A NEW DETERMINATION OF e/m FROM THE ZEEMAN EFFECT

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ABSTRACT

Values of e/m have been determined from the Zeeman separations of the Cd line 6439 and the Zn line 6362. For these lines the g-factors can be accurately determined from the theory. A magnetic field of 7300 gauss was produced by an air core solenoid in which the variation of field strength over a length of 6 cm at the center was less than 0.1 percent. The field to current ratio, K, of this solenoid was determined in terms of the calculated ratios of three single layer standard solenoids. The field strength during an exposure was then given by the product of this constant and the current flowing. Evaporation of Zn and Cd in the short (6 cm) positive column of a helium discharge tube gave the desired lines. The Zeeman patterns were photographed with a Fabry-Perot interferometer. The result is

$$e/m = 1.7579 \pm 0.0025 \times 10^7 \text{ e.m.u. per gram.}$$

BECAUSE of the apparent discrepancy, emphasized by Birge,[1] between the values of the electronic specific charge obtained from experiments on cathode rays and those obtained from spectroscopic measurements, a precise determination of this constant has become of considerable importance.

Up to about 1927 the various determinations made with cathode rays, although rather widely scattered, seemed to be converging to a value near 1.768×10^7 e.m.u., and most reviews[2] of the subject ignored the value of 1.761 obtained by Babcock.[3] In 1927 a determination made by means of an interferometric measurement of the Rydberg constants of H and He$^+$ agreed so well with Babcock's result,[4] and a careful cathode-ray measurement by Wolf[5] showed such good agreement with the expected higher value, that Birge was led to suggest that the two types of measurement were really measuring different things. Subsequent theoretical work[6] has failed to show any justification for this difference on the basis of the present theory, so that it remained an open and important question as to what is the correct value of e/m.

This investigation was undertaken to make a precise measurement of the Zeeman effect under conditions which differed as much as possible in detail from those of Babcock. The result which has been obtained is

[1] R. T. Birge, Phys. Rev. Supplement **1**, 47 (1929).

[2] W. Gerlach, Handbuch der Physik **22**, 41–82 (1926).

[3] H. D. Babcock, Astrophys. J. **58**, 149 (1923); **69**, 43 (1929).

[4] W. V. Houston, Phys. Rev. **30**, 608 (1927).

[5] F. Wolf, Ann. d. Physik **83**, 849 (1927).

[6] See among others L. D. Huff, Phys. Rev. **38**, 501 (1931).

$$e/m = 1.7579 \pm 0.0025 \times 10^7 \text{ e.m.u. per gram}$$

which confirms the previous spectroscopic values, although it is a little lower than either of them.

Since this work was started, two papers have appeared which describe new cathode-ray measurements of the specific charge.[7] These results agree so well with the spectroscopic measurements that one is led to suspect that the "spectroscopic value" is correct, and that the discrepancy between the two methods was due to overestimation of the accuracy of the deflection methods.

GENERAL REQUIREMENTS OF A ZEEMAN EFFECT DETERMINATION

In order that a precise measurement of e/m can be made by means of the Zeeman effect it is necessary that the following three conditions be fulfilled.

(1). The magnetic field must be such that it can be accurately measured, reproduced, and kept constant during an exposure. It must also be uniform over a space large enough to contain the source of light. This was attained by the use of an air core solenoid.

(2). The spectroscopic measurements must be made with the desired precision. In the case of very simple patterns this can be done with a Fabry-Perot interferometer. The Zeeman pattern of a singlet line, when the source is viewed parallel to the magnetic lines of force, is a doublet, and so is ideal for use with this instrument.

(3). The Zeeman pattern of the lines used must be known with sufficient accuracy.

This, combined with the above requirement for the use of an interferometer, and the general requirement that the lines shall be sharp and as free as possible from hyper-fine structure, considerably restricts the selection of possible lines. Those finally selected were the $^1P - {}^1D$ lines, Cd $\lambda6439$, and Zn $\lambda6362$. The fairly high atomic weight of these elements tended to keep the Doppler broadening within reasonable limits, even though the temperature of the source was fairly high. At no time was any trace of hyperfine structure observed, although of course the source was not particularly adapted to showing it.

For these singlet lines the Zeeman pattern can be determined with considerable accuracy.[8] It is affected slightly by the nearness of the ^3P and ^3D levels, so that the g-factor of each term is slightly more than one. The amount of this correction can be determined with more accuracy than is needed. For cadmium $g_D = 1.00049$, and $g_P = 1.00216$. For zinc $g_D = 1.00003$, and $g_P = 1.0002$. States assigned to other electron configurations might be expected to produce a slight additional change in these quantities, but since the nearest states are from configurations with a different parity and hence have no effect at all, and since the effect of the next states certainly would be very small, these can be neglected.

[7] C. T. Perry and E. L. Chaffee, Phys. Rev. **36**, 904 (1930); F. Kirchner, Phys. Zeits. **31**, 1074 (1930).

[8] W. V. Houston, Phys. Rev. **33**, 297 (1929).

When the g-factors of both the initial and final states are unity, the longitudinal effect shows a doublet. When, however, these factors differ slightly from this value, the various components of the line do not overlap exactly, and the pattern becomes very complex. Since this complexity is not resolved, the position of the center of gravity must be calculated in order to interpret the measurements. For each elementary component of the line, the displacement due to the magnetic field is

$$\Delta\nu_{ij} = aeH/4\pi mc \text{ where } a = m_i g - m_i'g'. \tag{1}$$

g and g' are the splitting factors for the initial and the final states respectively, while m_i' and m_j are the corresponding magnetic quantum numbers. For the whole complex line the expression for the displacement of the center of gravity is

$$\Delta\nu = \bar{a}eH/4\pi mc \text{ where } \bar{a} = \sum_{ij}I_{ij}(m_ig - m_j'g')/\sum_{ij}I_{ij}. \tag{2}$$

In this expression I_{ij} is the intensity of the transition $m_i - m_j'$. These can be determined from the usual formulae.[9] For the cadmium line used, $\bar{a} = 0.99966$, and for the zinc line $\bar{a} = 0.999945$.

The result is then that

$$e/m = 4\pi c\Delta\nu/\bar{a}H. \tag{3}$$

This equation gives e/m in absolute electromagnetic units of charge per gram. Here e is the velocity of light in a vacuum, H is the magnetic field in absolute gauss, and $\Delta\nu$ is the displacement of the center of gravity in cm^{-1} reduced to vacuum.

Apparatus

1. The solenoid

The solenoid constructed for this work was designed to fulfill the following requirements.

(a) The variation of the field strength over a light source 6 cm long, at the center of the coil must not exceed 0.1 percent.

(b) A method of cooling must be provided to permit continuous operation at full power.

(c) Subject to the demands of (a) and (b), the maximum possible field strength should be obtained from the available power supply.

Fig. 1 shows a cross section of the solenoid as it was built in the laboratory shop. The winding is continuous and consists of 2449 turns of No. 4 (5.2 mm) square, cotton-covered, copper wire in 18 layers. The coil proper is 80 cm long and has an outer diameter of 39.7 cm, and an inner diameter of 7.6 cm. The coil was wound on a heavy brass inner tube, between cast brass spiders. It was insulated from the tube by a layer of 3/16″ micarta strips laid longitudinally, and from the spiders by micarta strips attached to them. Between each two layers of the coil was placed a layer of black fiber spacers, 6.5 mm × 3.2 mm × 80 cm, parallel to the axis of the tube and spaced so as to

[9] See for example, Pauling and Goudsmit, "Structure of Line Spectra," p. 142.

leave passages through the length of the coil. A similar layer of spacers insulated the wire from the brass tube into which the completed coil was forced. The ends of the shell were closed by cast brass plates which were screwed to the inner tube and to a flange on the outer tube. The entire assemblage weighed 1200 lbs., and was supported by a wooden platform. In order to obviate the accidental presence of iron in the spiders and the end plates they were cast from freshly alloyed copper and zinc.

The solenoid was cooled by pumping kerosene through the passages left between the fibre spacers. A centrifugal pump maintained an estimated flow of about 200 liters per minute through a circuit consisting of the solenoid and eight automobile radiators. The radiators were assembled in a unit and immersed in a tank of running water. To protect the oil against contamination in case of leaks, the radiators were placed on the discharge side of the pump, so that the pressure of the kerosene was always higher than that of the water
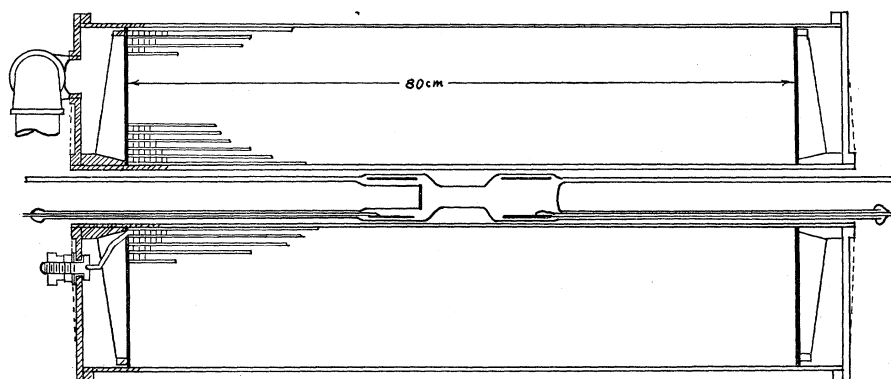

Fig. 1. Construction of solenoid.

outside. To test the insulation offered by the kerosene we frequently measured the resistance between the winding and the shell, and found it always between 10^5 and 10^6 ohms. The leakage resistance across the coil was necessarily of the same magnitude. As the resistance of the coil was 1.3 ohms between the terminals, the difference between the measured current and the effective current was negligible.

Two compound generators connected in series supplied the solenoid with a full-load current of over 200 amperes at 270 volts. The field circuit of one of the generators was controlled by means of a rheostat near the solenoid. A field of 7300 gauss, which required 54 kilowatts, could be maintained continuously without exceeding a temperature of 50°C in the circulating kerosene. During an exposure the exciting current was measured by means of a 0.001 ohm Leeds and Northrup shunt and a deflection potentiometer, and was controlled by adjusting the generator field current.

It was at first expected that it would be possible to determine the field-current ratio of this solenoid simply by careful measurement of the winding. For this purpose, each layer was carefully measured as it was wound. The

result obtained from this measurement was $K = 36.68$ gauss per ampere. This result is interesting, however, only in the fact that it is 0.5 percent lower than the value obtained by measuring the field in the completed coil. The difference is in the right direction to be explained as the result of the compression of the inner layers by the tension of those wound over them.

2. The source of light

The light was produced by evaporating zinc and cadmium in the positive column of a direct current discharge through helium. The tube, which is shown in Fig. 1, was constructed entirely of quartz. The illumination could be confined to a constriction, 6 cm long and elliptical in cross section (1 cm $\times 3$ cm), which was placed at the center of the solenoid. The electrodes were short sections of copper tubing 2 cm in diameter. A re-entrant window extended through the anode to within 2 cm of the end of the constriction, and the proportions of the tube were such that neither of the electrodes was visible from the lens used to focus the light on the slit of the spectrograph. Then by adjusting the pressure of helium so that the cathode glow was confined to within a few mm of the cathode, no light entered the spectrograph except that emitted from the region of maximum magnetic field.

The helium was continuously circulated and purified by means of charcoal and liquid air. The heat of the discharge was sufficient to vaporize zinc and cadmium shavings placed in the constriction or in the cathode. The whole tube was wrapped with copper foil and asbestos to maintain a relatively uniform temperature throughout. At currents above 600 m.a. the spectra of zinc and cadmium were produced with such intensity as to completely suppress the helium lines. All exposures were made, however, at a lower current, since the zinc and cadmium lines were somewhat sharper when their intensities were roughly one-third of that of He $\lambda5876$. The current for the discharge was furnished by four 500 volt generators connected in series, and was regulated by a series resistance.

3. Optical apparatus

The Fabry-Perot interferometer was the one previously described in connection with the study of the hydrogen fine structure,[10] and was placed between the collimator and the prism. The plate surfaces were sputtered with gold which was sufficiently dense to show about twenty visible reflections of the filament of a ten watt lamp. If the resolving power is roughly estimated as half the number of visible reflections multiplied by the order of interference, it was about 10^6, since most of the exposures were made with an order of interference something over 10^5. This corresponds to a plate separation of 3.5 cm. The spectroscopic apparatus was mounted on a heavy concrete slab supported by twelve tennis balls. This arrangement proved stable, and effectively protected the interferometer from the unavoidable vibration of the cooling pump. The supports of the collimator, interferometer, and camera

[10] W. V. Houston, Astrophys. **64**, 81 (1926).

were independently fastened to the concrete base, so that each part could be separately alligned. To protect the interferometer and prism from temperature variations a tight wooden box was used, from which the slit and the camera projected through felt gaskets. The box rested on a felt pad covering the concrete base and could be removed without disturbing the interferometer. The temperature was observed with a Beckman thermometer and was regulated by electrical heating to within 0.05°C. The spectroscopic equipment was placed 2 meters from the solenoid to prevent disturbance either of the interferometer or of the solenoid field.

The aperture of the interferometer was reduced to about 1 cm by means of a diaphragm, and the necessary exposure times ranged from 30 to 90 seconds with Ilford Extra Rapid Panchromatic plates.
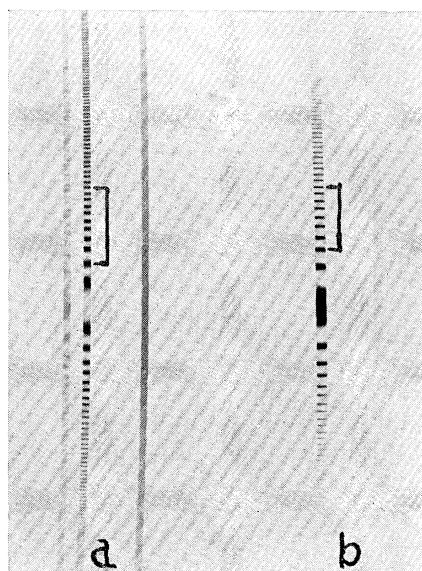

Fig. 2. Interferometer patterns. The brackets indicate pairs of fringes between which the Zeeman separation was measured.

Care was always taken to see that the fringes due to one component of the doublet lay half way between those due to the other. This prevents any displacement of the maxima due to slight overlapping. At the order of interference used, the normal Zeeman separation, with the maximum field of 7300 gauss, caused the components to overlap with a separation of 4.5 orders. In Fig. 2a the Zn and Cd red lines are shown at this separation. Two components, between which the separation would be measured, are indicated with a bracket. Because of the uneven spacing, this exposure was not measured. Fig. 2b shows the pattern of the Cd red line with the components separated approximately 3.5 orders at a field of 6860 gauss.

Magnetic Measurements

The intensity of the magnetic field during an exposure was determined by the relation

$$H = KI = KP/R \tag{4}$$

where K is the constant of the solenoid in gauss per ampere and I is the current in amperes. This current was determined from the reading P of a potentiometer connected across a shunt of resistance R. The measurements were made in international amperes, and the small correction to absolute units was made in the final result. The value of K was determined by comparing the field produced by a measured current in the large solenoid, with the field in a single layer standard solenoid. The magnetic constant of the standard solenoid could be calculated from its dimensions.

The comparisons between the standard solenoids and the large solenoid were made by two methods. The first is a null method, which affords a direct comparison between the two constants. It is limited, however, to measurements in which I is less than one ampere. On account of the possible influence of ferro-magnetic surroundings, heating, and slight distortion of the coil due to magnetic forces, it is undesirable to place too much dependence on the constancy of K over a large range of currents. Hence another method was also used which permitted determinations of K to be made for all currents.

1. The standard solenoids

Three different standard solenoids were used in the calibration. Two of them consisted of a layer of No. 12 bare copper wire wound on a bakelite tube which had been threaded with ten turns per inch. The bakelite was of linen stock in order to avoid the reputed ferromagnetism of paper stock bakelite. The other solenoid was wound with No. 20 enameled wire on a brass tube which had been threaded with 28 turns per inch, and upon which a layer of insulating varnish had been baked. All were of such a size that they could be placed within the inner tube of the large solenoid.

The number of turns per cm in the winding of each solenoid was determined by measurement with one of two scales. One was a Starrett steel meter, and the other was a glass scale taken from a cathetometer. Both scales were calibrated against a Gaertner type M1001 standard meter at Pomona College. We are indebted to the members of the physics department at Pomona, for their kind cooperation in helping us to make this comparison. At a temperature of 20.5°C the glass scale showed an excess length of 0.032 percent and the steel scale an excess of 0.008 percent. The error was uniform as nearly as could be determined, and has been applied to all measurements made with these scales. The measurements were made by placing the scale next to the solenoid, and reading the points at which a coincidence was observed between a scale division and the edge of a turn. The turn density in the various intervals was then evaluated and averaged over all the intervals. In no case was an appreciable deviation from uniformity noticed.

TABLE I. *Measurements of the standard solenoids.*

A. Dimensions, for the end correction.

Solenoid	Effective diameter in cm	Effective length in cm	End correction (cos α)
Bakelite No. 1	6.00	89.7	$0.99776 \pm 0.005\%$
Brass	5.86	90.0	$0.99788 \pm 0.002\%$
Bakelite No. 2	5.97	89.49	$0.99777 \pm 0.001\%$

B. Measurements of turn density, showing change with time.

Solenoid	Date	Scale	Number of readings	Number of intervals	Turns per cm	K_s	Mean deviation
Bakelite No. 1	10/10/30	Steel	20	10	3.93219	4.93028	0.02%
	5/ 4/31	Glass	32	70	3.93190	4.92990	0.01%
	7/13/31	Glass	12	36	3.92814	4.92519	0.01%
	8/29/31	Glass	38	116	3.92710	4.92389	0.01%
Brass	4/ 5/31	Steel	8	4	11.0236	13.8233	0.003%
	5/14/31	Glass	12	36	11.0235	13.8231	0.003%
	8/30/31	Glass	26	97	11.0230	13.8225	0.002%
Bakelite No. 2	7/ 8/31	Glass	66	174	3.93517	4.93406	0.03%
	8/26/31	Glass	12	36	3.93204	3.93014	0.02%

Table I gives the data from the measurements of the two solenoids to-gether with the values of the constants computed from the relation

$$K_s = 0.4\pi n \cos \alpha \qquad (5)$$

where n is the number of turns per cm and α is the angle subtended at the center by the radius at one end.

2. The null method calibration

The arrangement of apparatus for determining the ratios between the constant of the large solenoid and those of the standards is shown in Fig. 3.
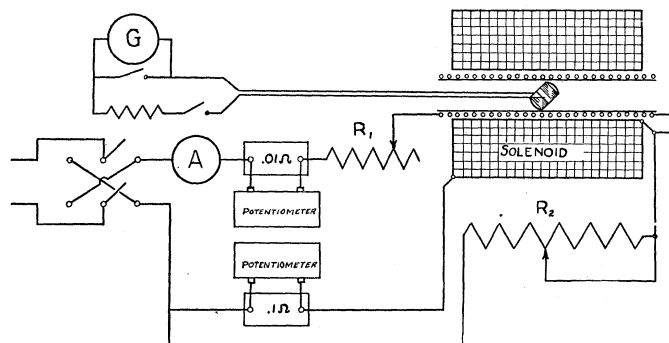

Fig. 3. Diagram of electrical connections for null calibration.

The standard solenoid was placed within the inner tube of the large solenoid and was connected so that the two fields were opposing. A large flip coil

wound with 10,000 ohms of No. 40 wire was placed at the common center of the solenoids and connected to a Leeds and Northrup wall-type ballistic galvanometer. The ratio of the currents in the two solenoids was then varied until a balance was indicated by a zero deflection in the galvanometer when the coil was turned over. The ratio of the constants is then given by

$$R = K/K_s = I_s/I \tag{6}$$

where the subscript $_s$ denotes the standard. The currents were measured with potentiometers and shunts.

In practice the currents were read at a series of values giving small galvanometer deflections. A plot of the deflections against the current ratios gave the balance point. The influence of the earth's field was removed by reversing both currents. The potentiometers were interchecked and showed a maximum departure of 0.02 percent. The same standard cell was used with both potentiometers. The shunts used were checked by the Southern California Edison Company's testing laboratory against resistances certified by the Bureau of Standards. In addition, the 0.1 ohm and the 0.001 ohm shunts were tested by the Bureau of Standards. The values of the resistances are given in Table II. The 0.1 ohm shunt was usually used at about 6 amperes so that the average of the values for 1.5 and 15 amp. has been used. The 0.001 ohm shunt was used at about 200 amp. At this current the heat developed is about half that at 300 amp. so the average of the two values was taken as being about correct.

TABLE II. *Calibration of shunts used.*

Nominal resistance (Ohms)	Edison Co. value	Bureau of Standards	Adopted value	Correction to be added to nominal value
0.001	0.00100050	60 amp. 0.0010004 300 amp. 0.0099994	0.0010000	0.000%
0.01	0.0099941		0.0099941	−0.059%
0.1	0.100024	1.5 amp. 0.10004 15 amp. 0.10003	0.100035	+0.035%
1.0	0.99955		0.99955	−0.045%
10.0	9.9969		9.9969	−0.031%

Table III gives the results of the null method calibration. There is no evidence of systematic change of the large solenoid between the summers of 1930 and 1931, so that all the observations were averaged together to get the final result. Each value of K in the last column was given a weight equal to the number of individual complete determinations from which it came. This number is given in column 3. The mean deviation is about one part in three thousand. During the summer of 1931 both bakelite standards changed considerably as can be seen from Table I. The value of K_s in Table III was obtained on the assumption that the change was linear with the time, at least between the times at which the solenoid was measured.

Since the current used in the standard solenoids caused them to become somewhat warmer than they were when measured, a small temperature cor-

TABLE III. *Calibration by the null method.*

Standard solenoid	Date	Number values	Mean P_s/P	Shunt corr.	I_s/I	K_s (Table I)	K
Bakelite No. 1	7/29/30	5	7.4766	+0.094%	7.4836	4.9302	36.896
	3/ 8/31	1	7.4739	−0.080%	7.4679	4.9300	36.817
	8/ 1/31	3	7.4899	+0.014%	7.4909	4.9247	36.890
Brass	8/ 9/31	3	2.6671	+0.014%	2.6675	13.823	36.873
	8/14/31	1	2.6701	−0.069%	2.6683	13.823	36.884
Bakelite No. 2	7/22/31	7	7.4781	−0.080%	7.4721	4.9328	36.858
	7/29/31	2	7.4769	+0.014%	7.4799	4.9325	36.887
	8/22/31	2	7.4830	−0.080%	7.4770	4.9305	36.865

Weighted Mean $K=36.872$ Temperature correction$=-0.011$ K Corrected$=36.863$
Mean deviation $=0.018$

rection must be applied to these values. The rise in temperature was about 15°, so that the correction to be applied is −0.03 percent. This is no more than the mean deviation of the various measurements.

3. Calibration with a mutual inductance.

In order to permit determinations of the solenoid constants to be made under conditions similar to those prevailing during an exposure, the arrangement shown in Fig. 4 was adopted. The current in the primary of the mutual
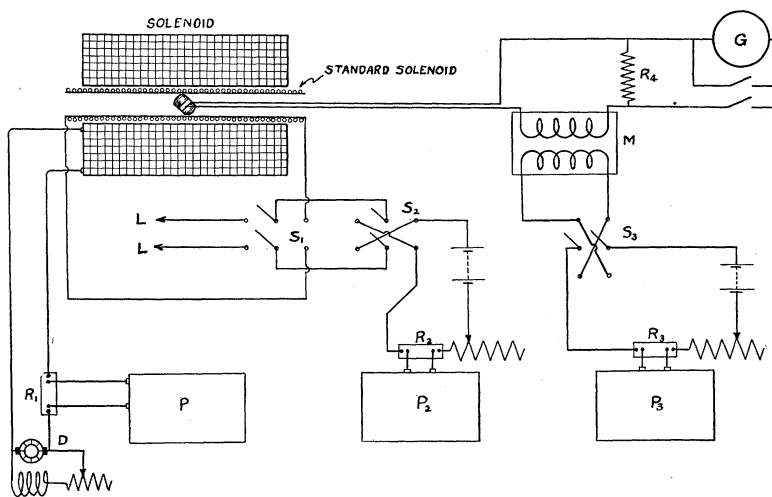

Fig. 4. Diagram of connections for calibration with mutual inductance.

inductance, M, was varied until its reversal gave a deflection of the galvanometer equal to that produced when the flip coil was operated in the field of the standard solenoid alone. The galvanometer shunt was such that full scale deflections were obtained. Since the total resistance in the galvanometer cir-

cuit was the same for both deflections, we had, in effect, a calibration of the mutual inductance and the flip coil in terms of the standard solenoid constant and the ratio of the currents. Let K_s = constant of the standard solenoid, gauss per ampere, I_s' = current in standard solenoid in amperes, $H_s = K_s I_s'$ in gauss, F = magnetic area of the flip coil, M = mutual inductance, I_m' = current in mutual inductance.

Then $2FH_s = 2MI_m'$ and $FK_s I_s' = MI_m'$, whence

$$M/F = K_s I_s'/I_m'. \tag{7}$$

After calibration, the mutual inductance could be used to determine the constant of the large solenoid at full current. The galvanometer shunt R_4 was increased until a suitable deflection was obtained when the flip coil was operated in the field of the large solenoid. The current in the primary of the mutual inductance was then raised until a reversal of this current with the switch S_3 gave the same deflection. The currents I and I_m in the solenoid and the mutual inductance respectively were then read from the potentiometers. Then

$$K = MI_m/FI \tag{8}$$

With Eq. (7) this gives

$$K = K_s I_s' I_m/I_m' I. \tag{9}$$

The accuracy and reliability of this method are dependent on several factors, which may be listed as follows:

(a) The galvanometer constant must not change during the calibration of the inductance, nor during the calibration of the solenoid from the inductance. The galvanometer used was a Leeds and Northrup Type P ballistic instrument. It was found necessary to found necessary to place it at least 70 feet from the solenoid, in order that the stray field would not change the constant. Although the galvanometer reading had to be reproducible, almost no dependence was placed upon the proportionality of the deflection to the charge which passed through the secondary. A series of deflections, covering a small range, was made using the flip coil. Intermingled with these was a second series of readings, in the same small range, obtained by reversing the current in the primary of the mutual inductance. Each deflection was then plotted against the current at which it was read. Graphical interpolation to the same deflection in both series gave the desired current ratio, I_m/I or I_m'/I_s'.

(b) The method also requires that the mutual inductance have the same value when being compared with the large solenoid as it has during the comparison with the standards. This requires constancy of inductance over a wide range of primary current values. Comparisons with the standards were intermingled with comparisons with the large solenoid so that any permanent change in inductance would show itself in the results. No such change was observed as can be seen from Table IV. Errors due to ferromagnetic surroundings were minimized by suspending the inductance midway between the floor and the ceiling of a large adjacent room. The construction of the inductance was such as to guard against possible changes with time or current.

A low resistance primary of No. 12 wire was wound on a micarta tube 8 inches in diameter and 15 inches long. The secondary consisted of 12 lbs. of No. 32 S.C.C. and enameled wire wound in three separate coils and rigidly supported inside the primary.

TABLE IV. *Calibration of the mutual inductance.*

	No. 1 Bakelite solenoid	Brass solenoid
Measured P_s/P_m	39827	14225
	39834	14234
	39842	14230
	39834	14223
	39851	14234
	39829	14227
	39840	
	39813	
	39846	
	39852	
	39854	
	39833	
Shunt	39838±8	14229±4
Correction −0.066%	26	+0.014% 2
	39812	14231
After temperature correction K_s	4.9284	13.819
K_sI_s'/I_m'	1962.09±0.4	1966.58±0.2
Weighted mean from the two solenoids		$K_sI_s'/I_m' = 1963.59 ± 2.0$

(c) The required constancy of the flip coil was indicated by the coherence of the readings.

(d) The galvanometer shunt resistance needed to remain constant only during each half of the calibration. Separate coils of Chromel wire were prepared for each required value of this resistance.

Table IV gives the results of the calibration of the mutual inductance against the standard solenoids. The surprising thing is the large difference between the results obtained with the two different standards. This is far larger than the accidental errors in the measurements, and is far larger than one has a right to expect, from the results of the null method calibration. This unexplained discrepancy indicates that the uncertainty is at least 0.1 percent.

Table V gives the results of the calibration of the large solenoid against the mutual inductance. Readings were taken for three different ranges of current. One was in the neighborhood of 1 amp., the next between 100 and 125 amp., and the third was near 200 amp. The slight systematic trend shown by the results is less than the probable uncertainty of the readings and so has been neglected. The mean of these three determinations was averaged with the mean of the null method readings to get the final value of the solenoid constant.

In this calibration three potentiometers were used which were checked against each other. The maximum disagreement was 0.03 percent while the average was about 0.001 percent. All potentiometers were always used with

TABLE V. *Calibration of large solenoid against mutual inductance.*

Current range	200 amp.	100 amp.	1 amp.
Observed	53282	53278	53266
Potentiometer	53356	53322	53315
Ratios	53311	53311	53289
P/P_m	53311	53311	53297
	53305		53285
	53289		
	53262		
	53282		
	53300		
	53283		
	53299		
Mean	53298 ± 17	53305 ± 14	53290 ± 12
Shunt correction $+0.035\%$	19 $+0.015\%$	08 $+0.014\%$	7
I/I_m	53316 ± 17	53313 ± 14	53297 ± 12

$$K = \frac{K_s I_s' I_m}{I_m' I} = 36.829 \pm 0.041 \qquad 36.831 \pm 0.041 \qquad 36.842 \pm 0.041$$

Mean value of $K = 36.834 \pm 0.041$

the same standard cell, so that no corrections were necessary and small changes of the cell with time were of no account.

Although the calibration with the mutual inductance shows some four times the uncertainty of the null method calibration, the two values have been averaged with equal weight, since the calibration under actual operating conditions is rather to be preferred to the null method where only low currents could be used. This gives for the adopted value

$$K = 36.846 \; gauss \; per \; ampere.$$

This value has been reduced by 0.005 percent from the mean of Tables IV and V to change the units from international to absolute gauss per ampere.

SPECTROSCOPIC MEASUREMENTS

In terms of the solenoid constant and the current the expression (3) for e/m is

$$e/m = (4\pi c/\bar{a}K)(\Delta\nu/I). \qquad (3')$$

The difference between the wave numbers of the two Zeeman components is found from the difference in order of interference at the center of the fringe pattern.

The relation for the fractional order of interference at the center of the pattern, in terms of the diameters of the fringes, is

$$p = \frac{D_i^2}{D_i^2 - D_{i-1}^2} - i \qquad (10)$$

where D_i is the linear diameter of the i-th ring from the center of the pattern. Two approximations are involved in Eq. (10). Tan θ has been substituted for θ, and $1 - \theta^2/2$ has been substituted for cos θ. The errors thus introduced are quite negligible since θ, the angular diameter of the largest fringe meas-

ured, was less than 0.04 radians. The error is further reduced by the fact that only the differences between values of p are used.

The diameters, D_i, of roughly twenty fringes of each component were measured on a comparator. From the table of values of D_i^2 a mean value of $(D_i^2 - D_{i-1}^2)$ was obtained. Dividing this into each D_i^2 gave, by Eq. (10), a table of $(i+p)$ for each component. The mean of the fractional parts was then the desired order of interference at the center. From the difference in order, $(i+p) - (i'+p')$, between the fringes corresponding to $+\Delta\nu$ and $-\Delta\nu$ in the Zeeman pattern, the separation in cm^{-1}, or Balmers, is given by

$$2\Delta\nu = (i + p - i' - p')/2dn \qquad (11)$$

where n is the index of refraction of air. This use of the index of refraction of air is necessary since the value of the velocity of light in vacuum is used. The method of Lord Rayleigh was used in evaluating d which could be determined with an accuracy of one or two parts in a million.[11] The order of interference was changed after every three or four exposures.

During the exposures the solenoid current was measured by the same shunt and potentiometer that were used in the 200 ampere calibration. By constant regulation of the generator field the current was maintained with average fluctuations of less than 0.1 percent. A week before the exposures were started, the standard cell was checked with a new cell having a Bureau of Standards certificate. Several checks indicated that the cell was constant and known to at least one part in ten thousand.

TABLE VI. *Spectroscopic measurements.*

Cadmium 6439				Zinc 6362			
Plate	$2\Delta\nu/I$	Order diff.	Wt.	Plate	$2\Delta\nu/I$	Order diff.	Wt.
1	0.0034386	1.4587	1	9	0.0034379	4.5130	3
2	344	4.5109	2	11	78	4.5127	1
3	412	4.5195	2	12	66	4.4882	2
4	382	4.5157	2	14	74	4.5132	2
5	416	3.5122	2	16	70	4.5127	1
6	355	4.5082	2				
7	338	4.4819	1				
8	402	3.5080	1				
10	329	4.5088	1				
12	396	4.4921	1				
13	399	4.5190	2				
14	336	4.5080	2				
15	357	4.5108	2				
17	341	2.4877	1				
Mean	0.0034372 ±28				0.0034374 ±5		
$2\pi c/aK'$	51149				51134		
e/m	1.7581 ±0.0025				1.7577 ±0.0025		

Thirty-one separate reductions of nineteen interferometer patterns gave the values of $2\Delta\nu/I$ listed in Table VI. More than half of the patterns were independently measured and reduced by both of us. In these cases the results showed an average discrepancy of 0.03 percent and a maximum discrepancy

[11] Rayleigh, Phil. Mag. 9, 685 (1906).

of 0.13 percent. The mean of each pattern is given a weight equal to the number of times it was measured. In determining e/m the value of c was taken as 2.99796×10^{10} cm per sec. K' is the value of K adopted above, but reduced by 0.016 percent to take account of the fact that the field in the solenoid was not entirely uniform and had to be averaged over the light source.

As a result of these measurements we may say with fair certainty that the spectroscopic value is

$$e/m = 1.7579 \pm 0.0025 \times 10^7 \text{ e.m.u. per gram.}$$

This is rather lower than the previous spectroscopic values, although the accuracy is not great enough to make certain that it is really different. However it is certainly far lower than the high deflection values.
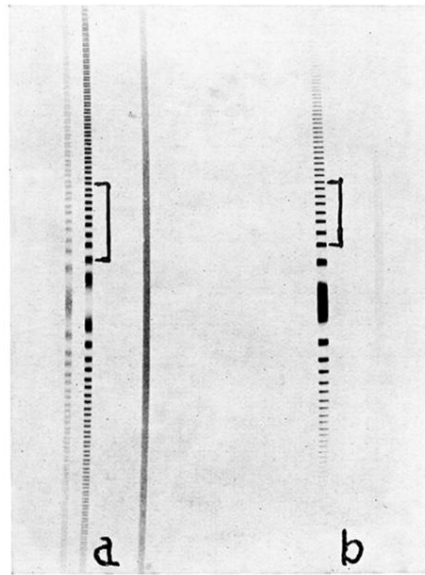
Fig. 2. Interferometer patterns. The brackets indicate pairs of fringes between which the Zeeman separation was measured.