SHOT EFFECT IN SPACE CHARGE LIMITED CURRENTS

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Abstract

The shot effect in space charge limited currents from tungsten and thoriated tungsten has been investigated. The effective fluctuation level is recognized, in general, as the combination of a depression due to pure space charge, and an elevation due to the liberation of electrons by positive ions. The magnitude of the elevation may be such as to carry the fluctuation level far above that for the same current under temperature limitation, or it may simply alter the form of the depression curve. The abnormal fluctuations observed have been analyzed, and their cause traced to inherent mixed emission from the metal surface. The effect of age and heat treatment has been studied for specimens of tungsten. A double grid tube has been used for obtaining a space stream consisting solely of electrons, and the conditions determined under which this stream is governed by the laws of pure chance applying in the simple shot effect theory. With this arrangement space charge measurements of fluctuations can be made over a considerable range with the same degree of precision attainable with temperature limited currents. Considerations involving statistical correlation between instantaneous values of the anode current provide a possible theoretical basis for the space charge depression of shot effect. This treatment yields an expression for the ratio of the mean square value of the fluctuation voltage under given space charge conditions to that under strict temperature limitation of the current:

$$\bar{V}_s^2/\bar{V}_t^2 = f(i_0/i)e^{-\omega^2/2\alpha^2}$$

i_0 is the space current, i, the saturation current, or total emission associated with a particular emitter temperature, and their ratio characterizes the space charge situation. It has been shown that in the absence of abnormal effects, the depression of the mean square fluctuation voltage which results is independent of the frequency, $\omega/2\pi$, at which it is measured. This conclusion provides information regarding the spread of the correlation function defined in the theoretical section. A relation showing the fluctuation depression proportional to the square of the current depression under space charge is indicated.

§1.

The study of spontaneous current fluctuations which occur in a circuit containing a thermionic element has given rise to new problems of considerable interest concerning the nature of the emission, and the passage through space of the charged particles. Treated theoretically first by Schottky[1] and later by Furth,[2] Ornstein and Burger,[3] and Fry[4] these manifestations of the atomic nature of electricity have come to be grouped under the general designation of the "shot-effect".

[1] Schottky, Ann. d. Physik **56**, 541 (1918).
[2] Fürth, Phys. Zeits. **23**, 354 (1922).
[3] Ornstein and Burger, Ann. d. Physik **70**, 622 (1923).
[4] Fry, Jour. Frank. Inst. **199**, Feb. (1925).

A technique of experimental investigation first successfully carried out by Hull and Williams[5] involves the use of a frequency selective screen grid vacuum tube amplifier of four or five stages. The thermionic current is caused to flow through an external tuned circuit of high impedance, and the fluctuations in potential across this unit are impressed upon the grid of the first amplifier tube. With a modification of the formula derived by Schottky for the mean square value of the fluctuation in potential, adapted to the particular circuit in use, Hull and Williams succeeded in determining the charge of the electron within a probable error of less than 1 percent.

Extension of this work and improvement of the technique has been reported in several recent papers. Particular mention should be made of the introduction of the aperiodic shot circuit,[6] the measurement of the charge carried by the K+ thermion,[7] and the studies in abnormal shot effect which have been carried out during the past year.[8,9]

It has been the practice to refer to those effects as abnormal where the conditions underlying the theory are not fulfilled, and where, as a result, the value of the electronic charge computed in the customary way departs from the accepted value.

The results of Johnson[10] at low frequencies have been interpreted by him, and further analyzed by Schottky[11] on the basis of an independent effect due to secular changes in the emission characteristics of the cathode.

Following are enumerated in brief form the essential postulates which must be fulfilled by the thermionic discharge under the normal shot effect theory. Experiment should provide:

(1) Emission consisting entirely of similarly charged carriers (electrons or ions as the case may be).

(2) Complete evacuation of the space surrounding the emitter.

(3) Random distribution of emission over the surface of the emitter.

(4) Random distribution of emission in time. (The number of electrons escaping from any section of the emitter in a small element of time should be independent of the number escaping from other sections and during adjacent time intervals. Only under these conditions do the simple probability relations apply to the emission.)

(5) Full reproduction at the collector of the random distribution of the emission. This involves the absence of interaction between carriers during their passage, and is realized only when the thermionic current is strictly "temperature limited".

(6) Freedom from sources of fluctuation in the circuit other than those due to the discreteness of charge of the particles.

[5] Hull and Williams, Phys. Rev. **25,** 147 (1925).
[6] Williams and Vincent, Phys. Rev. **28,** 1250 (1926).
[7] Williams and Huxford, Phys. Rev. **33,** 773 (1929).
[8] Donal, Phys. Rev. **36,** 1, 172 (1930).
[9] Kozanowski and Williams, Phys. Rev. **36,** 1,314 (1930).
[10] J. B. Johnson, Phys. Rev. **26,** 71 (1925).
[11] W. Schottky, Phys. Rev. **28,** 74 (1926).

The complete realization of the situation outlined by these postulates is, of course, impossible. By the use of well-evacuated tubes, low current densities, and high fields, the assumptions may be met to a degree of approximation within the limits of the experimental error. This was accomplished in the original work on the electronic and ionic charge.

It was early observed that when either the current density was large or the potential difference small, the mean square voltage fluctuations $\overline{V^2}$ fell below the value indicated for the normal shot effect.

The presence of even a small amount of gas in the tube was found to be responsible for abnormal effects of enormous and rather uncertain magnitude. With some types of emitters, particularly those employing a coating of barium and strontium oxides, elevations rather than depressions of the value of $\overline{V^2}$, were also observed when space charge limitation obtained. This effect was traced to the liberation of electrons from space charge by positive ions. By the device of mounting an electron emitter and a positive ion emitter in the same tube, the source of these abnormal fluctuations was definitely established by Kozanowski and Williams.[12]

It has been the aim of this research to investigate the effects of electron space charge on the fluctuations in a circuit carrying a thermionic current from tungsten and thoriated tungsten emitters.

In part, the observed decrease in fluctuation level with increasing space charge density finds its explanation in the cutting off of a certain fraction of the electron stream. Electrons whose velocities of emission are insufficient to carry them against the reversed field to the potential minimum of the space charge region will eventually be returned to the cathode. These, of course contribute nothing either to the average value of the anode current or to the fluctuations in the external circuit.

However, it appeared that the reduction in fluctuation level was more rapid than the change in current ratio would warrant. A theory proposed by Uhlenbeck sought to explain this behavior on the basis of statistical correlation between instantaneous values of the anode current. One consequence of this hypothesis is the prediction of a change of the space charge depression as the frequency of the amplifying system is changed.

Experiments made during the summer of 1930 by the writers and Kozanowski seemed to provide some evidence of a trend with frequency. The range between 54,000 and 190,000 pps was covered in these tests. Commercial tubes having thoriated tungsten filaments of the inverted "V" type were used, the grid and plate being connected together so as to form a two-element tube. From the inconsistencies and wide variations in results however, it became evident at the start that it would be necessary to find ways of analyzing these variations, and elimination of the errors which they introduce. Discrepancies as wide as five to ten percent were observed under experimental conditions which were reproducible certainly within two percent.

Commerical tubes with tungsten filaments were next investigated, and

[12] Kozanowski and Williams, Phys. Rev. **36**, 1314 (1930).

irregularities of the same general character observed. It appeared that the net depression might be the resultant of opposing influences, at least one of which was not consistent from one test to another. From previous work it was thought that the tendency to elevation of the fluctuation level might be traceable to the action of positive ions in electron space charge. Since the effects persisted at potentials below the lowest possible ionizing potentials, and since they were observed in notably "hard" commercial tubes, an impure emission was suspected as the underlying cause. A program of experiments was then undertaken with three main objects in view:

(1) To demonstrate the presence of positive ions in the emission and to show that the observed irregularity was produced by their action on the field in space charge.

(2) To devise a means of obtaining pure electronic emission governed by the laws of probability.

(3) If the second object could be realized, to study the effect of pure electron space charge on the normal shot effect with especial reference, as indicated in the theory, to the frequency dependence of the depression ratio and its connection with the total emission or saturation current.

§2

Apparatus and Experimental Procedure

It is necessary here to make brief reference to the experimental arrangement used in this work and the type of measurements undertaken. A five stage, screen-grid amplifier with tuned impedance coupling formed the essential unit. The amplifier with all auxiliary circuits is mounted in a double mesh screened cage within which the experimenter works. An oscillator for calibration and impedance measurement is located in a separate smaller cage several meters distant.

The amplifier response is detected by a 1000 ohm thermojunction coupled with a few turns of wire to the coil in the plate circuit of the last stage. The associated galvanometer reading is proportional to the square of the junction current, and therefore to the square of the output voltage. For a standard input, the current from the oscillator is passed through an inductance potentiometer of accurately concentric cylinders. The potential drop may be computed from the length used, and provides high frequency voltages of the required order of magnitude. The current in this circuit is read by a Western Electric type 20-D vacuum thermocouple and galvanometer. Strict proportionality between this reading for various currents and that of the output galvanometer indicates a linear characteristic for the amplifier. If this is maintained and checked at intervals, the absence of appreciable regeneration is assured.

When measurements involve absolute determinations, as of the electronic charge, the amplifier must be calibrated and the area under its resonance curve accurately determined. It is of course essential that this factor remain unchanged during the course of a series of readings.

The fluctuation circuit is enclosed in a copper box shield. The plate potential is supplied by a bank of dry cells in convenient 45 volt blocks with continuous variation provided by a potentiometer arrangement. The space current, indicated by a Weston four range microammeter, is passed through a wire resistor, R of about 30,000 ohms. The emitter is heated by a storage battery of 2–12 volts. The cathode, the high potential side of the anode circuit, and the shield are all maintained at ground potential for high frequency currents by the use of suitably located by-pass condensers, each of 1 mf capacitance.

The observed magnitude connected with the fluctuations is the deflection of the output galvanometer, G_0. This is a measure of the mean square output voltage, $\overline{E^2}$, and $\overline{E^2}$ is directly proportional to the mean square input voltage, $\overline{V^2}$. It was shown by Williams and Vincent that for the aperiodic shot circuit

$$\overline{E}^2 = 2A_0{}^2A\epsilon i_0 Z^2 \tag{1}$$

where A_0 is the voltage amplification at the resonant frequency, A is the area under the relative response curve of the amplifier, ϵ the electronic charge, i_0 the mean emission current, and Z the parallel impedance of the input circuit, measured at the natural frequency of the amplifier.

If the fluctuation current be expressed in a Fourier series, we find for the mean square value of the kth component of the input voltage,

$$\overline{V}_k{}^2 = \tfrac{1}{2}C_k{}^2Z_k{}^2 = 2zZ_k{}^2\epsilon i_0 \frac{\omega_k}{2\pi k}$$

(see appendix, note 1, Eq. (9))

Since in Eq. (1), the factors A and A_0 are constant

$$\overline{V}_k{}^2 \sim \epsilon i_0 \sim \frac{\overline{E}^2}{Z^2} \sim \frac{G_0}{Z^2}\,.$$

The "depression ratio" is then given by

$$\frac{(\overline{V}_k{}^2)_s}{(\overline{V}_k{}^2)_t} = \frac{(G_0)_s}{(G_0)_t}\left[\frac{Z_t}{Z_s}\right]^2, \tag{2}$$

where the subscript s refers to a particular space charge limited current, and t to the temperature limited current at the same temperature of the emitter.

In practice the deflections of the galvanometer are not constant, but show a continual variation covering usually less than 5 percent of the scale. Readings may be determined either by watching the path of the scale in the telescope for perhaps 30 seconds and making a mental average, or by arithmetically averaging a large number of independent readings taken at random. The former method has a considerable advantage in the amount of time required, and was used in most of the work. The difference between the values of the deflection arrived at by the two methods is insignificant. The estimated maximum error for either is about 1 percent.

The impedance of the shot circuit is measured in essentially the following way. Current from the oscillator tuned to the peak frequency of the amplifier is passed through a standard variable condenser, and measured by means of a W. E. vacuum thermocouple. The drop across this unit is impressed upon the shot circuit, and the impedance calculated from the resulting current. The thermocouples used in these measurements are calibrated on the alternating current bridge described by Williams and Huxford.[13]

For temperature limited currents the maximum error estimated for this last determination is $\frac{1}{2}$ of 1 percent. In space charge measurements there seems no reason to question the validity of the method unless the impressed alternating electromotive force used in the impedance measurement becomes comparable to the potential difference between the electrodes of the tubes.

§3.

Abnormal Fluctuations in Space Charge Limited Currents

The fulfillment of the requirements mentioned in the first section for the study of fluctuations in space charge limited currents has been accomplished only through the introduction of additional means of control of that current. It was at first thought that the solution to the problem lay in simplification. In the preliminary work with commercial tubes of the 201-A type the grid and plate were connected together, forming essentially a two-element tube. The shape of these elements was recognized as unfavorable and the resulting effect on the field uncertain. It was expected however that any particular space charge condition, whatever its nature, could be reproduced by the suitable re-setting of the filament current, space current and plate potential.

As a matter of interest, and for the sake of continuity, mention is made here of the first attempts in the direction of simplification. A two-electrode tube was constructed with concentric cylindrical elements of the following dimensions:

		Diameter	Length
Filament	Tungsten	0.1 mm	24 mm
Plate	Nickel	16 mm	29 mm

The tube was sealed onto an evacuating system and the pressure reduced to 10^{-5} mm of mercury. The unit was then baked with an electric furnace for two hours at about 350°C. The metal parts were outgassed by heating to a bright red heat with a high frequency induction furnace. During the process the filament was heated continuously, and in the final stage of the treatment a current twenty percent greater than any planned for subsequent operation was used. Two magnesium "getters" were mounted at right angles to each other so that one could be flashed after the outgassing had been completed, while the tube was still on the pump, and the other after it had been sealed off. The abnormal effects observed—namely, elevation of the fluctuation level in space charge rather than the depression indicated by the theory—were attributed

[13] Williams and Huxford, Phys. Rev. **33**, 773 (1929).

at first to the evaporation of the sputtered film of magnesium on the filament, which resulted from flashing the getters.

The obvious means of avoiding this difficulty was to eliminate the use of a getter entirely. When this was done the desired pressures could be maintained only by continuous operation of a mercury pump. Tubes of identical construction with the one described above were sealed to a pumping system which was connected by a large tube to an outlet within the shield box of the shot circuit. A coil type liquid air trap prevented the diffusion of mercury vapor from the pump or the McLeod gauge used to indicate the pressure. Outgassing and baking were carried out as before and pressure readings less than 10^{-6} mm of mercury were maintained during the periods of observation.

It became apparent at once that even with the possibility of gaseous ionization and evaporation of sputtered films eliminated, abnormal space charge effects remained. The character of the abnormality was such as to indicate strongly an emission of mixed nature, and further study was made in verification and enlargement of this view. Similar effects have been observed by Kozanowski in the emission from oxide-coated cathodes, and here the disturbance was definitely traced to the simultaneous emission of positive ions and electrons. The influence of an ion upon the field in its vicinity is connected with its mobility. A simple calculation based on the kinetic theory yields the result that this effect for a positive ion of mass M is on the order of 140 $(M)^{1/2}$ times the effect of an electron. As a consequence, it can be shown that fluctuations of many times the normal level are to be expected as a result of the release from space charge by a single ion, of many electrons.

In reports by L. P. Smith[14] and H. B. Wahlin[15] the emission of positive ions by hot metals has been presented. Mass-spectrograph measurements with tungsten and molybdenum show that as the temperature is raised, first, ions of atomic mass corresponding to the alkaline earth metals are given off. Later, ions of greater atomic mass, and finally, ions of the metal itself, are evolved. It is difficult to explain the appearance of ions other than those of the metal specimen under observation unless those substances are present as impurities. Wahlin reports the persistence of the specific ion in a large group of metals while the emission of other ions is of a more transient character.

The tungsten obtained for the emitters in the present work was specified as chemically pure by the manufacturers. This apparently indicates only that precautions are taken during its manufacture to avoid contamination. Wire from two different sources was tried with no marked difference apparent in behavior.

A set of curves showing the effect of age and heat treatment of the tube is given in Fig. 1. A constant space current of 500 microamperes was maintained by raising the filament temperature as the plate voltage was reduced. Above thirty volts the curves are all seen to converge toward the common level for temperature limited currents. In the early stages of the tube history

[14] Smith, Phys. Rev., **33**, 381 (1930).
[15] Wahlin, Phys. Rev. **37**, 467 (A) and 473 (A) (1931).

no depression below this level is indicated. This simply means that the elevation due to release of electrons by positive ions is greater than the depression which might occur in the absence of such disturbing influence.

During the interval indicated between C and D the filament was operated at 1.2 amperes with a *negative* potential of 200 volts applied to the plate. After this treatment an actual depression was observed for the first time. It will be noted however that as the filament current was increased (to maintain constant space current) the trend of the curve turns upward. This is also apparent in the later curves. In general as the temperature approached that at

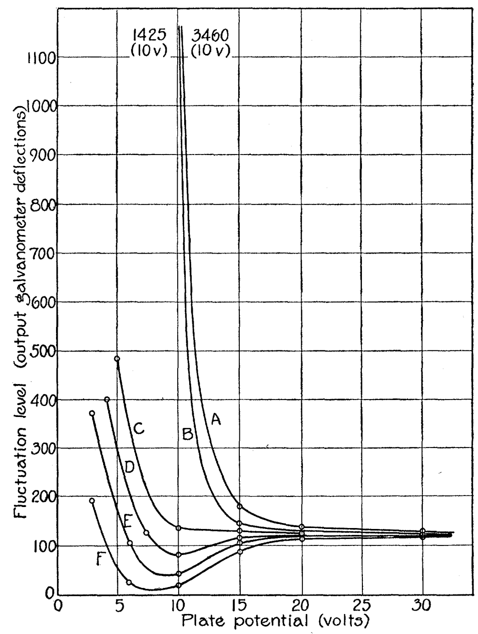
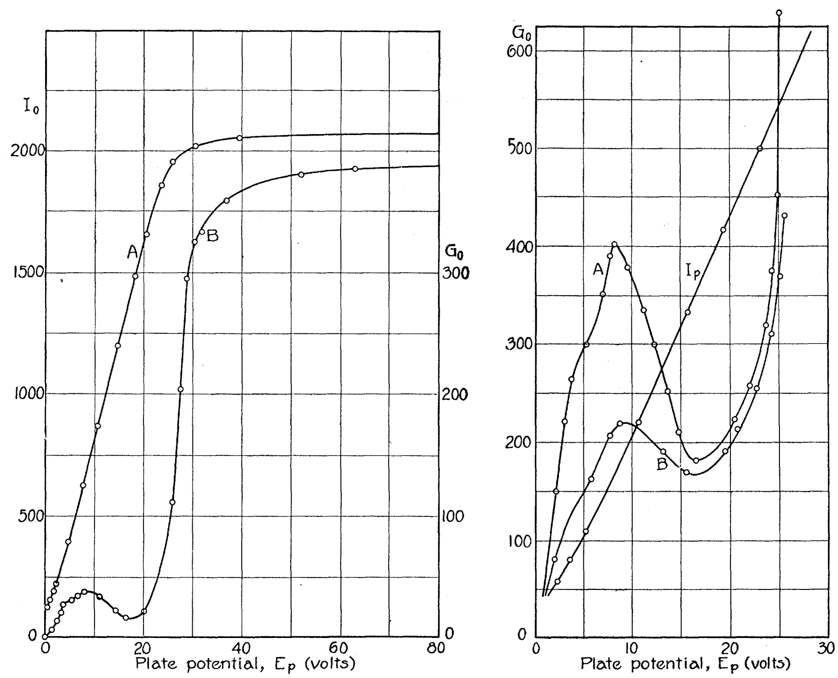


Fig. 1. Fluctuations in space charge limited currents from tungsten. Space current maintained constant by raising filament temperature. Curves $A-F$ represent successive stages in aging and heat treatment of specimen. All approach normal shot effect level when current becomes temperature limited. A—First heating of filament. B—After first outgassing. C—After repeated outgassing, pressure 10^{-6} mm—20 hrs. D—Filament aged 30 minutes, $E_p=200v$, $I_f=1.2$ amp. E—Filament aged 3 hours. F—Filament aged 9 hours.

which the aging process was carried on the depression gave way to an elevation. Curve F represents somewhat of a limiting situation, and subsequent data clustered around the level indicated at this stage after nine hours aging.

From the form of the curves of Fig. 1, it was recognized that changes in the emission characteristics of the filament were in large measure responsible for the elevation of the fluctuation level observed at low voltages. It should be pointed out that the excessive positive ion emission indicated in these tests results from the abnormally high temperatures necessary to maintain constant current under heavy space charge limitation. Indeed, it is probable that in this range, appreciable currents can be secured largely by virtue of neutralization of space charge by positive ions. Emission of this character, while of interest in connection with the present work would rarely be met in the normal operation of tungsten emitters. The possibility was considered that high temperatures alone might bring about changes in the emitting surface which could be the sole cause of the large fluctuations.

As a test of this question the transition between temperature limitation and space charge was carried out another way—with constant emitter temperature. The space current in this case varies with the applied voltage as shown in curve A, Fig. 2. On this account alone the fluctuation level as shown by the



Figs. 2 and 3. Abnormal fluctuations observed in space charge region when filament temperature is held constant. The persistence of a peak is evidence of mixed emission at normal operating temperatures. Curves A and B, Fig. 3, correspond roughly to curves C and D, Fig. 1.

output galvanometer would be expected to decrease proportionately as E_P is decreased. Previous experience under space charge conditions indicated a still more rapid decrease of $\overline{V^2}$. Interest in this case centers on the peak which appeared to break the regular fall of the G_0 curve when plotted against E_P. Since no change was made in the filament temperature we are forced to the conclusion that this is due to some cause inherent in the emission. In Fig. 3 two curves are shown which correspond in a rough way to curves C and D in the "constant current" group. A definite decrease in the height of the peak is coordinated with a period of aging the filament with a large negative potential on the plate. It is significant that the *position* of the maximum is not changed by this treatment and is therefore a *function only* of the space charge situation in the tube and not of the emission characteristics.* The magnitude of the peak on the other hand is associated, as was expected, with the rate of emission of positive ions. From this, correlated with evidence presented in the preceding sections, we feel that the *only contribution* of an increase in emitter temperature is the increase in specific emission, *both of electrons and positive ions*.

Emission from thoriated tungsten

For comparison with the results on tungsten, curves obtained with emission from thoriated tungsten are reproduced here. Fig. 4 is drawn to show the fall in the fluctuation level for a constant space current as E is decreased from a value providing temperature limitation. The two curves represent actual data taken on the same tube at different times. The difference in the shape of the elbow is characteristic of emission from thoriated tungsten.

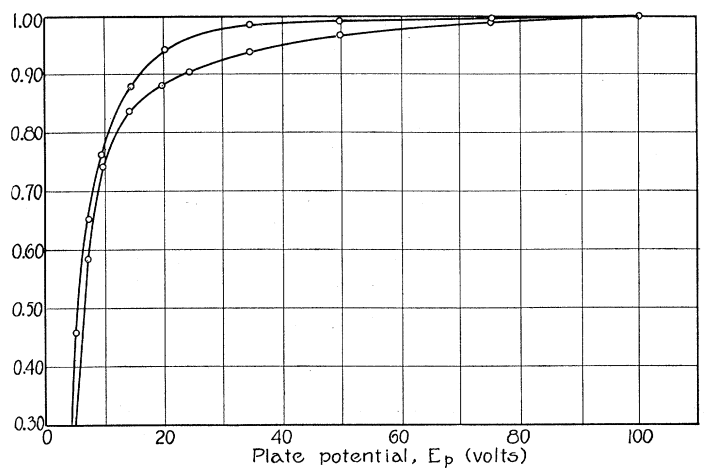


Fig. 4. Shot effect in emission from thoriated tungsten. Ordinates represent fractions of the normal fluctuation level. The spread between the two curves, taken under identical conditions is attributed to variation in the emission of positive ions. Constant space current $i_0 = 500\,\mu$.a.

There seems little doubt that the elevation of the fluctuation level due to positive ions in space charge is responsible for this irregularity in form. The abnormal fluctuation is simply superposed on the level resulting from the normal depression. The number of ions released at the temperature covered in these experiments is apparently insufficient to cause a resultant elevation. Further, the quantity varies with the time and previous treatment of the filament. This results in the region of uncertainty mentioned in connection with the preliminary work. It appears from the curves that this is greatest for a space current of 500 microamperes in the region of 20 volts on the plate. Below this, the curves seem to show greater uniformity. It must be borne in

* The filament current was actually held constant during runs of this type. The change in resistance due to the cooling effect of evaporation of electrons was less than one part in one thousand.

mind, however, that, due to the steepness of the slope and the low plate potentials used, experimental accuracy is considerably reduced.

Grid control of space charge

The effect of a space charge grid on fluctuations was next investigated. Tubes of cylindrical symmetry were built, with tungsten emitters, and provided, at first with grids 3 mm in diameter, and about 3 mm spacing between turns. Plates were of nickel, 16 mm in diameter.

Let us suppose, in line with the indication of previous results, that the emission consists mainly of electrons with a small quantity of positive ions. If we further assume that both types of emission occur with a chaotic distribution in time, we should expect, when potentials are adjusted to obtain continuous fluctuations. This was accomplished by applying a positive potential of 10–15 volts to the grid and 200 volts to the plate.

The establishment of space charge at any point in the tube resulted in a departure from this value. As an analysis of the fields in the tube will show, this may be due to an internal or external space charge referred to the grid. It is the latter, in which we are primarily interested, but it is essential to know the effect of internal space charge and the interaction with positive ions on the fluctuation levels. The use of a large positive potential on the grid would certainly result in the elimination of the inner space charge. The field would also be a retarding one for positive ions and would eventually cause the return of many of them to the cathode. Electrons, however, would suffer acceleration in the same field and their average velocity of appearance in the outer space would be much greater than the range of the original emssion velocities.

It is convenient to consider the region of the grid as a virtual emitter and to visualize the external space charge as produced by "emission" from the grid. It is at once apparent that for a given potential difference and current a larger emission velocity will result in a decreased space charge density. This follows from the relation $i=\rho v$ where i is held constant.

An experimental difficulty is encountered here in the extremely low voltages between grid and plate necessary to produce appreciable space charge density. This is concerned mainly with the accuracy of the measurements—in particular, the impedance determination, and is discussed in another section.

On the other hand, a *low* grid potential introduces space charge conditions in the inner space provided its field acts independently of that due to the plate potential. It was found necessary to cut the grid turn separation to about 0.4 mm to accomplish this shielding.

Since the *grid* and *filament* are connected together with only the small *fixed* potential difference between them, fluctuations between these two are logically assumed to be negligible. Both are held at earth potential for high frequency currents. We have to deal then only with grid or filament, whichever we choose. On this view the filament serves only as a source of electrons —the effective emission coming from the region of the grid.

It might appear that the existence of space charge in the inner space would not be of any significance in a measurement of these fluctuations. The design of a tube in which the external field was effectively screened from this space made possible a test of this question.

With the plate potential at 200 volts so that temperature limited conditions existed between plate and grid, the grid potential was carried in small steps over the range in which inner space charge was established. The variation of the $\overline{V^2}$ with E_g is shown in Fig. 5. The explanation of the eight-fold

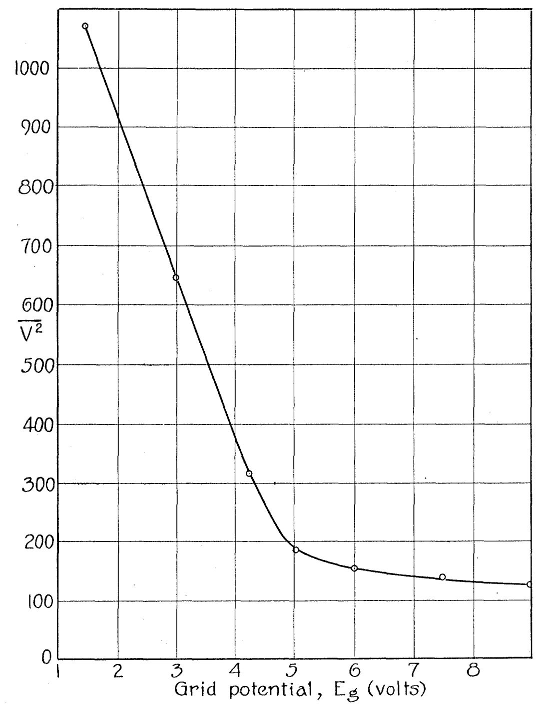


Fig. 5. Influence of positive ions in inner space charge in three element tube. Fluctuation level shown as a function of grid voltage, measured with respect to the negative terminal of the filament. The bend occurs at a value of E_g about equal to the filament drop in potential. $E_p = 200$ volts.

increase observed between $E_g = 6$ volts and $E_g = 1.5$ volts lies in the release of groups of electrons from the inner space charge by positive ions emitted from the filament. Moving toward the grid, some of these electrons are collected and some pass through into the grid-plate space. The "emission" from the virtual cathode therefore departs from a purely random character and the effect is observed in an elevation of the fluctuation level.*

The conclusion seems inevitable that when space charge conditions exist at *any* stage in the passage of a thermionic current, the action of positive ions will produce abnormal fluctuation effects.

* Grid and plate potentials in Fig. 5 are given with respect to the negative terminal of the filament. The bend in the fluctuation curve occurs when the value of the grid potential becomes equal to the drop along the filament due to the heating current.

In view of the extreme difficulty experienced in obtaining *pure* electron emission of reasonable intensity, this amounts to a requirement for the study of pure space charge effects that *the space charge must not occur in the immediate neighborhood of the emitter.*

§4

PURE SPACE CHARGE EFFECTS

Field conditions in the double grid tube

These steps led to the introduction of a second grid, g_2 of Fig. 6, the function of which was two-fold:

1. To provide a retarding field for electrons after their passage through the inner grid space.
2. To shield the internal sections of the tube from the action of the external field.

The inner grid was held at a sufficiently high potential F_1 to assure the continuously positive field indicated in region A, Fig. 6. This serves the double purpose, first, of preventing the escape of positive ions into the surrounding space, and second, of maintaining within the region a temperature limited current whose fluctuations have been shown to be unaffected by positive ions. Electrons of zero initial velocity would arrive at g_1 with a velocity of E_1 volts. Some of course strike the grid and contribute to a measurable current which we shall call I_1. Others pass through the rather coarse mesh into the region B where a retarding field is maintained. The potential of g_2 is held only slightly positive with respect to the filament. As a result, the average emission velocity into region C is but slightly higher than the original emission velocity. Here again a portion of the electron stream is cut off constituting a current I_2.

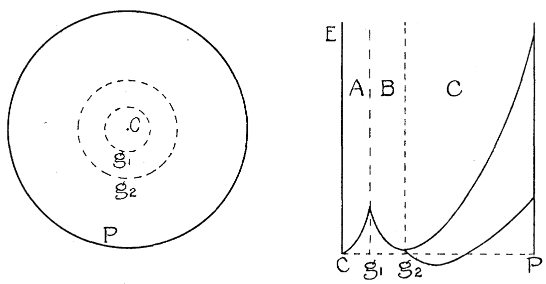


Fig. 6.

Let us now examine the nature of the remaining emission. If g_2 be considered as a virtual emitter, we have a pure electronic space current in which space charge limitation may easily be set up by a suitable choice of the potentials E_p and E_2. It was anticipated that the accumulation of space charge in region B would cause a somewhat lower fluctuation level in this current than that obtained under temperature limitation in the simple two electrode tube. Since the region in question, however, is free of positive ions, the only effect of this internal space charge will be an independent and constant depression. This is entirely different from the elevation produced by positive ions in an inner space charge which is of an irregular and uncontrollable magnitude.

Insofar, then, as our interest centers only on the influence of conditions in the outer space, an emission of the character described would appear to meet all the requirements. Gratifying results were immediately obtained in the increased accuracy of individual measurements. It became possible for the first time to repeat space charge measurements many times with an actual maximum spread in values of the depression ratio of about 2 percent.

The question as to whether the pure emission from the region of g_2 possesses the random character of true thermionic emission may be settled by measuring the mean square "shot" voltage with a high positive potential applied to the anode. A comparison of the values of $\overline{V^2}$ to those obtained with a two electrode tube under strict temperature limitation will immediately detect a departure from the ideal situation. For this comparison, tubes were chosen on which actual measurements of the electronic charge have been made. A value of $\overline{V^2}$ consistent with Eq. (1) and therefore a pure random emission fulfilling the postulates of the theory is assured. Tedious repetition of the complete calibration of the amplifier such as is necessary in the determination of ϵ were thus avoided since the same values of A_0 and A entered into each of the compared magnitudes.

Experiments were carried out to determine the optimum conditions for the production of space charge between the outer grid and the anode. A value of space current was chosen which produced sufficiently high electron density, and which could be maintained without danger to the filament. Grid potentials were then adjusted until the stream passing into region C exhibited fluctuations indentical with a temperature limited current from a metallic emitting surface.

The theoretical considerations outlined in the appendix Note A, lead to an expression for the fluctuation depression ratio:

$$\overline{V_s^2}/\overline{V_t^2} = f(i_0/i)e^{-\omega^2/2\alpha^2}$$

where $\overline{V_s^2}$ is the mean square voltage fluctuation under space charge conditions characterized by emission current i_0, space current i, and $\overline{V_t^2}$ the m.s. voltage fluctuation for a temperature limited current i. ω is 2π times the natural frequency of the amplifier curcuits, and α, a function defining the width of correlation.

Two questions are thus presented which invite investigation

(1) How great is the influence of the exponential term involving the frequency.

(2) In what manner does the depression of fluctuation level depend on the ratio of the collected current to the total emission current.

We will consider first the frequency effect.

The response characteristic of the amplifier has the form of a symmetric curve which falls off sharply on each side of the resonant frequency of the tuned circuits. The breadth of the curve can be controlled by the use of shunt resistors in these circuits. The area under the relative amplification curve is the factor A in the experimental Eq. (1). Thus from the wide range of frequencies represented in the Fourier analysis of the current fluctuations, only a narrow band is selected. Maximum amplification is afforded those components whose frequencies lie very close to the resonant frequency of the amplifier. By a choice of elements constituting the tuned impedance in the plate circuit of each stage, this band may be selected from various parts of the spectrum.

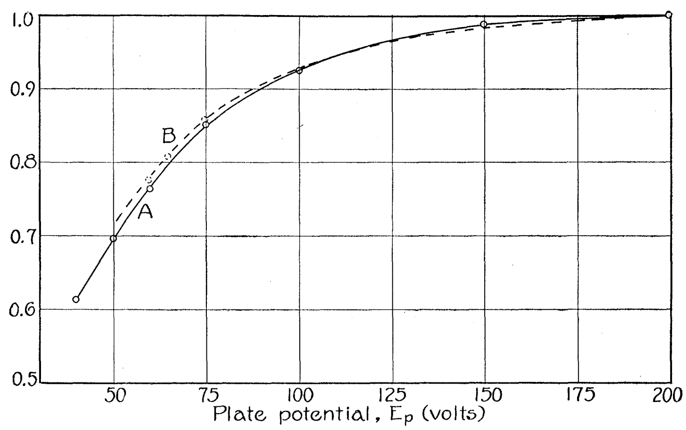


Fig. 7. Frequency dependence of space charge depression constant space current. Depression ratio as a function of E_p due to external space charge in double grid tube, E_1, E_2, and E_p. measured from negative terminal of filament. A, 54,000 p.p.s. B, 480,000 p.p s. $i_0=500$ microamps. $E_1=9$ volts. $E_2=1.5$ volts.

The potential E_p actually applied to the anode of the "shot" tube is obtained from the impressed battery voltage by subtraction of the voltage drop across the series load. Starting with 200 volts or more E_p is decreased in steps, and the space current held constant by a corresponding increase in the filament temperature. Space charge is thus increased with decreasing E_p. The ratio of $\overline{V^2}$ for a certain current and applied potential to $\overline{V^2}$ for temperature limitation of the same current is obtained from the ratio of G_0/Z^2 for the two cases. (See Fig. 7.)

Each point on the curves is the average of at least twenty independent observations. These were part of a series of experiments on two different tubes which extended over a period of about ten weeks. Within two percent the average curves at 54,000 pps. and 480,000 pps. coincide throughout their length. We therefore feel justified in believing that the frequency effect on space charge depression is vanishingly small.

This conclusion furnishes an indication of the maximum breadth of the correlation function. If over a range covering nearly a ten fold increase in the frequency, $e^{-\omega^2/2\alpha^2}$ does not depart appreciably from 1, $\omega^2/2\alpha^2$ must remain equal to zero. This can be true only if α is an extremely large quantity compared with ω. By reference to the role played by α in the correlation equation (Note A, Eqs. (4), (10))

$$\overline{j(t_1)j(t_2)} = \phi(t_1 - t_2) = \phi(w) = Pe^{-\alpha^2 w^2/2}.$$

We note that an extremely rapid decline of the function, ϕ is demanded. The width of the correlation band is thus shown to be *small compared to the shortest natural period of the amplifier*.

Since the frequency term reduces to unity, presumably $f(i_0/i)$ is a universal function representing the depression ratio of fluctuations in space charge. Let us follow the progress of a virtual emitting surface by considering its position in the tube to be that of the minimum potential between grid and anode. Under temperature limitations it coincides with g_2. As the space charge builds up the region expands and moves outward across the interelectrode space. A typical situation is represented by the lower potential curve in space C, Fig. 6. It is obvious that electrons will be retarded on entering this space and that those with initial velocities too low to reach the potential minimum will be returned in the direction of the cathode. Some of these returned charges strike the grid g_2. This number can be determined by the observed *change* in I_2. The field in the neighborhood of g_1 would lead one to expect a still greater change in I_1 and this is found when the collection current to this element is measured.

For any state, the total emission from g_2 is given closely by $I+\Delta I_1+\Delta I_2$ There is, of course, the possibility that a few electrons will reach the filament against the inner field. This number was thought to be negligible, and an independent check of the emission current by the application of a high potential to the plate showed this to be the case. Thus a reading of the emission current could be taken simultaneously with the determination of $\overline{V^2}$.

The requirement of a large applied potential in comparison to those used in the impedance measurement prevents the determination of the mean square voltage fluctuation level with accuracy for low plate potentials. Restrictions imposed in turn by the maximum safe filament temperature, and the portions of the electron stream intercepted by the grids set an early limit upon increase of the emission current. When these are considered together it is found that the range of trustworthy and significant results is confined from two directions.

The emission from the region of minimum potential was measured by a direct current meter in the anode circuit, since all the electrons reaching this position are accelerated toward the collector. There seems to be no reason to believe that the probability relations governing the escape of electrons from the "cathode grid" will be changed in any way by the external field conditions in the tube. It was to be expected therefore that when the region lay near the grid, the current fluctuations would be identical with those of the stream passing through the grid. As space charge is increased in the outer space with decreasing plate potential, the current reaching the anode falls off. The fluctuations, however, decrease more rapidly than the current. A comparison of these two effects is given in Fig. 8 in which f and its argument are both plotted against E_p.

The complete suppression of the shot effect fluctuations may be observed qualitatively by the behavior of the output galvanometer which falls to zero for very high space charge densities. The actual lower limit is fixed by the magnitude of the fluctuations due to the thermal agitation of electricity in

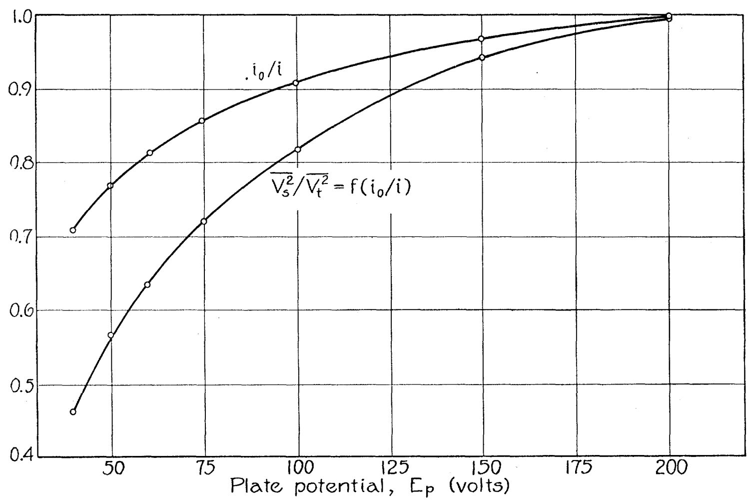


Fig. 8. Depression ratio of shot effect in space charge compared with ratio of anode current to emission current from the region of second grid. Between 50 and 200 volts. $i_0 = 500$ microamps. $E_1 = 22.4$ volts. $E_2 = 1.5$ volts.

the shot circuit. The mean square value of the voltage thus generated is of the order of 0.01 of the normal shot effect for a current of one milliampere. A correction can be made for this in the original galvanometer setting.

It may be of interest to record a rather striking empirical relation which may be obtained from the data presented in Fig. 8. Between $E_p = 50$ and 200 volts, it is true within experimental error that

$$\overline{V}_s{}^2/\overline{V}_t{}^2 = f(i_0/i) = (i_0/i)^2.$$

It is hoped that theoretical justification for this relation will be provided in the future.

Acknowledgment

It is a pleasure to acknowledge our indebtedness to Dr. G. E. Uhlenbeck for invaluable assistance during the investigation, and to Dr. A. W. Hull for helpful suggestions in the preparation of this report.

APPENDIX

Current Fluctuations

A simple thermionic device may consist of two electrodes sealed into a glass tube which is evacuated to such an extent that the residual gas plays no part in the discharge phenomena to be observed. One of the electrodes may be heated to any desired temperature below its melting point, and this is made the cathode in a circuit carrying the thermionic current.

Two fundamental sources of current fluctuation must be recognized in this circuit:

(1) The discreteness of the carriers of the current.

(2) The thermal agitation or "Brownian motion" of the electricity in the circuit.

In the case of a temperature limited thermionic current, the *first type* of fluctuation makes itself predominantly felt. Let us consider the flow of such a current in the simple circuit shown in Fig. 1. If the emission is a completely random effect, we may expect deviations from the long time mean value governed by the laws of probability. These fluctuations will be reproduced identically at the anode, with only the time of passage across the inter-electrode space intervening between the occurence of a particular event in emission, and its reproduction at the collector

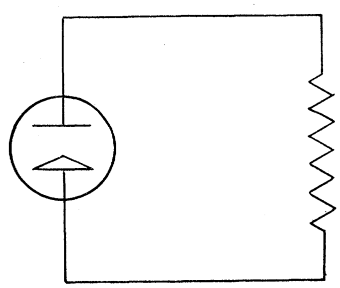


Fig. 1.

plate. As a result the potential of the plate will rise and fall chaotically about a certain mean value. The manifestation of the fluctuations in the electron stream, then, will be the development of small irregular alternating potential differences between separated points in the circuit.

It is clear in the normal shot effect that the fundamental fluctuating quantity is the number of electrons concerned in the emission from the cathode, and hence the *emission current.* The amplitudes of these particular fluctuations are of course independent of the frequency characteristics of the shot circuit or measuring device. The resulting potential fluctuations will involve the total impedance of the shot circuit which may be made practically free of frequency influence by the use of an aperiodic circuit.

Fluctuations of the *second type*—those due to the thermal agitation of electric charges—are always present in a conductor, whether carrying a current or not. Here the fundamental deviations from the long time mean situation are in the nature of *accidental electromotive* forces set up by random variations of charge density.

In section A we shall outline the theory of particle fluctuations in a thermionic current with particular emphasis on the effect of space charge. Following this a short discussion of thermal agitation is given, and the application to the present problem pointed out.

Note A. Amplitude of the Mean Square Deviation

Let $i(t)$ denote the instantaneous value of the thermionic current and $\bar{i}$ the long time mean value. The magnitude of the *deviation* from the mean may be developed in a Fourier series:

$$i(t) - \bar{i} = j(t) = \sum_{k=1}^{\infty}(A_k \cos \omega_k t + B_k \sin \omega_k t) \tag{1}$$

where

$$\omega_k = \frac{2\pi k}{\tau}$$

ω being the frequency in radians per second, and τ the fundamental period of the series. The coefficients of the kth term in the cosine and sine components of the general series are defined as follows:

$$\left.\begin{aligned} A_k &= \frac{2}{\tau}\int_0^\tau j(t)\cos\omega_k t dt = \frac{2}{\tau}\int_0^\tau j(\xi)\cos\omega_k\xi d\xi \\ B_k &= \frac{2}{\tau}\int_0^\tau j(t)\sin\omega_k t dt = \frac{2}{\tau}\int_0^\tau j(\xi)\sin\omega_k\xi d\xi \end{aligned}\right\} \quad (2)$$

The following customary assumptions[1] are made regarding $j(t)$:

(A). The mean deviation from the average current, taken at the time, t over a large number of similar, but independent circuits in which the same average current flows, is zero. This is denoted by

$$\overline{j(t)} = 0. \quad (3)$$

(B). If the values of $j(t)$ are considered at different instants, t_1 and t_2, there will be correlation between successive values only when $|t_2-t_1|$ is very small. This may be expressed by the following equation:

$$\overline{j(t_1)j(t_2)} = \phi(t_1 - t_2), \quad (4)$$

where the values of $j(t)$ at t_1 and t_2 are multiplied, and the average taken again over an ensemble of circuits. $\phi(t_1-t_2)$ is a symmetric function of the difference in time with a sharp maximum at t_1-t_2 equal to zero. This will frequently be referred to as the *correlation function*. It is well known from statistical mechanics that the ensemble of currents may be formed either by a great number of independent circuits, or by dividing the curve, $j(t)$, occurring in one circuit into many sections of length τ.

We are interested now in finding an expression for the *mean square* amplitude of the kth Fourier component of the current. This may be written from the Eq. (2)

$$\overline{A_k^2} = \frac{4}{\tau^2}\int_0^\tau\int_0^\tau \overline{j(\xi)j(\eta)}\cos\omega_k\xi\cos\omega_k\eta d\xi d\eta$$

where ξ and η are time variables of integration. Now $\cos\omega_k\xi\cos\omega_k\eta = 1/2[\cos\omega_k(\xi+\eta)+\cos\omega_k(\xi-\eta)]$ and by assumption (2) $\overline{j(\xi)j(\eta)} = \phi(\xi-\eta)$. Denoting $\xi-\eta$ by w, and $\xi+\eta$ by v, we have $dv\,dw = 2d\xi d\eta$ from the functional determinant.

Introducing these as new variables, the expression for $\overline{A_k^2}$ becomes

$$\overline{A_k^2} = \frac{1}{\tau^2}\int_0^{2\tau}\int_{-\infty}^{\infty}\phi(w)[\cos\omega_k w + \cos\omega_k v]dvdw.$$

The extension of the limits of the integration with respect to w is justified by the steep decline of the function $\phi(w)$.

$$\begin{aligned} \overline{A_k^2} &= \frac{1}{\tau^2}\int_0^{2\tau}\int_{-\infty}^{\infty}\phi(w)\cos\omega_k v dvdw + \frac{1}{\tau^2}\int_0^{2\tau}\int_{-\infty}^{\infty}\phi(w)\cos\omega_k w dvdw \\ &= \frac{1}{\tau^2}\int_0^{2\tau}\cos\omega_k v dv\int_{-\infty}^{\infty}\phi(w)dw + \frac{1}{\tau^2}\int_0^{2\tau}dv\int_{-\infty}^{\infty}\phi(w)\cos\omega_k w dw. \end{aligned}$$

Since $\omega_k = 2\pi k/\tau$, the first integral is seen to vanish at both limits, and there remains simply

$$\overline{A_k^2} = \frac{2}{\tau}\int_{-\infty}^{\infty}\phi(w)\cos\omega_k w dw. \quad (5)$$

A corresponding treatment yields an identical expression for $\overline{B_k^2}$.

Correlation

In fulfilling the postulates of the normal shot effect theory, the probability of the departure of an electron from the cathode is assumed to depend only on the thermal energy of the emitter,

[1] Uhlenbeck and Ornstein, Phys. Rev. **38**, 823 (1930).

and not at all upon the previous history of the region of the surface from which it leaves. It is further assumed that the arrival of electrons at the plate partakes of the same chaotic distribution.

The situation is represented graphically in A Fig. 2. Under temperature limitation the number, n_i, emitted during a certain time element Δt_i all arrive during a corresponding time $\Delta t_i'$ later than Δt_i by the time of passage across the intervening space.

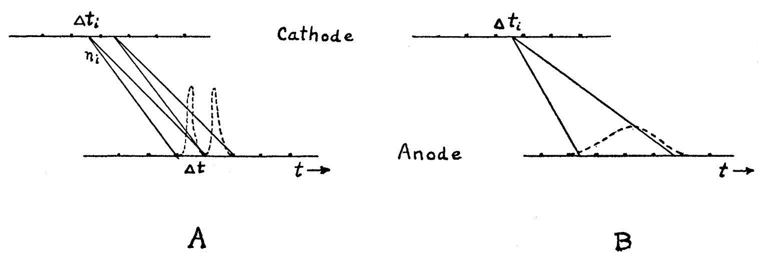


Fig. 2.

This statement has the effect of assigning an infinitesimally narrow width to the correlation function $\phi(w)$ as applied to the emission current. It follows that the integral $\int_{-\infty}^{\infty}\phi(w) \cos \omega_k w dw$ can be replaced by $\int_{-\infty}^{\infty}\phi(w)dw$ since the value of w for the range in which the integral $\neq 0$ is very small and $\cos \omega_k w = 1$. For temperature limited currents, therefore, we can write $\overline{A_k^2} = \frac{2\sigma}{\tau}$ and an identical expression for $\overline{B_k^2}$ (6)

where

$$\sigma = \int_{-\infty}^{\infty} \phi(w)dw.$$

The application of the principles of probability to the escape of electrons from a metal surface has given for the mean square deviation from the average current $\bar{i}$

$$\overline{j^2}\Delta t = \overline{(i - \bar{i})^2}\Delta t = \bar{i}\epsilon$$

where ϵ is the charge of the electron, Δt is a time element, short by comparison with the natural period of the measuring unit, and still long enough to allow the escape of many electrons. The role played by this factor has been clearly illustrated by Huxford.[2]

This may be identified with the function $\phi(w)$ by considering that the area under the correlation curve may be approximated by a rectangle of height $\overline{j^2}$ and width Δt. Thus

$$\sigma \cong \overline{j^2}\Delta t = \bar{i}\epsilon. \tag{7}$$

The mean square amplitude of the kth component of current fluctuation,

$$\overline{A_k^2} + \overline{B_k^2} = \overline{C_k^2} = \frac{4\epsilon\bar{i}}{\tau} \tag{8}$$

Adaptation of this relation to the experimental problem, and a solution applicable to all types of shot circuits have been carried out by Williams and Vincent.[3] Remembering that $\omega_k = 2\pi k/\tau$, the mean square voltage developed across the circuit load and impressed upon the amplifier becomes

$$\overline{V_k^2} = \tfrac{1}{2}C_k^2 Z_k^2 = 2Z_k^2\epsilon i_0 \frac{\omega_k}{2\pi k} \tag{9}$$

It was to be expected that one of the effects of the interaction of electrons in space charge would be the "spreading" of the correlation function as applied to their arrival at the anode.

[2] Huxford, Dissertation, University of Michigan, (1928), p. 10.
[3] Williams and Vincent, Phys. Rev. **28,** 1250 (1926).

The contrast between the two types of current limitation may be depicted as shown in Fig. 2. Under extreme space charge, the emission of n_i electrons during Δt_i may influence the arrival of n_i over a considerable range of time elements, beginning nearer Δt_i than was the case, due to long range forces.

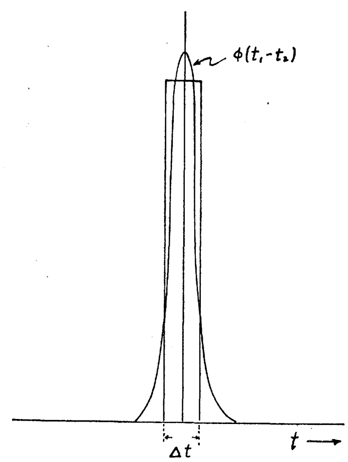


Fig. 3.

More formally, if we assign finite width to the correlation function, its value must be expressed as a symmetric, rapidly decreasing function of the difference in time between two events. This is the quantity which we have called w. An approximation which suggests itself is an exponential of the form

$$\phi(w) = Pe^{-\alpha^2 w^2/2} \tag{10}$$

where P and α are functions of the plate potential or of the space current.

Substituting this in the expression for $\overline{A_k^2}$

$$\begin{aligned}\overline{A_k^2} &= \frac{2P}{\tau}\int_{-\infty}^{\infty} e^{-\alpha^2 w^2/2}\cos\omega_k w\,dw \\ &= \frac{2P}{\tau}\,\frac{(2\pi)^{1/2}}{\alpha}\,e^{-\omega_k^2/2\alpha^2}\end{aligned} \tag{11}$$

for temperature limitation, we had, independent of frequency,

$$\overline{A_k^2} = \frac{2\epsilon i_0}{\tau}. \tag{6, 7}$$

This condition is fulfilled in the space charge equation by making

$$\alpha \to \infty \quad \text{and} \quad \frac{P}{\alpha}(2\pi)^{1/2} = \epsilon i$$

where i refers to the total emission or saturation current from the cathode.

It is apparent that P/α is finite and is itself a function of the space current i_0, which for $i_0 = i$ becomes $\epsilon i_0/(2\pi)^{1/2}$. Representing in general

$$\frac{P}{\alpha}(2\pi)^{1/2} \text{ by } \beta(i_0)$$

we may write

$$\overline{A_k^2} = \frac{2\beta}{\tau}\,e^{-\omega_k^2/2\alpha^2} \tag{12}$$

and reasoning as before,

$$\overline{V_k^2} = \tfrac{1}{2}Z^2C_k^2 = 2Z^2\beta\frac{\omega_k}{2\pi k}e^{-\omega_k^2/2\alpha^2}. \tag{13}$$

Then for the ratio of the mean square impressed potential fluctuation under given space charge conditions to the same quantity under temperature limitation.

$$\frac{(\overline{V_k^2})_s}{(\overline{V_k^2})_n} = \frac{\beta(i_0)}{\epsilon i}e^{-\omega_k^2/2\alpha^2}.$$

From dimensional considerations, we cannot escape the conclusion that the factor $\beta(i_0)/\epsilon i$ must be some function f of i_0/i which for $i_0 = i$ becomes unity. Making this substitution and denoting the depression ratio by δ, we have

$$\delta = f\left(\frac{i_0}{i}\right)e^{-\omega_k^2/2\alpha_i}. \tag{14}$$

The ratio on the left is frequently referred to in this paper simply as the *depression ratio.* Its value obviously approaches unity as strict temperature limitation is approached. For other situations indicated by the subscript s, the ratio is seen to depend on two factors, one a function of the space current and the saturation current, and the other an exponential function of the frequency.

This formal discussion of course cannot give the form of the functions α and β. For these a more detailed consideration of the mechanism is necessary. If the effect of space charge were only the spreading to which we referred, one can show that

$$\beta(i_0) = \epsilon i_0. \tag{14a}$$

The entire depression would then be caused simply by the fact that a certain fraction of the emission current is cut off by the potential minimum. A surface through the potential minimum would be considered as a virtual emitter with the same characteristics as the real cathode, but emitting fewer electrons per second. In this case also α could be determined.

One feels, however, that the spreading effect cannot be the only cause of depression. When there is space charge there will also appear fluctuations in potential due to density fluctuations of the space charge, which again is a consequence of the temperature motion. Therefore we can only expect (14a) to hold in the limit of very small depression.

Note B. Thermal Agitation

Under extreme space charge limitation, thermionic conduction is similar in many respects to metallic conduction. It has been pointed out that in this case only fluctuations due to the thermal agitation of electricity would be expected. This is analogous to the Brownian motion of particles in colloidal suspension. In an earlier section the current fluctuation was developed in a Fourier Series. We must now apply a similar development to the accidental electromotive force. Putting therefore:

$$E(t) = \sum_{k=1}^{\infty}(P_k \cos \omega_k t + Q_k \sin \omega_k t) \tag{15}$$

we find,

$$\overline{P_k^2} = \overline{Q_k^2} = \frac{2\rho'}{\tau} \tag{16}$$

where

$$\rho' = \int_{-\infty}^{\infty} \overline{E(t_1)E(t_2)} \cos \omega_k(t_1 - t_2)d(t_1 - t_2) \tag{17}$$

To find ρ' we must consider the differential equation of the current:

$$L\frac{di}{dt} + Ri = E(t). \tag{18}$$

The general integral is

$$i = i_0 e^{-Rt/L} + \frac{1}{L} e^{-Rt/L} \int_0^t E(t) e^{Rt/L} dt. \tag{19}$$

Squaring and taking the mean over an ensemble of currents

$$\overline{i^2} = \overline{i_0^2} e^{-2Rt/L} + \frac{\rho}{2RL}(1 - e^{-Rt/L}) \tag{20}$$

where

$$\rho = \int \overline{E(t_1)E(t_2)} d(t_1 - t_2)$$

which in the limit as $t \to \infty$ becomes

$$\overline{i^2} = \frac{\rho}{2RL}. \tag{21}$$

From the equipartition of energy theorem, we have, $1/2L\overline{i^2} = 1/2\kappa T$ where κ is the Boltzmann constant and T, the absolute temperature.

Then

$$\rho' = 2R\kappa T. \tag{22}$$

Assuming very sharp correlation, we have $\rho' = \rho$ so that

$$\overline{P_k^2} + \overline{Q_k^2} = \overline{V_k^2} = \frac{4\rho'}{\tau} = \frac{8R\kappa T}{\tau} \tag{23}$$

$$\overline{V_k^2} = 8R\kappa T \frac{\omega_k}{2\pi k} \tag{24}$$

This expression is to be compared with Eqs. (9) and (13).

It is important still to remark that we could integrate (18) also by developing i in a Fourier series in the same way as $E(t)$, and we find then for the mean square amplitude of the kth component,

$$\overline{i_k^2} = \frac{1}{2} \frac{\overline{P_k^2} + \overline{Q_k^2}}{R^2 + \omega_k^2 L^2} = \frac{2\rho'}{\tau R^2} \frac{1}{1 + \omega_k^2 \dfrac{L^2}{R^2}}. \tag{25}$$

Now, again summing over all components, we get

$$\overline{i^2} = \sum_{k=1}^{\infty} \overline{i_k^2} = \frac{2\rho'}{2\pi RL} \int_0^\infty \frac{dx}{1 + x^2} \tag{26}$$

where

$$x = \omega_k \frac{L}{R}; \quad \omega_k = \frac{2\pi k}{\tau} \quad \therefore \tau = \frac{L}{R} \cdot \frac{2\pi}{dx}$$

which gives

$$\overline{i^2} = \frac{\rho'}{2RL}. \tag{27}$$

Comparing this with (21) we see that ρ' is necessarily equal to ρ, or that even for temperature fluctuations the correlation *must be* very sharp. The result is that for thermal fluctuations the spectrum of $\overline{V^2}$ must be uniform as in the case of the temperature limited thermionic currents. This argument now provides an *a priori* estimate of the influence of the correlation factor in the depression ratio. When in the two limiting cases, though by quite different approaches, we meet the necessity of a constant spectrum, it seems doubtful that in the transition region we would find the frequency dependence.