It lies in the left lower corner of the diagram between $E_g \sim 100$ v and $E_g \sim 150$ v. It is considerably larger than formerly. Two other similar regions are seen within the same limits of E_g, the one between $L=40-60$ cm and the other between $L=65-72$ cm. These two regions must also belong to dwarf waves, as previously we expected to find them near $L=50$ cm and $L=70$ cm (see points M^{III}, N^{III} and M^{IV} Fig. 1).

The general properties of the regions of oscillations can easily be determined by means of the simplified diagram, shown on Fig. 7. Comparing it with the diagram of Fig. 6 we see that with the exception of the three regions of the dwarf waves, spoken of above, all the remaining regions have the same simple structure as those on the 3-dimensional diagram of Fig. 2. Each region has one ridge and a corresponding one line of maxima on the simplified diagram Fig. 7. Only the regions of dwarf waves mentioned above differ in appearance on account of the presence of several ridges going approximately in the same direction. The complex structure of the regions of dwarf waves has already been spoken of; now we see that this structure is characteristic just for these three regions.[9]

An inspection of the simplified diagram shows that the lines of maxima have different slopes. Among other things, this indicates that a change in the lengths of plate and grid circuits produces different effects on the oscillations of different regions. Of particular interest are the three regions of oscillations whose lines of maxima lie at $E_g \sim 200$ v, $E_g \sim 305$ v and $E_g \sim 530$ v. Unlike all the others, these regions go parallel to the axis of abscissas from one end of the diagram to the other. The width of the regions, and hence the amplitude of the corresponding waves, increases as E_g increases.

The (I_a, E_g)-characteristic of tube No. 20 with tube No. 18 as a ballast and at the circuits length $L=50$ cm, was obtained previously (see I, Fig. 7). It gives an idea of the amplitude of the oscillations in different regions up to $E_g \sim 300$ v. For other regions, lying at higher values of E_g we can use the values of the maximum plate current given in Table V.

The distance between the regions of oscillations on Figs. 6 and 7 indicates that at about $E_g \sim 200$ v and $E_g \sim 360$ v tube No. 20 generates waves considerably shorter than those we had previously. This is most clearly seen in that part of the diagram near $E_g \sim 360$ v where the regions of oscillations are disposed with particular regularity.

Table V gives the results of measurements of wave-lengths in different regions of oscillations. As previously, these measurements were made at maximum values of the plate current.

It is seen from the table that there are five different kinds of regions of oscillations. For one of them the product $\lambda^2 E_g$ has an average value 5.78×10^5. The corresponding waves can be called normal. The second group gives for $\lambda^2 E_g$ an average of 1.97×10^5, which is very near to the value previously obtained for dwarf waves. The corresponding waves can be called *dwarf waves of the 1st order*. Waves of the remaining regions, for which the product $\lambda^2 E_g$

[9] We shall see later that they are regions of the so-called dwarf waves of the 1st order.

TABLE V. *R5* (*No. 20*). $I_h=0.70A$.

E_g (volt)	L (cm)	I_a (mA)	λ (cm)	λ_b (cm)	$\frac{\lambda_b}{\lambda}$	λ^2E_g	E_g (volt)	L (cm)	I_a (mA)	λ (cm)	λ_b (cm)	$\frac{\lambda_b}{\lambda}$	λ^2E_g
105	33	2.2	74.0	85.1	1.15	5.75×10^5	247	69	0.15	18.4	55.5	3.02	0.836×10^5
121	29	2.3	69.6	79.3	1.14	5.86	327	13	0.80	16.1	48.2	2.99	0.848
142	26	2.3	64.0	73.2	1.14	5.82	334	21	0.50	16.0	47.7	2.98	0.855
150	53	2.5	62.2	71.2	1.14	5.80	528	22	0.62	12.7	37.9	2.98	0.852
158	19	2.8	60.2	69.4	1.15	5.73	530	15	0.65	12.7	37.9	2.98	0.855
182	18	2.6	56.0	64.6	1.15	5.71	532	5	0.65	12.65	37.8	2.99	0.851
187	17	2.8	55.4	63.8	1.15	5.74							
190	71	2.6	55.0	63.2	1.15	5.75					(λ^2E_g)	aver.	$=0.846\times10^5$
196	16	2.5	54.0	62.3	1.15	5.72							
200	69	2.4	54.0	61.6	1.14	5.83							
200	15	2.5	54.2	61.6	1.14	5.88	140	6	0.50	18.8	73.7	3.92	0.495×10^5
							150	19	0.30	18.0	71.2	3.96	0.486
					(λ^2E_g) aver.	$=5.78\times10^5$	160	9	0.30	17.35	68.9	3.97	0.481
							160	61	0.30	17.5	68.9	3.94	0.490
104	15	0.84	43.7	85.5	1.96	1.99×10^5	185	65	0.03	16.2	64.1	3.96	0.485
107	21	0.90	42.7	84.3	1.97	1.95	190	30	0.04	15.7	63.2	4.02	0.468
109	27	0.65	42.4	83.5	1.97	1.96	190	6	0.09	15.8	63.2	4.00	0.474
122	21	0.70	40.4	78.9	1.95	1.99	195	52	0.08	15.4	62.4	4.05	0.462
123	15	0.70	39.8	78.6	1.97	1.95	197	36	0.12	15.4	62.1	4.03	0.467
137	9	0.80	38.1	74.5	1.96	1.99	198	12	0.05	15.5	62.0	4.00	0.475
142	15	0.90	37.1	73.2	1.97	1.95	200	59	0.24	15.6	61.6	3.95	0.487
143	42	0.75	37.4	72.9	1.95	2.00	204	28	0.20	15.4	61.0	3.96	0.484
150	6	0.65	36.2	71.2	1.97	1.97	205	34	0.06	15.3	60.9	3.99	0.480
153	9	0.60	35.6	70.5	1.98	1.94	303	8	0.48	12.8	50.1	3.91	0.496
							303	21	0.45	12.65	50.1	3.96	0.485
					(λ^2E_g) aver.	$=1.97\times10^5$	305	15	0.46	12.55	49.9	3.98	0.480
200	39	0.10	20.7	61.6	2.98	0.857×10^5					(λ^2E_g)	aver.	$=0.481\times10^5$
212	8	0.60	19.8	59.9	3.02	0.831							
237	72	0.53	18.8	56.6	3.01	0.838	200	33,55	0.20	12.4	61.8	4.97	0.308×10^5
243	70	0.28	18.6	55.9	3.01	0.841	330	13	0.04	9.4	48.0	5.10	0.292
											(λ^2E_g)	aver.	$=0.300\times10^5$

has a still smaller value, can be called *dwarf waves of the 2nd, 3rd and 4th orders*, depending on their length and the magnitude of the product λ^2E_g. Dwarf waves of the 4th order are then the shortest. As seen from Table V some of them have a wave-length $\lambda=9.4$ cm. These are the shortest waves we observed with tubes of the type $R5$.[10] No independent region of oscillations could be found for these waves and therefore it is plotted neither on the working diagram Fig. 6, nor on the simplified diagram Fig. 7. As we shall see later, these waves were always found on the boundaries of the regions of waves of other orders. These waves were first discovered with tube No. 20. They were found on the boundary of the dwarfs of the 2nd and the 3rd orders when observations were being made on the dependence of the wave-length of the dwarfs of the 3rd order on the grid voltage (see below). Subsequently these waves were observed with other tubes.

It is seen that the average values of λ^2E_g for normal waves and for dwarf waves of various orders differ widely. At the same time, for waves of the same order the individual values of the product λ^2E_g are very close to the average value. Thus, the determination of the order of a wave presents no difficulties.

Comparing the data of Table V with those of Table II it is seen that in the earlier experiments there is a greater difference between individual values of the product λ^2E_g for normal waves; the same is also true for the dwarf waves

[10] A. Wainberg, Journn. appl. Phys. (russ) **7**, 97–104 (1930) in an investigation made at our suggestion, was separating short waves by screening the tube by means of metallic shields. Using a tube of the type $R5$ he succeeded in obtaining waves 8 cm long at $E_g\sim320$ v., i.e., where we found waves $\lambda=9.4$ cm.

of the 1st order. This is due not so much to the lesser accuracy of the early measurements as to the fact that the early measurements were made at different times, frequently after considerable intervals of time and the tube was not always sufficiently preheated before the beginning of measurements (cf. I, §4).

Together with the observed wave-lengths λ Table V gives the wave-lengths λ_b which were calculated by means of Barkhausen's formula (2). It is seen that the length of the normal waves is close to the theoretical value.[11] The value of the ratio λ_b/λ shows that the dwarf waves of the 1st order are approximately twice as short as those given by formula (2); dwarf waves of the 2nd order are approximately three times shorter than the computed value; dwarfs of the 3rd order four times; and finally dwarfs of the 4th order are approximately five times shorter than those given by formula (2). Hence it follows that the wave-lengths of the dwarf waves are given by the following simple relationships:

$$\lambda_1 = \frac{\lambda_0}{2} \text{ (dwarf waves of the 1st order)}$$

$$\lambda_2 = \frac{\lambda_0}{3} \text{ (`` \quad `` \quad `` `` 2nd ``)} \qquad (3)$$

$$\lambda_3 = \frac{\lambda_0}{4} \text{ (`` \quad `` \quad `` `` 3rd ``) etc.,}$$

where λ_0 is the length of the normal waves, corresponding to the grid potential at which dwarf waves are observed. The approximate value of λ_0 can be computed from Eq. (2).

In the particular case it is evident that at the same grid voltage E_g we should observe a series of waves whose lengths correspond to series (3). Whether *all* these waves are actually generated or only some of them depends on the absolute value of λ_0 and on the presence of oscillating circuits, one of the natural periods of which is equal or near to the members of series (3). Without going into a detailed discussion we shall give two examples from the data of Table V. At $E_g = 150$ v the tube generates the following waves, $\lambda_0 \sim 62.2$ cm (normal waves), $\lambda_1 = 36.2$ ($\sim\lambda_0/2$) and $\lambda_3 = 18.0$ ($\sim\lambda_0/4$). At $E_g = 200$ v we have: $\lambda_0 = 54.1$ (normal waves), $\lambda_2 = 20.7$ ($\sim\lambda_0/3$), $\lambda_3 = 15.6$ ($\sim\lambda_s/4$) and finally $\lambda_4 = 12.5$ ($\sim\lambda_0/5$). It is seen that the observed waves do not exactly correspond to the series (3). We shall see later the causes of this discrepancy. In all cases without exception, the observed wave-lengths exceed the values expected from series (3) by approximately 15 percent. Hence it follows that the discrepancies between the calculated and the observed wave-lengths are not accidental but are brought about by some as yet unknown cause.

Formula (2) was derived on the assumption that the frequency of the

[11] The observed waves are approximately 15 percent shorter than those given by formula (2).

observed oscillations is equal to the frequency of the electronic oscillations within the tube. The existence of waves 2, 3, 4, etc., times shorter than those computed on the basis of this formula leads to the following very important conclusion: *a vacuum tube can generate oscillations whose frequency exceeds many times the frequency of electronic oscillations.*

§10.

We investigated the oscillations generated by tubes Nos. 2 and 4 and found that these oscillations can be considered as *GM*-oscillations, since the wave-lengths of normal and dwarf waves[12] depend on the grid voltage and on the dimensions of the grid and plate circuits. Comparing the working diagrams Figs. 6 and 7 with the diagrams of Figs. 1 and 3 it is seen that nearly all the regions of oscillations are of the same type. This indicates that the oscillations of all these regions depend in the same manner on the grid voltage and on the dimensions of the oscillating circuits and hence, that they must be *GM*-oscillations.

There are, however, on the diagrams, Figs. 6 and 7, three regions different from the rest. These are regions going parallel to the axis of abscissas and having lines of maxima approximately at $E_g=200$, 305 and 530 volts. From the data of Table V it is seen that oscillations generated within these regions have approximately the same wave-length, irrespective of the dimensions of the oscillating circuits used. The latter circumstance is very important, for we know that one of the characteristic features of the *BK*-oscillations is the fact that their wave-length is independent of the length of the oscillating circuits.

In order to decide whether or not the oscillations of these three regions are *BK*-oscillations, we must determine how their wave-length depends on the grid potential. If they are BK-oscillations, then a change in the grid potential must produce a change in the wave-length according to the Eq. (1). Table VI

TABLE VI. *R5* (*No. 20*). $I_h=0.70A$.

E_g (volt)	L (cm)	I_a (mA)	λ (cm)	$\lambda^2 E_g$	E_g (volt)	L (cm)	I_a (mA)	λ (cm)	$\lambda^2 E_g$
285	15	0.06	12.9	0.474×10^5	490	15	0.01	12.8	0.803×10^5
295	"	0.28	12.7	0.476 "	500	"	0.04	12.8	0.819 "
305	"	0.46	12.55	0.480 "	510	"	0.20	12.75	0.829 "
315	"	0.32	12.4	0.484 "	520	"	0.35	12.75	0.846 "
325	"	0.06	12.3	*0.492* "	530	"	0.65	12.7	0.855 "
330	"	0.02	9.4	*0.292* "	540	"	0.40	12.7	0.871 "
335	"	0.01	16.2	0.878 "	550	"	0.30	12.7	0.887 "
					560	"	0.20	12.65	0.896 "
					570	"	0.02	12.65	0.912 "

gives the results of measurements of wave-lengths at different grid potentials. Measurements were made for two regions only. No data are given for the third region because the narrowness of that region and its close proximity to the regions of other waves introduces an uncertainty in the measurements.

[12] They were dwarf waves of the 1st order.

Table VI shows that the wave-length changes with the grid potential. The nature of that change is quite analogous to that previously obtained for normal and dwarf waves of the 1st order. The product $\lambda^2 E_g$ changes when the grid potential is changed, which shows that these oscillations cannot be considered BK-oscillations. This conclusion can be extended to the oscillations of the third region, lying at $E_g \sim 200$ v as there is no reason to consider it different from the other two.

The results obtained permit us to assert that the observed *normal waves and all the dwarf waves of all orders belong to the same type of GM-oscillations.* It is of interest to know why the oscillations in these three regions are different from all the others and why their wave-length does not appreciably depend on the lengths of the plate and of the grid circuits. This can be readily understood if we assume that a circuit with a natural wave-length close to $\lambda = 12.5$ cm, in which these oscillations originate, lies somewhere within the tube[13] and that the plate and grid circuits are coupled with this circuit so that their oscillations are forced oscillations. In such a case a change in the

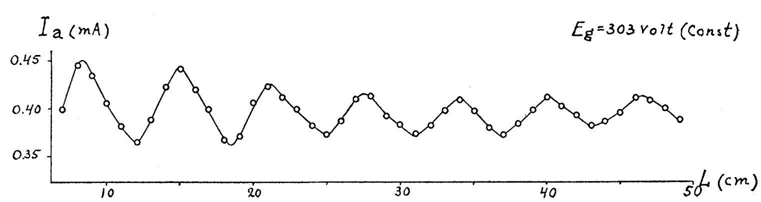

Fig. 8. (I_a, L)-characteristic showing the dependence of the amplitude of oscillations of dwarf waves of the 3rd order on the length of the grid and plate circuits.

wave-length of the oscillations in the primary circuit, which depends on the change of grid potential, will produce a corresponding change in the wave-length of the oscillations in the plate and grid circuits. As the oscillations in the primary circuit are not affected by the forced oscillations, a change in the length of the plate and grid circuits cannot produce a change in the wave-length of the oscillations in the primary circuit. This is what we observe.

At present it is difficult to say what is the cause of the dependence of the wave-length in the primary circuit on the grid potential and the problem requires a special investigation. It is clear, however, that this dependence cannot be entirely ascribed to a change in the time of passage of the electrons, since Eq. (1) is not satisfied when E_g changes.

If our assumption is correct, that there exists within the tube a circuit with a natural wave-length $\lambda = 12.5$ cm and that the plate and grid circuits are coupled with it, then a change in the length of these circuits must produce a change in the amplitude of the oscillations. The curves of Fig. 8 shows that this is verified by experiment. This curve represents the relation between the plate current and the lengths of the plate and grid circuits. The grid voltage was kept constant at a value corresponding to the line of maxima of the second (i.e., the middle one) region of oscillations. The curve, as we see, is

[13] As we shall see later, this assumption is supported by a number of other considerations.

the usual curve of standing waves.[13a] The wave-length found from this curve is very near to the value found by direct measurements.

§11.

During the above described measurements we succeeded in discovering waves $\lambda=9.4$ cm, which are the shortest ones we found with tubes of the type *R*5. As shown in Table VI, with a change of grid voltage from $E_g=325$ v to $E_g=330$ v the wave-length decreased abruptly from $\lambda=12.3$ cm to $\lambda=9.4$ cm, with a simultaneous decrease in the plate current. Such an abrupt transition from one kind of waves to another is usually accompanied by transitions from one region of oscillations into a neighboring one. At $E_g=330$ v we did not succeed in finding a special region of dwarf waves of the 4th order to which waves $\lambda=9.4$ cm belong. When E_g was increased by $3-5$ v, the wave-length increased abruptly to $\lambda=16.2$ cm which corresponds to the boundary of the region of dwarf waves of the second order which exists in this part of the diagram. The waves $\lambda=9.4$ cm disappeared when the length of the plate and grid circuits was changed by $1\frac{1}{2}-2$ cm. They reappeared at $L=6-7$ cm and $L=23-24$ cm, that is, again in those points where the region of dwarf waves of the 2nd order passes into the region of dwarfs of the 3rd order.

Subsequently, waves near to $\lambda=9.5$ cm were obtained with other tubes. For example, tube No. 5 gave waves $\lambda=9.4$ cm. These observations shall be presented later. Such waves were obtained at $E_g=330$ v and at $E_g=335$ v. In both cases we had $L=13.5-15$ cm, i.e., in both cases the waves were found on the boundary of the dwarfs of the 2nd and 3rd orders. Again, in the very beginning of our investigation, tube No. 2 gave waves $\lambda=9.6$ cm at $E_g=90$ v and $L=35-40$ cm.[14] The working diagrams show that this point corresponds, to the boundary of the region of normal waves.

All these observations show that waves approximately 9.5 cm long are characteristic for tubes of the type *R*5. They also indicate that the appearance of these waves is in some way connected with the unstable "regime" existing in the tubes at the boundaries of regions of oscillations. This should be mentioned because other types of tubes give sometimes very short waves at unstable operating "regimes."

§12.

The working diagrams show that tube No. 20 began generating oscillations at $E_g \sim 55$ v. The measurements given in Table V do not go below 105 v. The discrepancy is not accidental. At the heating current at which tube No. 20 was operated, the upper bend of the usual static characteristic of that tube was located between $E_g=50-80$ v. Thus all the points of the diagram for which $E_g<80$ v went beyond the region of the saturation current. Since the space charge must have a distorting influence in all points outside of the region of saturation current, all these points were excluded from our observations.

[13a] Cf. M. O. J. Strutt, Ann. d. Physik **4**, 26 (1930).

[14] In this case $\lambda^2 E_g=0.083\times10^5$. It is evident that we are dealing with dwarf waves of a fairly high order (8^{th} or 9^{th}).

Curve (a) Fig. 9 gives an idea of the results obtainable in the region $E_g=50-80$ v. The dotted line corresponds to the equation $\lambda^2E_g=5.78\times10^5$. It is seen that all points within the region $E_g=105$–200 v (former observations) fit the curve very well, while within the region $E_g=55-80$ v the points follow an entirely different curve. As a result the product λ^2E_g for these points differs considerably from the average value found previously, as is seen from curve (b) Fig. 9 and also from the data given in Table VII.

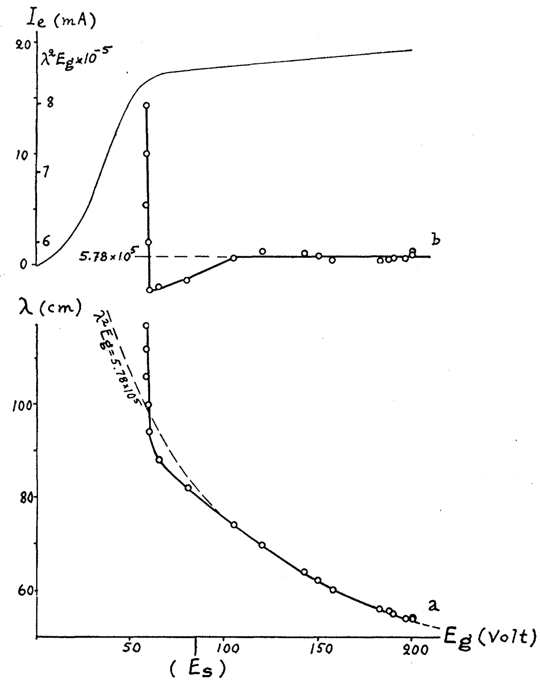

Fig. 9. Curves showing the dependance of the length of the normal waves, the value of the product λ^2E_g and the emission current on the grid potential. The dotted line (a) corresponds to Barkhausen's Eq. (1).

It is seen that at grid voltages $E_g<100$ v, the wave-length is below that given by the Eq. $\lambda^2E_g=5.78\times10$ and the value of the product λ^2E_g falls off rapidly. This phenomenon is due to the fact that space charges appear near the filament. Analogous results were obtained by N. Kapzov and S. Gwosdower[15] who worked with vacuum tubes of a different type. Our observations shows that the product λ^2E_g has a minimum value at $E_g=60$ v and $L=50$ cm when the line of maxima comes close to its asymptote which lies at $E_g=58$ v at the heating current used. In the region of this asymptote we have a very interesting phenomenon—at constant E_g the wave-length increases as the length of the circuits is increased (see Table VII) and the corresponding points of curves Fig. 9 lie above the dotted lines. As the grid voltage remains practically unchanged, the case is evidently quite analogous to the one we

[15] N. Kapzov u. S. Gwosdower, Zeits. f. Physik **45**, 114–134 (1927).

TABLE VII. *R5* (*No. 20*). $I_h=0.70A$.

E_g (volt)	L (cm)	I_a (mA)	λ (cm)	$\lambda^2 E_g$
105	33	2.2	74	5.75×10^5
81	39	2.0	82	5.45 "
65	45	1.9	88	5.34 "
60	50	1.3	94	5.30 "
60	55	0.97	100	6.00 "
58	60	0.77	106	6.52 "
58	65	0.63	112	7.27 "
58	70	0.53	117	7.95 "

had previously, when the wave-length depended on the lengths of the grid and plate circuits—see Table III. In the present case, however, an increase in the circuit length produces a considerably greater increase in the wave-length than before. This is apparently due to the fact that at $E_g \sim E_s$ the oscillations are considerably less stable than at $E_g > E_s$ and all the external factors exercise a greater influence.

§13.

Measurements made with other vacuum tubes of the same type *R*5 gave results very similar to those described above.

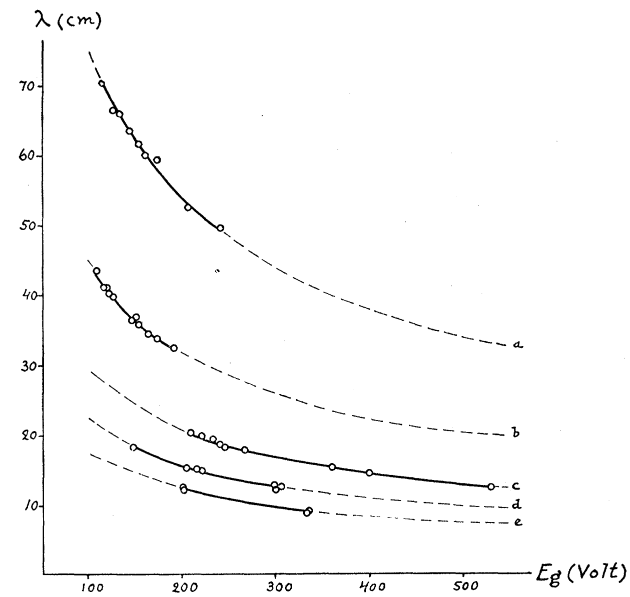

Fig. 10. The lengths of normal and dwarf waves at different grid voltages. The dotted lines correspond to the equation $\lambda^2 E_g$ = Const. Curve (a) $-\lambda^2 E_g = 5.81\times10^5$ (normal waves); curve (b)—$\lambda^2 E_g = 2.01\times10^5$ (dwarf waves of the 1st ord.); curve (c) $-\lambda^2 E_g = 0.856\times10^5$ (dwarf waves of the 2nd ord.); curve (d)—$\lambda^2 E_g = 0.495\times10^5$ (dwarf waves of the 3rd ord.); curve (e)—$\lambda^2 E_g = 0.303\times10^5$ (dwarf waves of the 4th ord.).

We shall give as an example the results obtained with tube No. 5 (type *R*5). Tube No. 20 which has been investigated earlier, was used as a "ballast." All measurements were made at a higher heating current than before, the emission current reaching as high as 41 mA. All the measurements were made within a very brief period of time, as at this heating current the disintegration of the filament proceeded rapidly.

We shall omit the tables of data and the working diagrams which are very similar to those of Figs. 6 and 7. Fig. 10 gives the results of measurements of wave-length in different points of the diagram, corresponding to the maximum values of the plate current. It is seen that the observations fit well the curves $\lambda^2 E_g = \text{const}$. The curves differ in the numerical value of the constant. The numerical values are very close to those previously obtained with tube No. 20. We are evidently dealing with five different kinds of waves: normal waves and dwarf waves of the 1st, 2nd, 3rd and 4th orders, corresponding to the numerical values of $\lambda^2 E_g$.

It could be easily shown that the waves generated by tube No. 5 almost coincide with those generated by tube No. 20. They lie now at slightly higher values of E_g than before. In other words, a comparison of the working diagrams for these two tubes would show that the regions of oscillations of tube No. 5 lie above those of tube No. 20. We shall see later that such a translation of regions is a result of the increase in the heating current.

We shall mention the following details. Due to the increase in heating current, the region of normal oscillations reaches now to higher values of E_g than before. On the curve of normal oscillations (Fig. 10) we see points up to $E_g = 242$ v, the boundaries of the region reaching almost up to $E_g = 300$ v. The region of dwarf waves of the second order, lying near $E_g \sim 350$ v, shows an appreciable increase in size, reaching up to $E_g = 400-410$ v. All the other regions remained practically unchanged. Observations with tubes Nos. 5 an 20 were made at a relatively high value of the heating current. We shall give also the results obtained with tube No. 7 (type $R5$), when the heating current was considerably lower, so that the emission current was about 12 mA.[16] Exactly similar results were obtained, the average value of $\lambda^2 E_g$ being: 5.85×10^5 for normal waves, 1.90×10^5 for dwarf waves of the 1st order, 0.800×10^5 for dwarf waves of the 2nd order, 0.466×10^5 for dwarf waves of the 3rd order, and, finally, 0.300×10^5 for dwarf waves of the 4th order. These values agree well with those found previously.

All our findings point to the following conclusion: as far as the generation of ultra-short waves is concerned, tubes of the same type differ only with respect to the number and the size of the regions of oscillations on the working diagrams. Moreover, if a region exists it will be located in nearly the same part on all the diagrams and the corresponding wave-lengths and values of $\lambda^2 E_g$ will be nearly equal for all the tubes. It should be borne in mind that observations should be made at a constant heating current and in those points of the diagram for which $E_g > E_s$ and which correspond to the maximum values of the amplitude of oscillation (maximum plate current).

[16] The results of our earlier observations, see Zeits. f. techn. Physik **10**, 542–548 (1929), Table III, are not given here, because they are less accurate and because tubes of the older series were used which had a slightly different diameter of the plate.

There is one exception to this rule. It occurs in those infrequent cases when two kinds of waves are generated at the same time and we have simultaneously dwarf waves and weak normal waves. This is sometimes observed in those regions of dwarf waves, which lie at $E_g < 300$ v, i.e., where normal waves can generally be observed. In such a case the wave-length and the value of $\lambda^2 E_g$ are higher than usual. An explanation of this fact can be given, after an investigation is made of the dependance of the length of the waves on the heating current and the amplitude of oscillation.

§14.

It is of importance to know, why some tubes have working diagrams rich in regions of dwarf waves (Figs. 6 and 7) and generate such waves easily, while other tubes of the same type have diagrams almost devoid of regions of dwarf waves and these waves are generated with difficulty.

In the case of normal waves it is known that their production is facilitated by a symmetry in the construction of the tube. This fact is supported by direct observations on the generation of these waves by tubes with destroyed symmetry[17] and also by the well-known fact that normal waves are nearly always generated by tubes with cylindrical electrodes, while tubes with plane electrodes produce them only in exceptional cases. The effect of symmetry is easily understood. If the symmetry of a tube with cylindrical electrodes is destroyed, i.e., the filament does not coincide with the axis of the grid and of the plate, then the time required for the electrons coming off the filament to pass to the plate will be different for different electrons, depending on the direction of their motion. If the difference in the time of passage is sufficiently large in comparison with the period of the generated oscillations, the oscillations will be broken up. Hence it follows that the condition of symmetry is essentially *a condition of the equality of the times of passage of the electrons* in different directions within the tube. In the case of a tube with cylindrical electrodes of the usual construction, i.e., with a filament inside of the grid, the two conditions are equivalent. In the case of tubes of some other design it is necessary and sufficient to satisfy the second condition only.

In a recent paper H. E. Hollmann[18] has indicated the possibility of obtaining oscillations using asymmetric tubes, with the anode placed within the cylindrical grid and the filament (one) located outside of the grid. This is not in contradiction with our statement. In these tubes the majority of electrons will follow approximately the direction of the straight line connecting the filament and the plate. Thus the condition of the equality of times of passage will be satisfied for the majority of electrons, in spite of the asymmetry of the construction. Our statement is further supported by the well-known fact that oscillations can be obtained with cylindrical tubes having spiral shaped filaments.

The condition of the equality of the times of passage must also have an influence on the generation of dwarf waves. Also, the higher the order of the

[17] See, for example, G. Breit, J. Opt. Soc. Am. **9**, 907–922 (1924).
[18] H. E. Hollmann, Phys. Zeits. **31**, 56–63 (1930).

dwarf waves, the more they are affected by an inequality in the times of passage, because the greater will be the relative difference between the period of oscillation and the times of passage of the electrons in different directions. Following our suggestion A. Wainberg[19] has systematically investigated the effect of the position of the filament in tubes of the type *R*5 on the generation of dwarf waves. These investigations[20] show that shifting the filament by 0.5 mm from the axis of the plate produces a decrease in the amplitude of the normal waves from 10 percent to 30 percent depending on their length and completely prevents the generation of dwarf waves of the 1st and 2nd orders[21] which normally are generated by the tube.

These results explain why working diagrams obtained with tubes of the same type can differ so widely with respect to the number of regions of dwarf waves.

For tubes of the type *R*5 we can calculate, using formula (2), that displacing the filament by 0.5 mm relative to the plate will cause a maximum difference of 20 percent in the times of passage of two electrons moving in opposite directions along the same diameter. This difference amounts to 40 percent of the period of the dwarf waves of the 1st order and to 80 percent of the period of the dwarf waves of the 2nd order. This explains why a small displacement of the filament has such a great effect in preventing the generation of dwarf waves.

In the commercially available tubes the electrodes are frequently displaced from their normal position by more than 0.5 mm. Therefore, very few of these tubes are capable of generating dwarf waves and especially dwarf waves of higher orders.

§15. On the Nature of Dwarf Waves

Series (3) is very similar to the series of overtones of harmonic oscillators. The resemblance is purely superficial, however, and dwarf waves cannot be considered as overtones of the normal waves.

The main difference between dwarf waves and overtones, as they are usually understood, consists in the fact that dwarf waves are completely independent of the normal waves. This is shown in Figs. 6 and 7. Dwarf waves are observed, in fact, even at such grid potentials E_g at which no normal waves can be produced at all. For example, at grid voltages of 300 v and higher, at which we have no traces of normal waves we can easily obtain dwarf waves of the 2nd and 3rd orders.

This does not mean, of course, that the wave-length of the dwarfs may not coincide with that of one of the overtones of the grid and plate circuits. All the points of lines of maxima will always correspond to one of the natural

[19] A. Wainberg, Journ. appl. Phys. (russ) **8**, 97–104 (1930).

[20] In tubes of the type *R*5 the plate is so arranged that it can easily be shifted from its normal position. Thus, the same tube could be used in all measurements. This guaranteed constancy of vacuum, filament, etc.

[21] A. Wainberg, reference 19, see Figs. 3 and 4, region IV (dwarf waves of the 1st order) and region V (dwarf waves of the 2nd order).

periods of these circuits (fundamental or overtone) and this coincidence may occur. But in this case the oscillations will not have the corresponding fundamental period, i.e., they will be forced.

§16.

According to Barkhausen's theory, the period of the oscillations in the circuits coupled with the tube must be equal to the period of the oscillations of the electrons about the grid. In other words, it must be equal to the time required for the electrons to pass from the filament to the plate and back. The derivation of Eq. (2) is based on this assumption. Hence it follows that the lengths of the normal and of the dwarf waves can be represented by the following relationships, which are analogous to the relationships (3):

$$\begin{aligned} \lambda_0 &= c_0\tau \quad \text{(normal waves)} \\ \lambda_1 &= c_0\frac{\tau}{2} \quad \text{(dwarf waves of the 1st order)} \\ \lambda_2 &= c_0\frac{\tau}{3} \quad (\text{" \quad " \quad " \quad " 2nd \quad "}) \\ \lambda_3 &= c_0\frac{\tau}{4} \quad (\text{" \quad " \quad " \quad " 3rd \quad "}) \text{ etc.,} \end{aligned} \tag{4}$$

where c_0 is the velocity of light and τ the time required for an electron to pass from the filament to the plate and back, corresponding to the value of the grid potential at which waves are observed.

If the periods of the oscillations generated be denoted by T_0 (normal waves), T_1 (dwarf waves of the 1st order), T_2 (dwarf waves of the second order) etc., series (4) gives

$$\tau = T_0 = 2T_1 = 3T_2 = 4T_3 = \cdots . \tag{5}$$

This means that during τ, the time of electronic passage, the oscillating circuit can perform not one oscillation only, but two, three, four and so on.

This answers the question regarding the nature of dwarf waves. Since dwarf waves are GM-oscillations and since oscillations of this type orginate in the circuits coupled with the tube, it is clear that *dwarf waves are oscillations of the circuits within the tube or coupled with the tube and which are excited in such a maner that during the time of passage of electrons from the filament to the plate and back, they perform two complete oscillations (dwarf waves of the 1st order) three complete oscillations (dwarf waves of the 2nd order) etc.*

The difference between the normal and dwarf waves lies in the difference in the excitation of the oscillating circuits. The question as to how the circuits are brought into oscillations corresponding to dwarf waves is evidently analogous to the question as to the mechanism of the excitation of normal waves. The solution of this question involves an investigation of the mechanism of the electronic oscillations in a vacuum tube. In a preliminary paper[22] we have indicated one of the possible ways.

[22] G. Potapenko, Zeits. f. techn. Physik **10**, 542–548 (1929).

There remain some small systematic discrepancies between the results of observations and series (3) and therefore series (4) and (5). An investigation of the relation between the wave-length and heating current and the amplitude of the oscillations will explain the cause of these discrepancies.

§17.

We shall now consider to what regions of the spectrum will belong the normal and dwarf waves generated by tubes of the type *R*5. Fig. 11, which is based on the data of Fig. 10 and of Table V, shows that the oscillations generated cover only certain regions of the spectrum. Between these regions there are considerable gaps where no oscillations are generated at all. The normal waves cover a region of the spectrum corresponding to a wave-length $\lambda \sim 50$ cm and longer. The dwarf waves of the 1st order cover a region between $\lambda \sim 34$ cm and $\lambda \sim 44$ cm. Dwarf waves of the 2nd, 3rd and 4th order lie within still narrower limits around $\lambda = 18.5$ cm, $\lambda = 15.5$ cm, $\lambda = 12.5$ cm and $\lambda = 9.5$ cm. It is of interest that waves close to $\lambda = 18.5$ cm and $\lambda = 15.5$ cm are generated

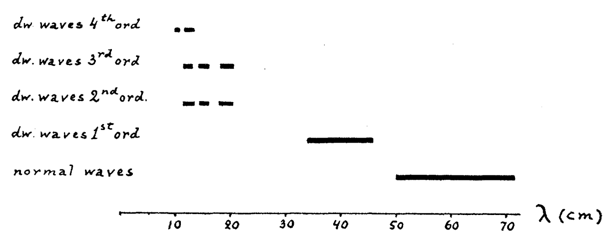

Fig. 11. Regions of the spectrum corresponding to the oscillations generated by vacuum tubes of the type *R*5.

both as dwarf waves of the 2nd order and as dwarf waves of the 3rd order. Waves close to $\lambda = 12.5$ cm are generated as dwarf waves of either 2nd, 3rd or 4th orders.

The fact that only certain waves whose length lies within definite limits can be generated as dwarf waves of the 2nd, 3rd and 4th orders shows that somewhere within the tube there exist oscillating circuits whose natural periods (fundamental or overtones) are close to the periods of the observed oscillations. These circuits exist somewhere within the bulb or the base of the tube, because the length of the corresponding waves (waves near to $\lambda = 12.5$ cm) is little or not at all dependent on the length of the grid and plate circuits. The question as to what these internal oscillating circuits really are is rather complicated and must be treated separately. In connection with this question we had constructed a special tube in which one of the internal circuits was variable. The design of this tube was described in a previous paper.[23]

We have seen that waves of the same length can be generated as dwarf waves of different orders. This is in complete agreement with our previous considerations. It shows that the same circuit can be made to oscillate in several different ways, so that during the passage of electrons from the fila-

[23] G. Potapenko, reference 22, see Figs. 7 and 8.

ment to the plate and back to the filament it will make three, four or five oscillations depending on whether we have dwarf waves of the 2nd, 3rd or 4th orders respectively.

From Eq. (1) it is easily seen that the values of the grid potentials at which waves of the same length can be observed must satisfy the following series

$$(E_g)_0, \frac{(E_g)_0}{4}, \frac{(E_g)_0}{9}, \frac{(E_g)_0}{16}, \ldots \tag{6}$$

where the first term corresponds to normal waves, the second to dwarf waves of the 1st order, etc. The correctness of the above series can be seen from the data of Table V.

Fig. 11 shows that waves corresponding to the oscillations of the internal circuits of the tube can be observed as dwarf waves only and only as dwarf waves of certain definite orders. For example, none of them appear as dwarf waves of the 1st order. As dwarf waves of the 4th order we have waves near to $\lambda = 9.5$ cm and $\lambda = 12.5$ cm. We do not find here any waves of greater length which, one would think, could be more easily obtained. An explanation of this can be obtained from a consideration of Table VIII, which gives the grid potentials at which we can expect the appearance of waves corresponding to internal circuits in the form of dwarf waves of different orders. These potentials are calculated using formula (1), the numerical values of the product $\lambda^2 E_g$ being taken from Table V. Voltages at which waves were actually observed are separated by a thick line from the other figures.

TABLE VIII.

Order of the waves	$\lambda^2 E_g$	$\lambda=9.5$	$\lambda=12.5$	$\lambda=15.5$	$\lambda=18.5$
Dwarf waves 1 ord.	$1.97 \cdot 10^5$	2200 v	1300 v	820 v	580 v
" " 2 "	0.846 "	940 "	540 "	450 "	250 "
" " 3 "	0.481 "	530 "	310 "	200 "	140 "
" " 4 "	0.300 "	330 "	190 "	120 "	90 "
" " 5 "	$(0.21 \cdot 10^5)$	230 "	130 "	90 "	60 "

From Table VIII it is seen that waves $\lambda = 15.5$ cm and $\lambda = 18.5$ cm could be generated as dwarf waves of the 4th order at grid potentials $E_g = 120$ v and $E_g = 90$ v. The working diagram Fig. 6 shows, however, that longer and more powerful normal waves and dwarf waves of the first order are generated at these voltages. Their presence precludes the appearance of the dwarf waves of the 4th order which are shorter and hence generated with greater difficulty. Waves $\lambda = 9.5$ cm could be obtained as dwarf waves of the 3rd order at a grid potential $E_g = 530$ volts. This is hindered, however, by the longer waves $\lambda = 12.5$ cm, which, as we have seen before, occupy the whole region of the diagram which lies between $E_g \sim 500-550$ v. We cannot expect to obtain waves $\lambda = 9.5$ cm as dwarf waves of the 2nd order, in view of the high potential required. For the same reason, evidently no dwarf waves of the first order $\lambda = 9.5-18.5$ cm were observed.

At $E_g = 190 - 200$ v it is theoretically possible to obtain dwarf waves of the 4th order $\lambda = 12.5$ cm and dwarf waves of the 3rd order $\lambda = 15.5$ cm. These waves were actually observed. In this case their appearance was not prevented by longer waves. This is explained by the fact that waves $\lambda = 15.5$ cm are generated only at certain definite lengths of the plate and grid circuits, while waves $\lambda = 12.5$ cm are independent of the length of the circuits. Thus at certain circuit lengths the waves $\lambda = 12.5$ can appear as they are not hindered by the longer waves. That is where they were observed.

From series (3) it is easy to calculate the probable value of $\lambda^2 E_g$ and the potentials at which dwarf waves of the 5th order could be generated. These calculations are also given in Table VIII. Comparing them with the working diagram Fig. 6 it would seem that waves $\lambda = 9.5$ and especially waves $\lambda = 12.5$ might be observed as dwarf waves of the 5th order. However so far all attempts to find them were unsuccessful. Apparently, this is due to the fact that a higher degree of symmetry in the arrangement of the electrodes is required for the generation of dwarf waves of higher orders.

The production of dwarf waves of higher orders is of interest because it enables us to obtain short waves at relatively very low voltages. The latter ensures a steadiness in the operation of the tube and renders possible the application of short waves for exact measurements. A few simple computations will show the advantages of dwarf waves as far as a decrease in grid voltage is concerned. To obtain normal waves $\lambda = 18.5$ cm a grid voltage $E_g \sim 1900$ v is required. To obtain normal waves $\lambda = 9.5$ cm, a grid voltage $E_g \sim 6400$ v is necessary. These figures are so large that the production of normal waves of this length is entirely out of the question. Besides, at such values of the grid voltage the emission current would have to be increased approximately 150 times (see I, §2). This is also quite impossible.

In conclusion the author wishes to express his gratitude to the Rockefeller Foundation for the grant of a Fellowship and to Professor R. A. Millikan for the facilities of the Norman Bridge Laboratory.

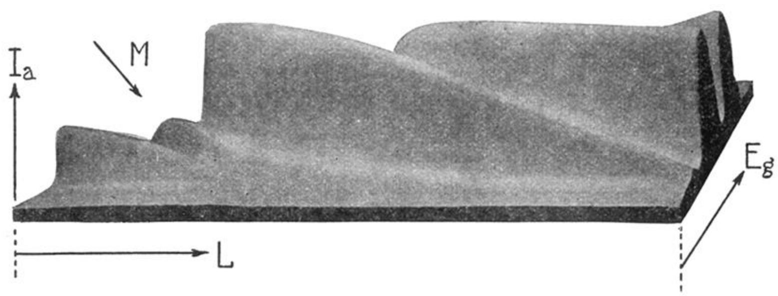

Fig. 2. A 3-dimensional working diagram for vacuum tubes of the type $R5$.
The ridge M corresponds to dwarf waves.