Substituting Eq. (1), and introducing W as new dependent variable, one obtains for W the partial differential equation:

$$\left(\frac{\partial W}{\partial v}\right)_T = \frac{2}{3}\,\frac{T}{v}\left(\frac{\partial W}{\partial T}\right)_v$$

of which the general solution is, that W is an arbitrary function of $Tv^{2/3}$. From (4) and (2) follows, that for an adiabatic change also:

$$Tv^{2/3} = \text{const.} \tag{5}$$

Thus W itself is an adiabatic invariant.

Perhaps the most remarkable consequence of these equations is, that the velocity of sound (neglecting heat conduction and viscosity) in a gas of point molecules for all statistics and for all degrees of degeneration is given by:

$$C = \left(\frac{5}{3}\,\frac{p}{\rho}\right)^{1/2} \tag{6}$$

when ρ is the density. This result follows immediately from the general formula:

$$C = \left(\frac{dp}{d\rho}\right)^{1/2}_{\text{adiab.}}$$

and Eq. (3). We were led to Eq. (6), when we observed that the derivation of the equation for the velocity of sound from the hydrodynamical equations in first approximation does not require the introduction of the equation of state. One uses only the virial theorem (1). In the case of no degeneration, that is for large values of $y = Tv^{2/3}$, W approaches unity. Then, and only then, the velocity C becomes a function of the temperature alone.

G. E. Uhlenbeck
E. A. Uehling

Department of Physics,
University of Michigan,
February 25, 1932.

Excited Electronic States of Li_2

The writer has recently made a Heitler-London calculation of those states of Li_2 which dissociate into a 2^2S atom and a 2^2P atom.[1] Of these eight states there are only two, namely $^1\Sigma A^N$ and $^1\Pi S^N$, which can make transitions to the ground state and hence are readily amenable to observation. Both are known to be attractive and their molecular constants have been accurately determined.[2] In the above calculation the $^1\Pi S^N$ state is found to be repulsive, in definite qualitative disagreement with experiment. The purpose of this note is to describe a revision of the calculations for this state.

Using the notation of the previous paper, one can choose two-electron product functions Ψ_i for the present case in such a way that the energy expressions for the states of various symmetries are of the same form as those found previously, and one then finds:

$$H_{11}' = J(pp;\,ss);\quad H_{21}' = J'(sp;\,sp);$$
$$N_{11} = 1;\quad N_{21} = S_{ps};$$
$$H_{31}' = J'(pp;\,ss);\quad H_{41}' = J(sp;\,ps)$$
$$N_{31} = S_{pp;ss};\quad N_{41} = 0.$$

One would expect to obtain qualitative results by taking for the atomic p function simply the s function with a suitable surface harmonic affixed.[3] If this is done one can easily obtain the energies, since the needed integrals have already been tabulated. The $^1\Sigma A^N$, $^1\Sigma S^N$, $^3\Sigma S^N$, and $^3\Pi S^N$ curves all come out markedly attractive, and the $^1\Pi A^N$ has a slight minimum far out. The other states are repulsive, and the $^3\Sigma A^N$ shows erratic behavior similar to that shown in the case of Be_2 by $^3\Sigma S^N$. The $^1\Sigma A^N$ has heat of dissociation 1.29 volts and internuclear distance 2.55A in good agreement with the experimental values[2] of 1.25 volts and 3.13A. For the $^1\Pi S^N$ we compare calculated values with readings from a curve drawn from the experimental results, and obtain ($\alpha = \kappa R$, $\kappa = 1.26$):

$\alpha =$		3	4	5	6	7	8	9	10
$(E-E_0)/\kappa$	calc.	0.196	0.083	0.036	0.0186	0.0124	0.0111	0.0102	0.0097
	obs.			+0.010	−0.021	−0.027	−0.024	−0.017	−0.013

There is complete disagreement between the two curves.

The revision of the calculation consists in the use of a more accurate atomic $2p$ function

---

[1] See end of article by Furry and Bartlett, Phys. Rev. **39**, 210 (1932). This paper must be seen for notation used here. Symmetry designations for Σ states are in sense of correction at end, not of body of paper.

[2] Loomis and Nusbaum, Phys. Rev. **38**, 1447 (1931).

[3] Assumption tacitly made in previous article (reference 1). Its success here for the $^1\Sigma A^N$ state tends to justify its use for Σ states of Be_2.

obtained from the variation method. Following Guillemin and Zener,[4] we take for the 1*s* wave function $u_{1s}=c'e^{-\gamma r}$, $\gamma=z-(5/16)$, and for the 2*p* functions $c_2 r^{n^*-1}e^{-\delta r}\cos\theta$ and $c_1 r^{n^*-1}e^{-\delta r}\sin\theta\, e^{\pm i\phi}$, where δ and n^* are parameters subject to variation. In the notation of Guillemin and Zener the energy expression is

$$E-E_k=\Delta_2{}^2-Zs_2{}^2+2k_{12}{}^{12}-k_{22}{}^{11}$$

Taking $n^*=2$, one obtains:

$\delta=$	0.51	0.52
$E-E_\kappa=$	-0.25533	-0.25547

$\delta=$	0.53	0.54
$E-E_\kappa=$	-0.25543	-0.25522

The minimum is not far from $\delta=0.525$. The adjustment of n^* is more difficult, since $k_{22}{}^{11}$ cannot be written in closed form for nonintegral values of n^*. For nearly integral values a careful use of approximation methods gives ample accuracy, and for $\delta=0.525$ one obtains:

$n^*=$	1.9	2.0	2.1
$E-E_\kappa=$	-0.25536	-0.25547	-0.25422

The minimum occurs at a value of n^* quite near to, but slightly less than, 2. The result is exactly like that found by Guillemin and Zener for the 2*s* function. One must take $n^*=2$ to make the molecular calculations feasible.

The experimental value of the energy of the 2^2P state is -0.261. Thus the discrepancy is about $0.006Rh$, which is the same error as found by Guillemin and Zener for the ground state. There is good agreement of δ with $(-(E-E_K))^{1/2}$, and the values found are almost equal to the hydrogenic ones, i.e., $n^*=2$, $\delta=0.5$.

Writing $\delta=\kappa'/2$, we have $\kappa'=1.05$, which is just 5/6 of the value of κ in the *s* function. The integrals needed for the calculation of the Π states are $S_{11}{}^{1/2}$, $I(11)$, $I'(11)$, $j(11;ss)$, $j(s1;1s)$, $j'(s1;s1)$, $j'(11;ss)$. Of these $S_{11}{}^{1/2}$, $I(11)$, $I'(11)$, $j(s1;1s)$, and the large contribution to $j(11;ss)$ are the same in form as in the previous work,[1] but require changes in scale to allow for the new exponential in the *p* function. Graphical interpolation gave sufficient accuracy. The remaining contribution to $j(11;ss)$ was computed from a table of A_n and B_n made up for fractional arguments. For $j'(s1;s1)^{(1)}$ a few $w_1(m, n, \alpha)$ were obtained for suitable arguments by graphical interpolation of certain functions of their logarithms. The integrals with superscript 2 are so small that we can regard them as unchanged. We then have the values given in Table I

TABLE I

$\kappa R=\alpha=$	3	4	5	6	7	8	9	10
$S_{ll}{}^{1/2}=$	0.8670	0.7720	0.6761	0.5778	0.4853	0.4014	0.3269	0.2621
$J(ll;ss)/\kappa=$	0.1702	0.0616	0.0212	0.0049	-0.0002	-0.0017	-0.0016	-0.0013
$J(sl;ls)/\kappa=$	0.0594	0.0518	0.0448	0.0368	0.0302	0.0244	0.0196	0.0157
$J'(sl;sl)/\kappa=$	0.0530	0.0434	0.0327	0.0239	0.0168	0.0110	0.0071	0.0045
$\{J'(ll;ss)-j'(ll;ss)\}/\kappa=$	-0.1557	-0.2123	-0.1967	-0.1572	-0.1155	-0.0796	-0.0521	-0.0329

The values of $J(11;ss)/\kappa$, $J(s1;1s)/\kappa$, and $J'(s1;s1)/\kappa$ for the larger values of α do not differ much from those obtained with $\kappa'=\kappa$. The curve for $\{J'(11;ss)-j'(11;ss)\}/\kappa$, however, lies appreciably lower. Thus if $j'(11;ss)^{(0)}$ were smaller than its value for $\kappa'=\kappa$ we might get agreement with experiment. Being much harder to compute than the other integrels, $j'(11;ss)^{(0)}$ was calculated first at only two points. Since these show the curve for ${}^1\Pi S^N$ to be definitely repulsive, further calculation is not needed.

The integral $j'(11;ss)^{(0)}$ must be evaluated in terms of the quantities $H_0(m, n, \alpha_1, \alpha_2)$ described by Zener and Guillemin.[5] The computation of these functions for $\alpha_2\neq\alpha_1$ is quite laborious. It was found convenient to calculate $j'(11;ss)^{(0)}$ at the values $\alpha=6$ and $\alpha=9.6$.[6]

$\alpha=$	6	9.6
$j'(ll;\ ss)^{(0)}/\kappa$	0.1109	0.0259

Then at $\alpha=6$, $J'(11;ss)/\kappa=-0.0448$, and at $\alpha=10$, $J'(11;ss)/\kappa=-0.013$ approximately. One finds for the energy

$\alpha=$	6	10
${}^1\Pi S^N/\kappa=$	0.0153	0.006

These values are somewhat lower than the results of the preliminary calculation with $\kappa'=\kappa$, but the curve is still decidedly repulsive.

Thus it is shown that a straightforward application of the simple Heitler-London method

---

[4] Zeits. f. Physik **61,** 199 (1930).

[5] Phys. Rev. **34,** 999 (1929).

[6] I wish to thank Mr. C. E. Ireland for placing at my disposal numerical results which aided in calculating the integral at these points.

cannot give qualitatively correct results for the $^1\Pi S^N$ state of Li_2. For an adequate theory of the electronic states of molecules we must look to the development of other methods, that recently introduced by Hylleraas[7] being perhaps the most promising.

WENDELL H. FURRY

University of Illinois,
February 25, 1932.

---

[7] Hylleraas, Zeits. f. Physik **71**, 739 (1931).

Relative Abundance of the Isotopes of Lead in Uranium Bearing Minerals

According to Kopfermann[1] the spins of the isotopes of lead descended from uranium (206), actinium (207), and thorium (208) are respectively 0, $\frac{1}{2}$, 0. This is in agreement with the results obtained by us in the visible as well as in the ultraviolet. We have obtained Lummer[2] patterns on the same photographic plate showing the relative positions of the h.f.s. components of ordinary nonradioactive lead and of the single lines due to the 206 isotope. In this way the component which must be ascribed to this isotope in the pattern of ordinary lead can be definitely identified. Accepting Kopfermann's conclusion about the spin of 208 and Aston's[3] estimates of the relative abundances of the isotopes, the strongest component of the patterns observed by us for nonradioactive lead must be ascribed to the Thorium lead isotope.

Strong exposure to the Paschen lamp source used for uranium lead brought out also some of the components of the 207 and perhaps the 208 isotopes. The effect was particularly marked for the 4244.99A line of Pb II because this line may be brought out strongly without reversal. For it the component due to 208 does not appear on our plates even though the exposure was sufficiently strong to make the 207 components as heavy as in the comparison pattern for nonradioactive lead taken side by side on the same plate.

This shows conclusively that the ratio of the contents of the 207 to the 208 isotopes is much larger in the uranium bearing minerals (obtained from Belgian Congo) than in ordinary lead. The trace of 207 cannot be due therefore to contamination. This result is in agreement with Aston's[4] observation of the relative abundance of the isotopes of lead separated by C. S. Piggott from uranium bearing Norwegian bröggerite. The sources of the minerals as well as the methods used being different from those employed by Aston, it appears that the association of the actinium decay product with that of the uranium series may be universal and that the relative abundance of the isotopes may be estimated with ease by hyperfine structure observations.

J. L. ROSE
L. P. GRANATH

Department of Physics,
New York University,
University Heights,
New York, N. Y.,
February 26, 1932.

---

[1] H. Kopfermann, Naturwiss. **19**, 400 (1931); **19**, 675 (1931).

[2] Kindly loaned to us by Professor R. W Wood.

[3] F. W. Aston, Nature **120**, 224 (1927).

[4] F. W. Aston, Nature **123**, 313 (1929). E. Rutherford, Nature **123**, 313 (1929).

Thermomagnetic e.m.f.'s in Transversely and Longitudinally Magnetized Wires

Recently C. W. Heaps[1] in referring to an article by Willaim H. Ross,[2] has pointed out that, in addition to the thermo-junctions formed by the transversely and longitudinally magnetized parts of the wire used in Ross' experiment, there will also be present junctions of magnetized and unmagnetized material where the two long ends of the wire extend from the region between the poles of the magnet into the region outside which is practically unmagnetized. He also pointed out that Ross' results were partly due to these additional junctions which would be at different temperatures due to the conduction of heat along the wires.

From experiments performed in this laboratory it has been found that the effect of these two additional junctions may be considerable. The three set-ups shown in Fig. 1 were used: (a) This is the same type of arrangement as that used by Ross. A wire, bent in a U shape, is placed in a magnetic field so that the length between the two bends is longitudinally magnetized and the leads extending away from the magnet are transversely magnetized. The

---

[1] C. W. Heaps, Phys. Rev. **38**, 1391, 1931).

[2] William H. Ross, Phys. Rev. **38**, 179, (1932).