INELASTIC AND ELASTIC ELECTRON SCATTERING IN ARGON

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Abstract

The energy distribution and the angular distribution of electrons scattered by argon atoms were investigated both for elastically and inelastically scattered electrons. *Elastic scattering* was investigated over the angular range between 10° and 170°, and for the energy range between 50 and 550 volts. The scattering curves fell off steeply with increasing angle. In the case of the 50 and 100 volt electrons, however, maxima were found at 100° and 90° respectively. The 400 and 550 volt curves when plotted as a function of sin $(\theta/2)/\lambda$ (where θ is the angle and λ the de Broglie wave-length) were superposable, in agreement with Mott's theory. The number of elastically scattered electrons, integrated over all angles, depends on the colliding energy in much the same way as does the difference between the total electron absorption coefficient and the ionization efficiency. The energy distributions of electrons scattered *inelastically* in argon at 10° were measured for 50, 100, and 200 volt electrons. It was found that as the energy of the colliding electrons increased the probability of the larger energy losses became relatively greater than the probability of the smaller energy losses. Angular distribution curves (5° to 35°) for such losses (11.6 to 34.0 volts) were steeper the smaller the magnitude of the loss. For any one loss, the steepness increased with the speed of the colliding electron. Electrons which have lost more than half the energy left over after ionization are called *ejected* electrons. To each ejected electron there corresponds a *colliding* electron, and the sum of their energies amounts to the original energy before collision less the energy of ionization. The angular distributions of slow ejected electrons having various amounts of energy (1 to 8 volts) were studied for different collision energies (50 to 200 volts). Such distribution curves showed a general distribution of electrons over all angles with, in many cases, strong maxima at large angles (90° to 160°). The positions of the maxima depended in a definite way upon the energy of collision and the energy of the ejected electron.

The study of the scattering of electrons by gaseous atoms has received a new impetus in recent years (a) because of the remarkable experimental results of Davisson and Germer, of Thomson, and of Rupp, and (b) because some aspects of the problem have been interpreted successfully from a new theoretical standpoint, that of wave mechanics.

Mott[1] calculated the angular distribution of electrons scattered by gas atoms by means of wave mechanics. He arrives at the result that the "scattering coefficient", that is, the number of electrons scattered in a direction θ from that of the original beam through unit solid angle, from unit length of path of the original beam, per single electron in this beam, per single atom per unit volume, is

$$\alpha(\theta, v) = \left[\frac{e^2}{2mv^2}(Z - F)\operatorname{cosec}^2\frac{\theta}{2}\right]^2 \tag{1}$$

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[1] N. F. Mott, Proc. Roy. Soc. **A127**, 658 (1930).

when e, m, and v, are the charge, mass, and velocity of the electron, Z the atomic number and F the atomic structure factor. Mott pointed out that there is a striking similarity between the wave mechanical theory of the scattering of electrons and the ordinary theory of scattering of x-rays, the formula for the latter case being

$$\alpha'(\theta) = \left[\frac{e^2}{mc^2} Z \operatorname{cosec}^2 \frac{\theta}{2}\right]^2. \qquad (2)$$

Important experimental investigations of electron scattering have been carried out recently by Arnot[2] and by Bullard and Massey.[3] They find that, in the region between 30° and 120°, the scattering coefficient shows maxima and minima which are especially well marked for the lower velocities. Mott's theory predicts a rapid decrease in the scattering coefficient with increasing angle, but fails to predict the presence of the maxima and minima. This is due to the omission of certain effects which are negligible only at high velocities. These effects—electron exchange, polarization of the atom by the electron, and distortion of the incident electron wave—, which should be considered in a more complete theory, modify the theoretical scattering curve especially at low velocities, and qualitatively account for the presence of the maxima and minima.[4] At sufficiently high velocities, Mott's theory is in satisfactory agreement with experiment.

In addition to the elastically scattered electrons, there are many varieties of inelastically scattered electrons which have lost different amounts of energy as a result of exciting or ionizing the atom. A complete experimental exploration of the field of excitation losses would call for a study of the scattering of many distinct groups of electrons, each group corresponding to a particular energy loss which is determined by the energy level to which the excited atom is raised. Spectroscopy shows that there are many of these energy levels and it is probable that there is a characteristic angular distribution for the electrons which have been effective in raising the atom to the particular energy level considered.[5] We should also consider the scattering of electrons which have ionized atoms. In the absence of evidence to the contrary, we may assume that the *colliding* electron may lose any amount of energy *greater* than that just necessary for ionization. The electron *ejected* from the atom in the ionization process will, if conservation principles apply and if the positive ion has but one energy value, have an amount of energy just equal to that lost by the colliding electron over and above the amount necessary for ionization A complete analysis of the problem would call for a study of the angular distribution of both the colliding and ejected electrons over the full range of possible energies. As it is impossible to distinguish be-

[2] F. L. Arnot, Proc. Roy. Soc. **A129,** 361 (1930); **A130,** 655 (1931); **A133,** 615 (1931).

[3] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A130,** 579 (1931); **A133,** 637 (1931).

[4] F. L. Arnot, Proc. Roy. Soc. **A133,** 615 (1931); E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133,** 637 (1931).

[5] For an interesting theoretical treatment of this problem, see H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A132,** 605 (1931).

tween the identity of two electrons appearing after the ionization of an atom, it is convenient for the present to call the faster electron the colliding electron and the slower one the ejected electron.[6]

The study of the scattering of electrons is closely related to two other fields, (1) the absorption of electrons by a gas, and (2) the ionization efficiencies of electrons of various energies. The integral of all electrons, scattered elastically and inelastically, taken from a certain small angle up to 180°, measures the total number of electrons diverted from the beam and therefore the absorption of the electrons by the gas. The total number of ionizing and ejected electrons, integrated from 0° to 180°, should be equal to twice the number of ions produced. (The direct measurements of the absorption and of the ionization efficiencies, of course, give far more accurate results than could be obtained by integration of the scattering curves.)

Experimental Method

The apparatus used is shown in Fig. 1. It consists of two parts, a collision chamber B and an analysing chamber C, the latter sorting out electrons of

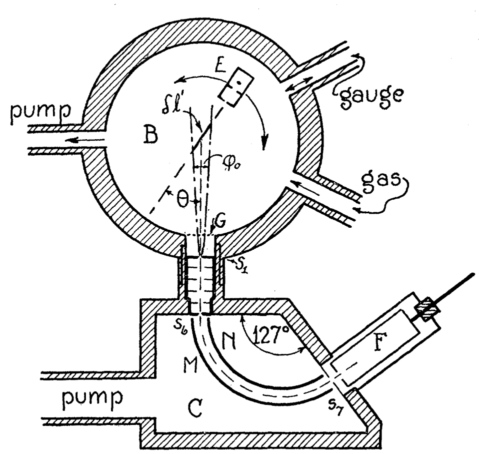

Fig. 1. Apparatus.

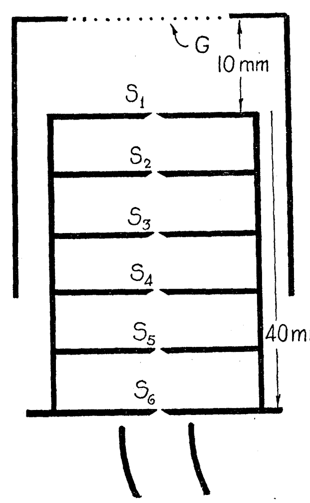

Fig. 2. Slit system.

different energies. The collision chamber is made of brass with three gas inlets, one connected to a diffusion pump, a second to a McLeod gauge and the third through a fine capillary to a reservoir containing pure argon. This constant renewing of the gas ensured a high degree of purity in the collision chamber. The pressure could be held constant at any desired value, the pressures actually used in the experiments to be described being between 0.0015

[6] If the ionization of a single atom by an electron could be studied, it would be interesting to find whether or not the angle of scattering of the ejected electron is uniquely determined by the angle of scattering of the colliding electron. It does not have to be uniquely determined, for the positive ion may share the energy and momentum lost by the colliding electron with the ejected electron in an infinite number of ways.

mm and 0.0030 mm. By means of a ground glass joint (not shown) the electron gun E could be rotated so that the main beam of electrons could be set at any angle θ to the beam of scattered electrons passing into the analyser. The analyser is one in which electrons of different energies are sorted out by a radial electrostatic field. As the analyser has been described at length in other papers, no further discussion will be given here.[7] The collision chamber and electron gun are substantially the same as those described in fuller detail by McMillen,[8] except for the fact that in the present apparatus the gun could be rotated as far as the 170° position. The analyser was separated from the collision chamber by a slit system (see Fig. 2) designed to offer considerable resistance to gas flow (because of the number of slits), so as to allow a very low pressure to be maintained in the analyser which was connected to a high speed diffusion pump. Since S_1 and S_6 were slightly smaller than the other slits, they defined the electron beam entering the analyser. The chief innovation in this apparatus is a grid in front of the entrance slits of the analyser which allows one, by means of a suitable field, to accelerate electrons up to the slits. The grid G is made up of fine platinum wires 0.001 inches in diameter, set about 0.01 inches apart. The wires run perpendicularly to the lengths of the slits and not parallel to them as implied in the diagram. The analysers used by McMillen and Van Atta could not be used for low velocity electrons. For some reason, not fully understood, it was not possible to get measurable effects when electrons of energy less than about 40 volts were directed on to the slits. It appeared as though the slits became sufficiently charged in some way to refuse admission to slowly moving electrons. Possibly electrons get entangled on an invisible film of grease or other insulating material over the slit surfaces and so abnormally large spurious contact potentials are formed. The obvious way to minimize this effect is to outgas the whole system at a high temperature, but this was impossible on account of the way the apparatus was assembled.[9] Consequently to study the distribution of slow electrons it is necessary to speed them up to an arbitrarily chosen speed. This was effected by applying the proper accelerating potential between the grid G and the slit S_1. The usual procedure in studying the energy distributions of a group of electrons was to set the potential between the deflecting plates M and N to deviate electrons of 100 volts energy from the lower entrance slit S_6 to the exit slit S_7. Then electrons of velocity V_0 approaching the grid perpendicularly in the collision chamber would have to be accelerated by a potential $(100-V_0)$ volts in order to register in the Faraday cylinder. The experimental variable was therefore the accelerating voltage $100-V_0$, from which the original energy of the electrons in the collision cham-

[7] A. L. Hughes and J. H. McMillen, Phys. Rev. **34**, 291 (1929); A. L. Hughes and V. Rojansky, Phys. Rev. **34**, 284 (1929); J. H. McMillen, Phys. Rev. **36**, 1034 (1930); L. C. Van Atta, Phys. Rev. **38**, 876 (1931); E. Rudberg, Proc. Roy. Soc. **A129**, 628; (1930); **A130**, 182 (1930).

[8] J. H. McMillen, Phys. Rev. **36**, 1034 (1930).

[9] See J. D. Whitney, Phys. Rev. **34**, 923 (1929); D. C. Rose, Canad. Journ. of Research **3**, 174 (1930).

ber could be inferred. The presence of the field between the grid and the first entrance slit has a focusing effect (when $V_0 < 100$), the result of which is to exaggerate the number of slower electrons present. It can be shown that the effect of an accelerating voltage V is to change the angle ϕ_0 between two electron paths into ϕ_1 where

$$\phi_1 = \phi_0 \frac{1}{(1 + V/V_0)^{1/2}}$$

and where V_0 is the initial energy of the electrons in volts (Fig. 3). (It is assumed that the angles are small enough to allow us to use the angles themselves in place of their sines and tangents.) The grid and slit system are drawn

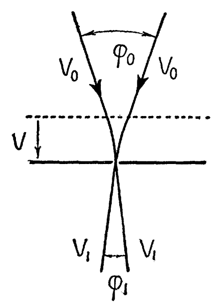

Fig. 3. Schematic representation of focusing effect.

approximately to scale in Fig. 2. From the dimensions given it is possible to show that ϕ_1 (width of S_6 divided by 40 mm) is 40 minutes. This angle ϕ_1 is fixed and consequently the angle ϕ_0 depends on the values of V and V_0. Since ϕ_0 in Fig. 1 determines $\delta l'$, the length of path of the original electron beam from which scattering into the analyser takes place, it is evident that in order to get comparable quantities we must *multiply* the experimentally measured electron current into the Faraday cylinder, N_θ'', by the factor

$$\frac{1}{(1 + V/V_0)^{1/2}}. \tag{3}$$

This gives us a corrected value N_θ'. This same length of path, $\delta l'$, also depends on the angle of scattering θ, and increases with $\sin \theta$ (Fig. 1). Hence to reduce the scattering to some constant path length, δl, we must multiply by $\sin \theta$. Thus the true value, N_θ, the number of electrons scattered from a constant length of path, δl, is

$$N_\theta = N_\theta' \sin \theta = N_\theta'' \frac{\sin \theta}{(1 + V/V_0)^{1/2}}. \tag{4}$$

The scattering coefficient is defined by the following equation

$$N_\theta = \alpha(V, \theta), \; N_0 \cdot \delta l \cdot \delta\omega \cdot n$$

where N_θ is the number of electrons scattered through an angle θ, within a solid angle $\delta\omega$, from a length δl of a beam of electrons containing N_0 electrons, when there are n atoms per unit volume. The scattering coefficient, $\alpha(V,\theta)$, defined in this way, is a function of the angle θ, and V, the energy of the electrons. It depends also on the nature of the scattering atom. Except when there is a statement to the contrary, the ordinates in the figures showing the experimental results graphically are proportional to this scattering coefficient $\alpha(V,\theta)$.

Elastic Scattering

The scattering coefficient for electrons scattered without loss of energy has been investigated over a range of electron energies between 50 and 550 volts. The results are shown in Fig. 4. For small angles, *i.e.*, for angles less

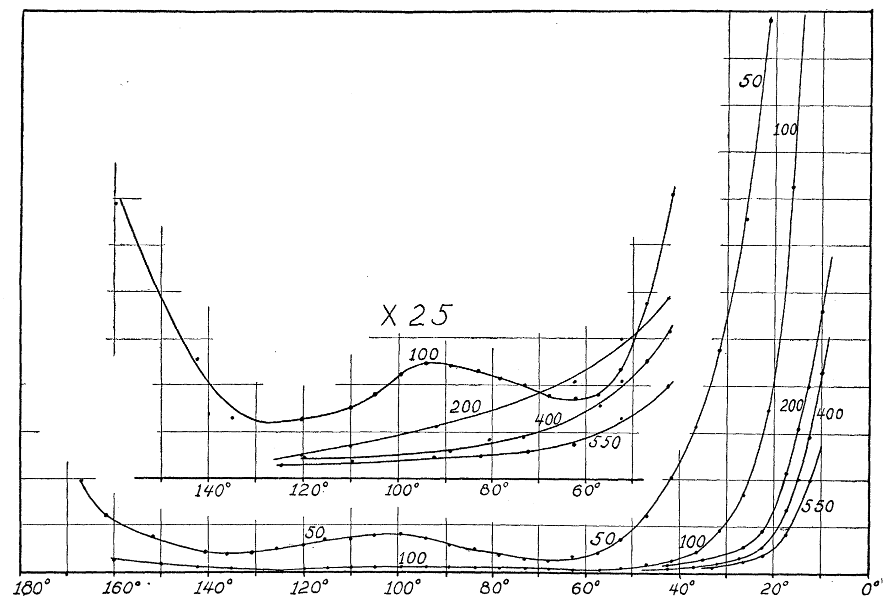

Fig. 4. Elastic scattering in argon.

than about 40°, the scattering coefficient decreases rapidly with increasing angle, the decrease being faster the greater the energy of the electrons. For the 50 and 100 volt energies we find a definite maximum in each curve in the neighborhood of 100°. The positions of these maxima are in satisfactory accord with the positions found by Arnot.[10] His range did not extend beyond 120°. In our experiments the range extended to 170°. It is interesting to note the marked increase in the scattering between 140° and 170°, a result predicted by Holtsmark's theory.[11]

Mott[12] derived the formula given in Eq. (1) for the scattering of electrons

[10] F. L. Arnot, Proc. Roy. Soc. **A133**, 615 (1931).

[11] See Fig. 7 in the paper by E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133**, 637 (1931).

[12] N. F. Mott, Proc. Roy. Soc. **A127**, 658 (1930).

by individual atoms using the method of wave mechanics. Since the electron wave-length $\lambda = h/mv$, the expression can be changed into

$$\alpha(\theta, v) = \left[\frac{e^2 m}{2h^2}(Z - F)\frac{1}{\mu^2}\right]^2 \tag{5}$$

where $\mu = \sin(\theta/2)/\lambda$, a quantity upon which x-ray scattering has been shown to depend. The scattering coefficient should therefore have the same value for the same μ, no matter what the velocity of the electron may be, provided we take such a value of θ to give the same μ. Our results are given in Fig. 5. It is clear that the higher the energy of the colliding electrons the closer do

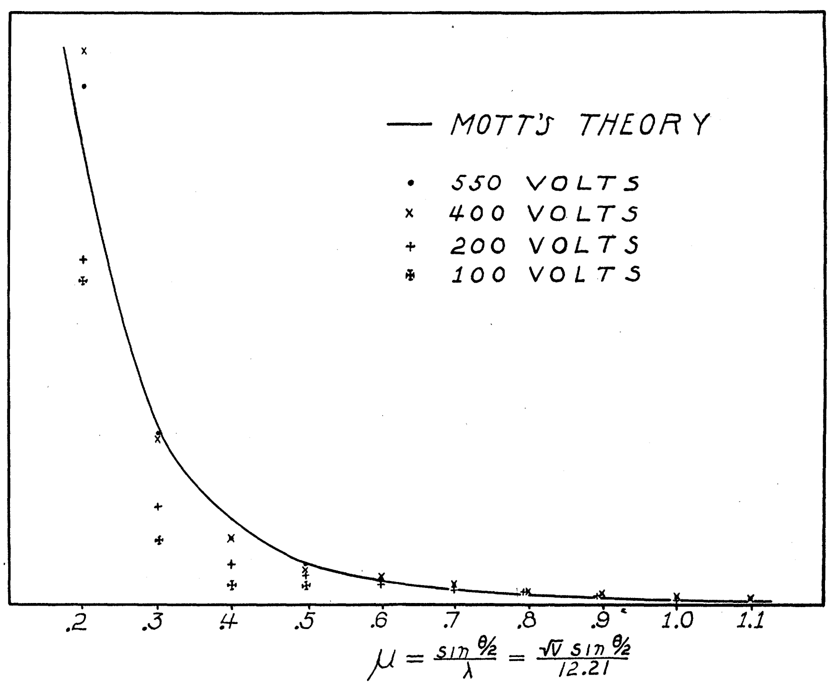

Fig. 5. Elastic scattering plotted as a function of μ. Mott's theoretical curve is plotted on half scale.

the experimental points come to the theoretical curve. Mott pointed out that his formula should agree better with experiment the higher the energy of collision. These experiments then indicate that, for scattering of electrons by argon atoms, Mott's theory gives an accurate description of the facts when the electron energy exceeds about 400 volts.[13,14]

[13] The deviation of the experimental results from the theoretical curve for 400 and 550 volt electrons, at $\mu = 0.2$, is explained by the fact that this corresponds to a very small angle where the experimental errors are considerable.

[14] The F values used in plotting the curves were computed by James using the Hartree field for the argon atom. We wish to thank Mr. N. F. Mott for communicating these unpublished results to us.

We attempted to determine the value of the scattering coefficient in absolute units. This is a matter of considerable difficulty, for it involves assumptions as to the geometry of the apparatus and especially as to the *effective* dimensions of the very narrow slits S_1, S_6, and S_7. The final result, however for the 500 volt electrons, gave a value only 50 percent less than that calculated by Mott, which is perhaps the best we can expect in view of the fact that the apparatus was certainly not designed to give good absolute determinations.

While the work was in progress the very interesting and comprehensive results of Arnot and of Bullard and Massey were published. We therefore temporarily postponed our studies of elastic scattering and turned to the hitherto unexplored territory of inelastic scattering.

Inelastic Collisions

The amounts of energy lost by electrons in inelastic collisions differ according to the type of collision. The least amount of energy which can be lost is that corresponding to the first radiation potential of argon, viz., 11.6 volts;

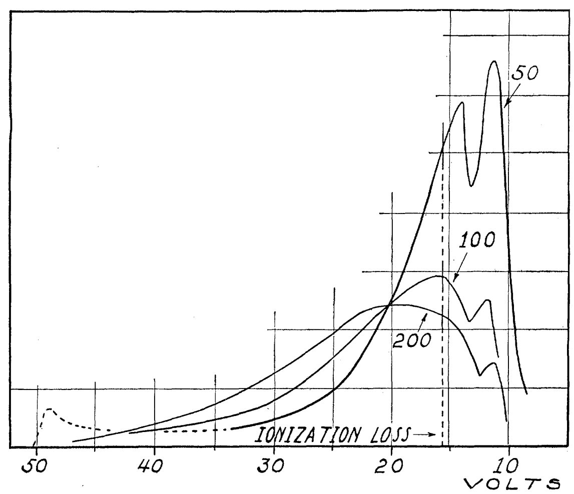

Fig. 6. Energy losses in argon. Angle of scattering 10°.

then follow a series of greater losses corresponding to excitation of the atom to higher levels. A "continuous spectrum" of energy losses is to be expected when ionization of the atom is effected, for presumably the energy left over after ionization may be divided in an infinite variety of ways between the original electron and ejected electron.

Typical distributions of inelastic energy losses for electrons colliding with different speeds, all scattered at 10^0, are shown in Fig. 6. The 11.6 volt loss stands out distinctly, but the resolution of the apparatus was not sufficiently good to isolate other excitation losses. The curves are arbitrarily fitted together at an energy loss of 20 volts (i.e., 4.5 volts above the ionization potential). The curves have been corrected for the focusing effect by means of Eq.

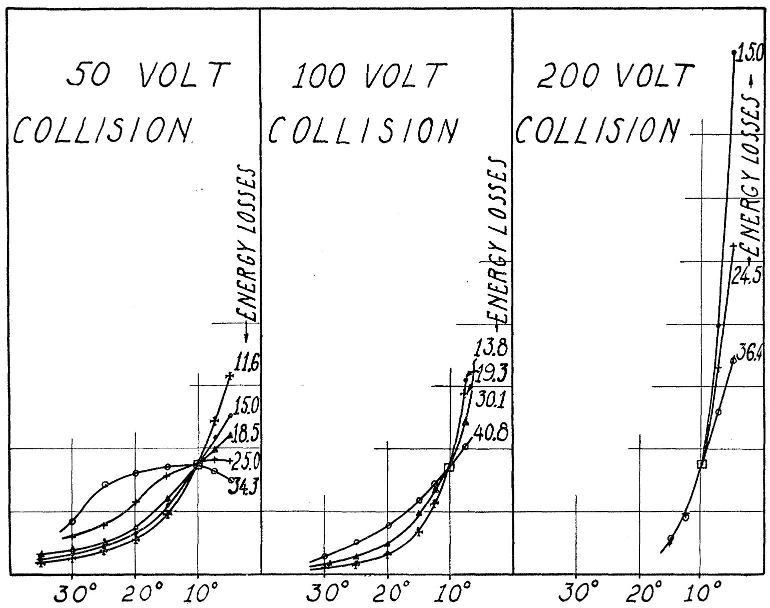

Fig. 7. Angular distributions of various energy losses for 50, 100 and 200 volt collisions.

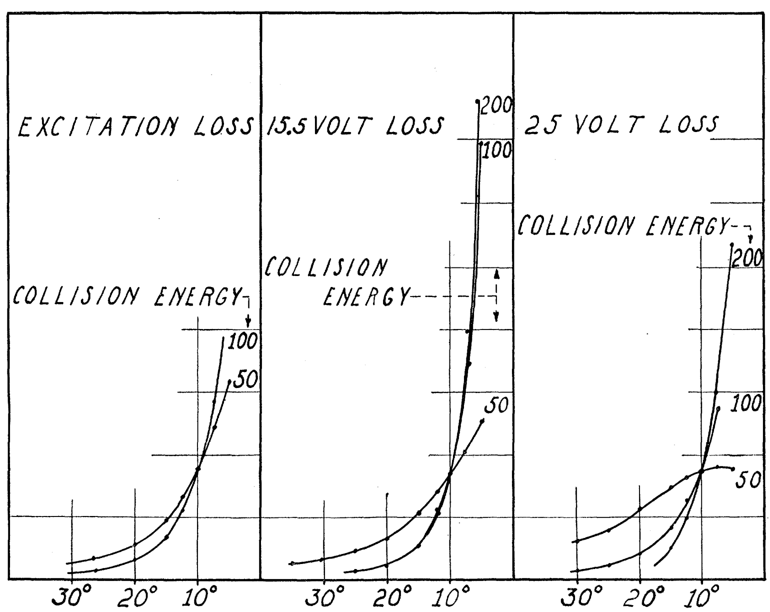

Fig. 8. Dependence of angular distributions on the collision energy for different energy losses.

(3), and for slight absorption of the electrons in the gas. Both corrections were small. It is clear that as the energy of the colliding electron increases, (1) the number of collisions producing the 11.6 volt excitation diminishes relatively, and (2) the number of collisions in which the colliding electron ionizes the atom and loses a considerable amount of energy to the ejected electron over and above that necessary for ionization, increases relatively to the total number of collisions.

The angle distributions of various losses were measured and the results are shown in Figs. 7 and 8. In Fig. 7 we compare the angular distribution curves for various energy losses, first when the electrons in the primary beam have 50 volt energy, then when they have 100 volt energy, and finally when they have 200 volt energy. In Fig. 8 we have picked out a particular energy loss and shown how the corresponding angular distribution curve depends on the

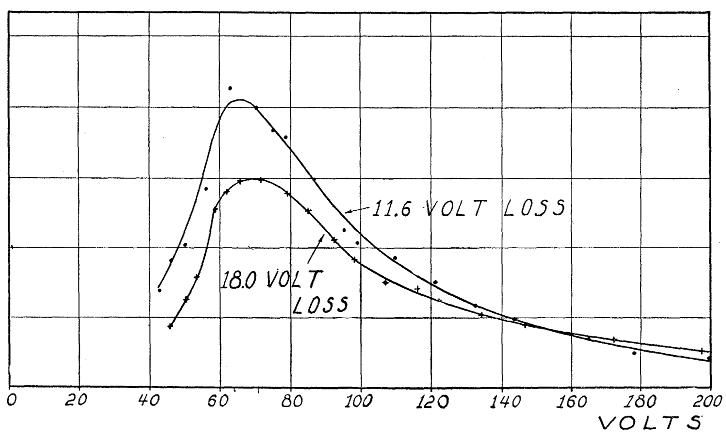

Fig. 9. The 11.6 and 18 volt energy losses, at 10°, plotted as a function of the collision energy.

energy of the electron before collision. The results may be summarized as follows: The *lower* the energy loss, the *steeper* is the angular distribution curve. (This is particularly true for the lowest collision energy, viz., 50 volts.) For any given energy loss, the angular distribution curves are *steeper* the greater the energy of impact.

The scattering coefficient for the 11.6 volt loss and for the 18.0 volt loss is plotted as a function of the energy of impact in Fig. 9, the angle of scattering in all cases being 10°. It is evident that primary electrons having 63 volts energy result in the maximum number of 11.6 volt energy losses, while primary electrons having 70 volts energy give the greatest number of 18.0 volt energy losses. (This statement covers only the case where the electrons are scattered through 10°; further systematic experiments are required to determine whether or not it is true for other angles of scattering as well.)

Attention is called to the fact that in our experiments the 11.6 volt loss has been found over a wide range of collision energies (Fig. 9) and over a wide range of angles (Figs. 7 and 8). Bullard and Massey[15] were unable to find this loss in their experiments and quoted several others (Dymond, Michels, Oppenheimer) to the effect that this particular loss should not occur since the corresponding energy level is metastable, and so could be appreciably excited only by voltages very close to the excitation potential itself. However, we have found strong excitation over a wide range of voltages and associated with a fairly wide range of scattering angles. Well marked peaks corresponding to the 11.6 volt loss were also found by Van Atta[16] for zero scattering angle, and for a range of colliding energies from 100 to 300 volts. Van Atta took especial care to identify the peak and concluded that the energy loss was 11.53 ± 0.05 volts, which is in close agreement with the value 11.57 volts obtained from spectroscopic data. This agreement makes it difficult to maintain that the identification of the peak in our experiments and in those of Van Atta's is open to question. We should like to have traced the 11.6 volt

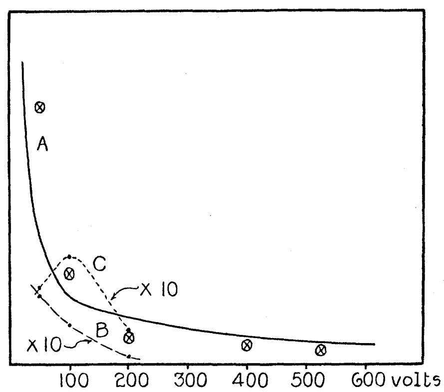

Fig. 9a. Absorption coefficients as computed from scattering curves. *A*, total absorption coefficient less ionization efficiency (Normand; Smith); *B*, "excitation" absorption coefficient; *C*, "ionization" absorption coefficient; *crosses*, "elastic" absorption coefficient. (×10 means that the real ordinates of *B* and *C* have been increased tenfold.)

energy loss curve in Fig. 9 right down to a colliding energy of that amount, as it is conceivable that a strong maximum would be found with colliding electrons of energy a volt or two above 11.6 volts, but experimental difficulties made it impossible to explore this region.

By multiplying the ordinates of the elastic scattering curves (Fig. 4) by $\sin \theta$ and then integrating over all angles, we get a measure of the total number of electrons lost from the initial beam through elastic collisions. The results of such an integration are plotted as a function of the electron energy in Fig. 9*a*. The total absorption coefficient of electrons as ordinarily measured gives the number of electrons lost from the beam through elastic scattering and inelastic scattering, the latter being associated with excitation and ionization. Since the number of collisions associated with excitation losses is rela-

[15] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A130**, 583 (1931).
[16] L. C. Van Atta, Phys. Rev. **38**, 876 (1931).

tively small, an absorption coefficient for elastic scattering could be obtained by subtracting from the total absorption coefficient as determined by Normand,[17] the total ionization efficiency as measured by Smith.[18] The result of such a subtraction is plotted on an arbitrary scale in Fig. 9a. The agreement with our results is fairly satisfactory, when the uncertainty involved in integrating over the very small and the very large angles is realized. A somewhat similar procedure allows us to calculate the total number of electrons associated with all the excitation losses and with all the ionization losses, leading to what may be called the "excitation absorption coefficient" and the "ionization absorption coefficient". The results are included in Fig. 9a. The ionization absorption coefficient so calculated has the same general shape as that determined by direct experiment. It is not possible to obtain accurate values of the various absorption coefficients in this way as considerable errors are possible in the graphical integration.

Ejected Electrons

An electron of energy V collides with an atom and loses energy amounting to V'. If $V' > V_+$, the ionization potential, then ionization takes place and the ejected electron goes off with energy $V' - V_+$. The energy left to the colliding electron is $V - V'$. (The colliding and ejected electrons have identical energies when they share, equally between them, the surplus left over after ionization, and this has a value $\frac{1}{2}(V - V_+)$.) We shall refer to the faster of the two electrons as the colliding electron and to the slower as the ejected electron. These are to be regarded as convenient labels, for we do not mean to insist that the electron going away from the atom with the greater speed after ionization is necessarily identical with the electron which hit the atom and caused the ionization. In a complete experimental description of the process of ionization we should need to include the angular distributions of all colliding and ejected electrons.[19]

In Fig. 6 we show all the possible energy losses after a 50 volt collision, resulting in electron energies after collision right down to zero. (Such low energies after collision are not shown for the 100 and the 200 volt collisions because we used a scale suitable for displaying certain other features.) We shall now consider the experimental distributions of the low velocity electrons appearing after various kinds of collisions. It will be remembered that N_θ'', the experimentally measured current of electrons entering the Faraday cylinder must be multiplied by the factor $1/(1 + V/V_0)^{1/2}$ in order to correct for the focusing action of the accelerating field between G and S_1. This factor becomes very different from unity for ejected electrons of very small energies. As a concrete case we may state that in one set of experiments electrons were ac-

[17] C. E. Normand, Phys. Rev. **35**, 1217 (1930).

[18] P. T. Smith, Phys. Rev. **36**, 1293 (1930).

[19] The ideal method would be to study single encounters and to measure θ for the colliding electron and ϕ for the ejected electron, and to find out whether or not ϕ is determined uniquely when θ is given. There is possibly no unique value for ϕ, because for a given θ there may be many combinations of ϕ and of ψ the angle of recoil of the atom.

celerated up to a constant energy of 100 volts. Electrons of 100 volts energy would require no accelerating voltage V, while electrons of 1 volt energy would require an accelerating voltage $V=99$ volts. In the first case, no correction would be needed, i.e., $N_\theta'=N_\theta''$, whereas in the latter case the correction would be given by $N_\theta'=1/10\ N_\theta''$. While we believe that the theory for the correction factor applies to the whole range, it is evident that if it is only approximate, the errors due to its use will be relatively greater the larger the ratio V/V_0. Thus it is possible that the numerical values for N_θ for ejected electrons of low energy are not strictly comparable with those for much faster electrons.[20]

In Fig. 10 we have plotted the distributions of energies of the slow electrons ejected by electrons of different speeds. The continuous lines represent the experimentally measured electron current to the Faraday cylinder, i.e.,

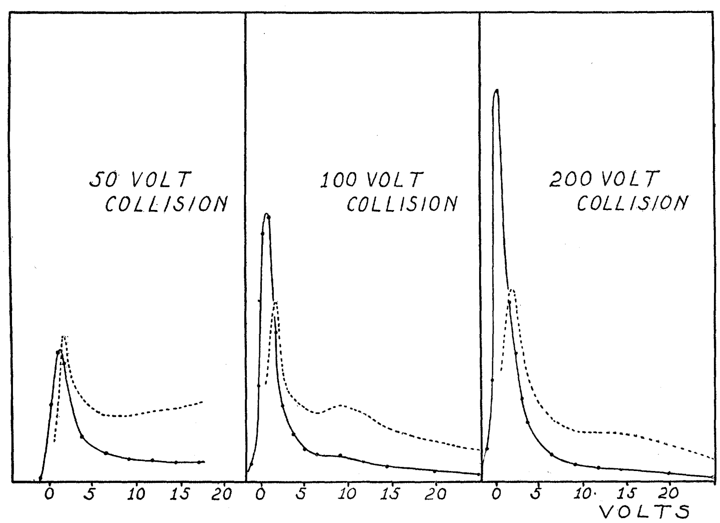

Fig. 10. Energy distribution (at 10°) for electrons ejected at 50, 100 and 200 volt collisions.

N_θ'', while the broken lines represent the values, N_θ', of the electron current after correction for focusing has been made. (The values of N_θ' have been increased by a scale factor of 10, otherwise the characteristics of the curve would hardly have been observable. The three pairs of curves are not to the same scale, however.) A survey of the curves shown in Fig. 10 shows that the slower electrons are the more plentiful. The peaks indicating the most probable energy of the ejected electrons occur between 0.5 and 1.0 volts. However, the resolution for slow electrons analysed by the method of introducing an

[20] It was possible to check the theory giving the factor $1/(1+V/V_0)^{1/2}$ over a moderate range by carrying out some experiments in which no accelerating field was applied between G and S_1. Since the analyser would not register electrons entering it if their energies were below about 40 volts, it was impossible to check the theory for slower electrons.

accelerating field between G and S_1 is poor, and it may well be that a more accurate investigation would show the peaks to come exactly at zero energy.[21]

To each colliding electron which has lost an amount of energy V', i.e., $V' - V_+$ over and above the energy necessary for ionization, there should correspond an electron ejected with energy $V' - V_+$. Hence the total number of each should be equal. Experiment gives us N_θ'' for each. As in Eq. (4) we get the number scattered per unit solid angle by multiplying by sin θ and by the focusing factor $1/(1+V/V_0)^{1/2}$. To get the *total* number scattered in *all* directions we multiply by sin θ again and integrate from 0° to 180°. On doing this we get pairs of curves analogous to those shown in Fig. 11. All that we can say is that the area under each curve is of the same order. One source of un-

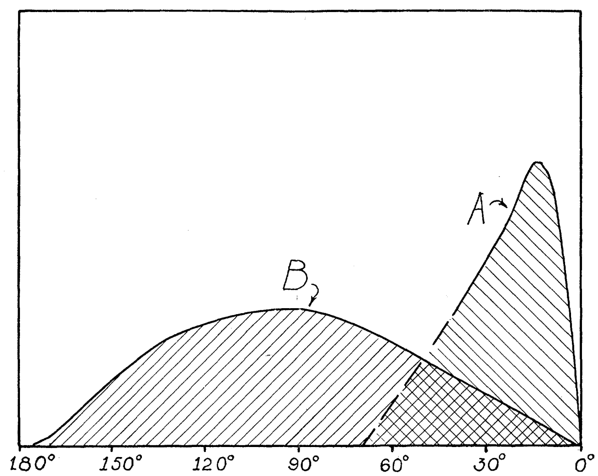

Fig. 11. A, Total number of electrons losing energy 3 volts more than energy required for ionization. B, Total number of ejected electrons having 3 volts energy. (The ordinates here measure the number of electrons scattered between the cones θ and $\theta + d\theta$.)

certainty lies in the fact that the smaller the angle, the larger is the number of colliding electrons scattered (see Fig. 7) and the measurements become increasingly inaccurate as we go to smaller angles, especially below 20°. Then

[21] F. L. Arnot, Proc. Roy. Soc. **A129**, 361 (1930); Proc. Camb. Phil. Soc. **27**, 73 (1931) has shown that when a beam of electrons is sent through a gas at a low pressure in a supposedly field-free space, a small field is set up between the inside of the beam and points at some distance from it, because the secondary electrons leave the beam with greater speeds than the positive ions. Such a field may distort the paths of electrons whose angular distribution we are investigating and this effect will clearly be larger the lower the energy of the scattered electrons. This effect should therefore be much more marked with our slowest ejected electrons than with the faster colliding electrons. Arnot found that the difference of potential set up between the inside of a beam and a point far outside it was proportional to the strength of the current in the beam and amounted to 2.0 volts for an electron current of 25 microamperes through mercury vapor at about 0.001 mm. As our currents were generally less than 0.5 microampere, it seems justifiable in a preliminary survey of inelastic scattering to assume that the effect discovered by Arnot could be neglected.

again it is not certain that we can assume that the focusing effect is completely taken care of by the factor $1/(1+V/V_0)^{1/2}$, when we apply it to a range of electron energy extending from, say, 100 volts down to 0.5 volt. However, the agreement is perhaps as good as can be expected under the circumstances.

It should be noted that the integral of the number of ejected electrons having energies from 0 to $\frac{1}{2}(V-V_+)$ volts, or the integral of colliding electrons having energies from $V-V_+$ down to $\frac{1}{2}(V-V_+)$ volts (integrated in each case over all solid angles) should give the ionization efficiency for electrons colliding with energy V. More direct methods of course give the efficiencies far more exactly.

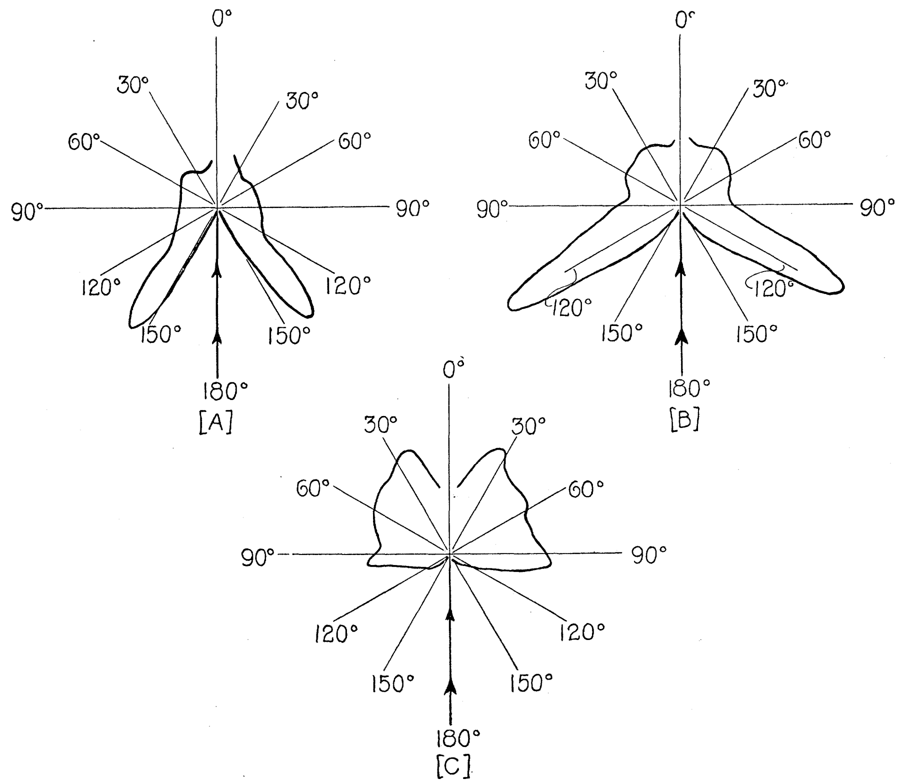

Fig. 12. Angular distribution of electrons ejected with 1 volt energy for (A) 50 volt collisions, (B) 100 volt collisions, and (C) 200 volt collisions.

We now proceed to discuss the angular distributions of ejected electrons of energies 1, 3, 5.5, and 8 volts, when these are produced by primary electrons having energies 50, 100, and 200 volts. The results for the 1 volt ejected electrons are most strikingly displayed on polar diagrams (Fig. 12). The angular distributions are symmetrical on both sides of the main beam, as they should be. The results for all are plotted in Fig. 13. Before going into detail we may make the following general deductions.

(A). In striking contrast with the angular distributions of colliding electrons, in which the concentration is very high at small angles, i.e., in the for-

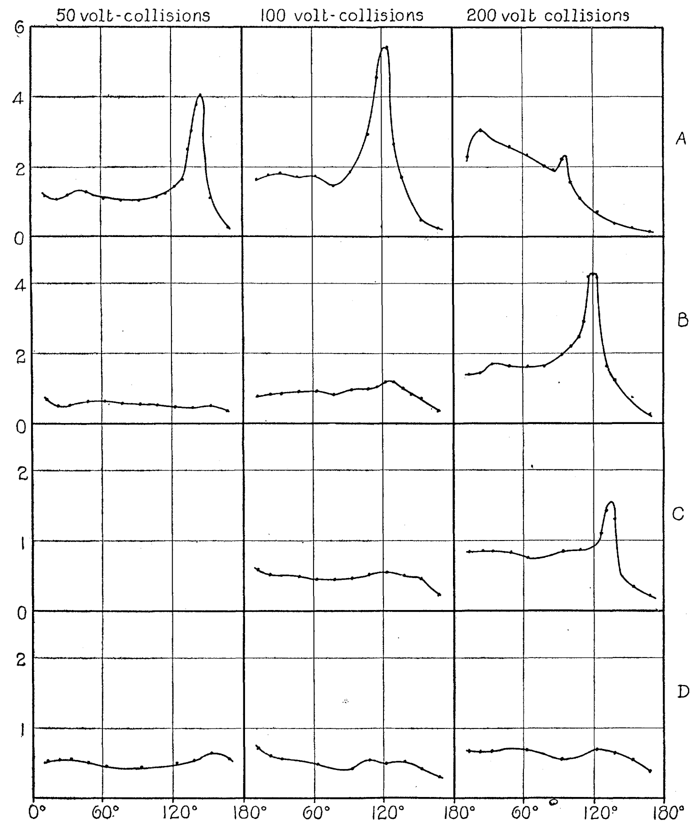

Fig. 13. Angular distribution of ejected electrons. *A*, 1 volt ejected electrons; *B*, 3 volt ejected electrons; *C*, 5.5 volt ejected electrons, *D*, 8 volt ejected electrons. (Note: vertical scale for *C* and *D* differs from that for *A* and *B*.)

ward direction (Fig. 7), the angular distributions of the ejected electrons are more evenly spread over a wide range of angles, with well-marked peaks at large angles in some cases.