THE SATELLITES OF M-SERIES X-RAY LINES

BY F. R. HIRSH, JR.
DEPARTMENT OF PHYSICS, CORNELL UNIVERSITY

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ABSTRACT

Four satellites of the diagram line $M\alpha_1$ have been measured in the range of atomic numbers from U (92) to Yb (70). Two of these satellites have not been reported previously. Three satellites of the diagram line $M\beta$ have been measured in the range of atomic numbers from U (92) to Gd (64). Two of these satellites have not been reported previously. Superposition of the semi-Moseley graphs for the satellites of $M\alpha_1$ and $M\beta$, gives rise to the suggestion that perhaps both x-ray lines are modified by the same optical lines. All these observed satellites obey the "Moseley" law,—the data giving a straight line within experimental error. The semi-Moseley graph of the satellite $M\beta^{ii}$ shows a significant break at atomic number 70. The satellites of $M\alpha_1$ vanish at Er (68), coincident with the splitting of the $M\alpha_1$ line into three components at this place, i.e. at atomic number 68. The range of values of $\Delta\nu/R$, between satellite and parent line, is from 0.1 to 2.0, in agreement with the results of previous workers, for the satellites of K and of L series lines. A peculiar continuous spectrum accompanies the satellites of $M\alpha_1$ and $M\beta$ for the elements U (92) and Th (90).

X-RAY "satellites," "spark lines" or "non-diagram" lines, as they have been variously termed, are those relatively faint lines accompanying certain diagram lines in the x-ray spectrum. Their presence cannot be accounted for by the usual energy-level diagrams, consequently they have been described as due to various inner-atomic processes. Among these Wentzel's hypothesis[1,2] of multiple ionization, with a single electron jump, was first advanced as an explanation.

The work of Druyvesteyn[3,4] has shown that the experimental verification of Wentzel's theory is not very satisfactory. Bäcklin[5] has shown that experimental observations of satellite excitation potentials do not confirm Wentzel's hypothesis.

In view of the defects of Wentzel's theory, and because of certain important experimental evidence, F. K. Richtmyer[6] advanced his "double-jump hypothesis" as an explanation of the origin of x-ray satellites. This double-jump hypothesis has an experimental basis in the existence of the quasi-Moseley graphs for x-ray satellites. The hypothesis is adequately explained in a later paper.[7] Briefly, his theory is, that in a doubly ionized atom, simultaneous jumps of two electrons occur. One jump releases an x-ray quantum;

[1] G. Wentzel, Ann. d. Physik **66**, 437 (1921).
[2] G. Wentzel, Zeits. f. Physik **31**, 445 (1925).
[3] M. J. Druyvesteyn, Zeits. f. Phys. **43**, 707 (1927).
[4] M. J. Druyvesteyn, Dissertation, Groningen (1928).
[5] E. Bäcklin, Zeits. f. Physik **27**, 30 (1924).
[6] F. K. Richtmyer, Phil. Mag. **6**, 64 (1928).
[7] F. K. Richtmyer, Jour. Frankl. Inst. **208**, 325 (1929).

the other jump releases the energy of a semi-optical quantum; the total energy of both jumps are emitted from the atom as a single modified x-ray quantum, or satellite quantum.

More data were essential for the testing of this double-jump hypothesis, and for providing an experimental foundation for the building of any explanation of the origin of x-ray satellites. Accordingly, F. K. Richtmyer and R. D. Richtmyer[8] made a survey of the L-series x-ray satellites. Recently, O. R. Ford[9] made a similar survey of K-series satellites. Hence a survey of M-series satellites seemed to promise new and interesting results.

M-series satellites of the diagram lines $M\alpha_1$, $M\beta$ and $M\gamma$ were first reported by Hjalmar.[10,11] Recently Lindberg[12,13] made a re-survey and an ex-

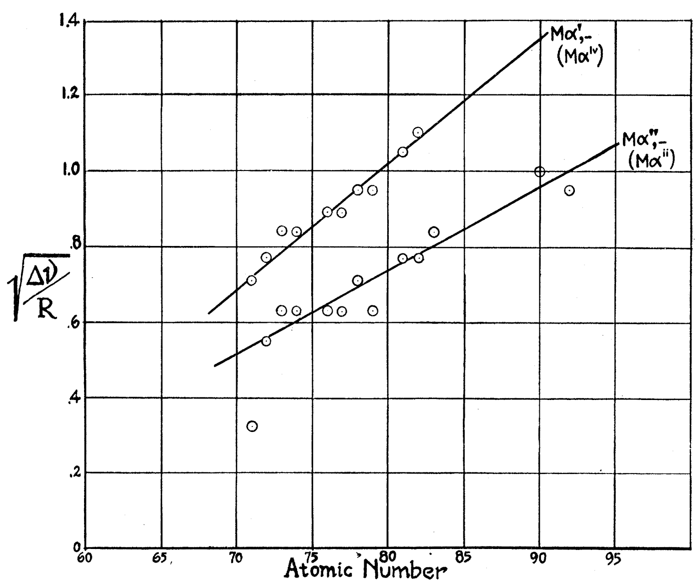

Fig. 1. $M\alpha_1$ satellites (from Lindberg's data).

tension of previous M-series precision wave-length measurements. He noted satellites of the diagram lines $M\alpha_1$ and $M\beta$ only.

The satellite data, secured from the data tables of the articles by Lindberg,[12,13] are plotted as a semi-Moseley graph in Figs. 1 and 2. Lindberg finds two satellites of $M\alpha_1$ and one satellite of $M\beta$, in agreement with the results of Hjalmar.[11] (Hjalmar apparently found three satellites of $M\beta$, but this was for one element only, U(92).)

Especially to be noted in Fig. 2 is *some sort of discontinuity in the semi-*

[8] F. K. Richtmyer and R. D. Richtmyer, Phys. Rev. **34**, 574 (1929).
[9] O. R. Ford, Phys. Rev. **37**, 1695 (1931).
[10] E. Hjalmar, Zeits. f. Physik **1**, 439 (1920).
[11] E. Hjalmar, Zeits. f. Physik **15**, 65 (1923).
[12] E. Lindberg, Zeits. f. Physik **50**, 82 (1928).
[13] E. Lindberg, Zeits. f. Physik **57**, 797 (1929).

Moseley graph at or about atomic number 70. Further mention will be made of this fact.

Experimental Work

In the present investigation a Siegbahn vacuum spectrograph was used. Spectra were recorded on Imperial eclipse plates cut specially for this spectrograph. Three different crystals were used to cover the wave-length range from 3.708A to 10.234A. Calcite was used for the range from 3.0 to 6.0A.; quartz, for the range from 6.0 to 8.0A., and gypsum (selenite), for the range from 8.0 to 10.3A.

The d.c. voltage for the x-ray tube was supplied by a standard high-potential circuit, using kenotrons and condensers in the usual manner. A small vol-

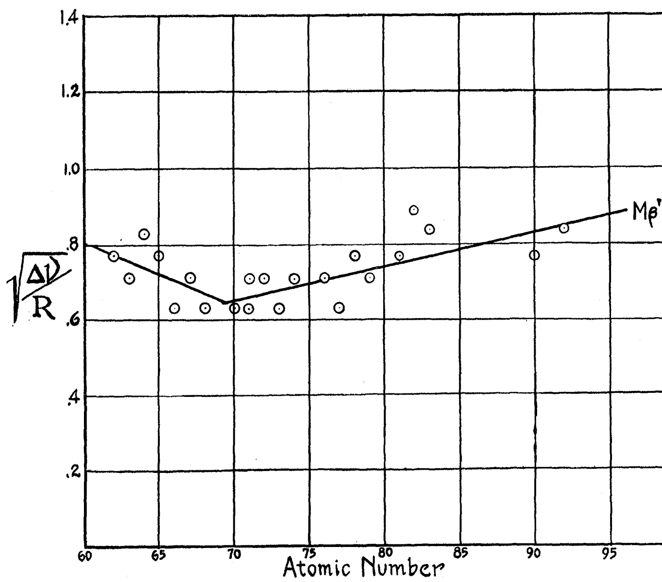

Fig. 2. $M\beta$ satellites (from Lindberg's data).

tage ripple was present. A third kenotron was used as a thermionic valve to limit the current through the x-ray tube to a safe value. This prevented the sudden high-current discharges which are disastrous to milliammeters. The current was read with a standard Weston milliammeter designed for x-ray work. Voltage was measured by means of a repulsion-type, electrostatic voltmeter, which was calibrated by a standard spark gap. The moving member of this voltmeter consisted of two ping-pong balls, coated with india ink and fastened at the ends of a threaded brass tube of small diameter. This device reduced the inertia of the moving member, and the india ink rendered the balls conducting.

Exposures of the x-ray plates, necessary to bring out the satellite structure, varied from one to eight hours, depending on the power which could be safely applied to the anti-cathode wedge and the material fixed on the wedge. The exposure time was considerably reduced in the case of the long wave-length x-rays, where aluminum leaf was used as the slit covering to exclude light from the plate-crystal chamber of the spectrograph. Ordinary typewriter carbon paper proved satisfactory for the slit covering, when wavelengths of from 3 to 5A were being used. However, in the region of from 7 to 12A this absorbed practically all the radiation which should have passed through to the crystal and plate.

In Table I, is given the form in which each element was used as an anti-cathode material, and the metal which composed the wedge supporting the material. Wedges were prepared by roughening the metal surface with a pointed tool, and the material to be used as an anti-cathode was forced on to the roughened surface. Such wedges would often last three hours, with the x-ray tube running at 8 kv and 12 to 13 ma.

TABLE I. *Form in which each element was used as anti-cathode, and the wedge material.*

Element	Used in form of	Wedge material
U	Yellow Oxide, UO_3	Cu
Th	White Oxide, ThO_2	Cu
Bi	Metal	Cu
Pb	Metal (commercial solder)	Cu
Tl	Metal	Cu
Au	Metal	Cu
Pt	Metal	Cu
Ir	Metal (powdered)	Al
W	Metal (flat spiral)	Al
Ta	Metal	Al
Hf	White oxide	Al
Lu	Oxide, mixed with BeO	Al
Yb	Oxide, mixed with BeO	Al
Er	Oxide	Al
Dy	Oxide, mixed with BeO	Al
Gd	Oxide	Al

The potential, at which the x-ray tube was run, was always maintained at least twice the excitation voltage for the parent line, for the purpose of having the satellites as intense as possible.

While most metals can be hard soldered to copper, for use as x-ray targets, tungsten and tantalum are notable exceptions. Neither metal is wetted by molten solder and hence cannot be secured by this method. A flat spiral of tungsten wire was made, and this was pounded into an aluminum wedge which served satisfactorily for an anti-cathode wedge. A small piece of sheet tantalum was likewise pounded into a piece of aluminum and thus firmly secured for use as a target.

Although tungsten spirals are convenient for use as x-ray tube filaments, they usually coat the anti-cathode with tungsten, a very objectionable feature. This can be avoided by using oxide filaments, which, however, are relatively short lived.

The wave-lengths of the lines accompanying a given diagram, (or "parent"), line, were determined by means of comparison with the known wavelengths of the parent lines.

Satellites of the diagram lines $M\alpha_1$ and $M\beta$ were measured for sixteen elements in the range of atomic numbers between U(92), and Gd(64).

Satellites of the diagram line $M\alpha_1$ were present at U(92) and vanished at Er (68). Satellites of the diagram line $M\beta$ were present at U(92) and continued to Gd(64), where the last remaining satellite was so faint that observations beyond this point must be considered as somewhat doubtful.

A careful search was made for satellites of the diagram line $M\gamma$ (previously reported by Hjalmar).[11] Faint lines accompanying $M\gamma$ were observed, but they were identified as higher order reflections of known L-series lines.

Another investigation was carried out in the M-series spectrum of Bi(83), for the purpose of detecting satellites accompanying diagram lines other than $M\alpha_1$ and $M\beta$. The results were negative. Satellites, if present, must be so faint as to avoid detection. Perhaps all diagram lines are modified by optical

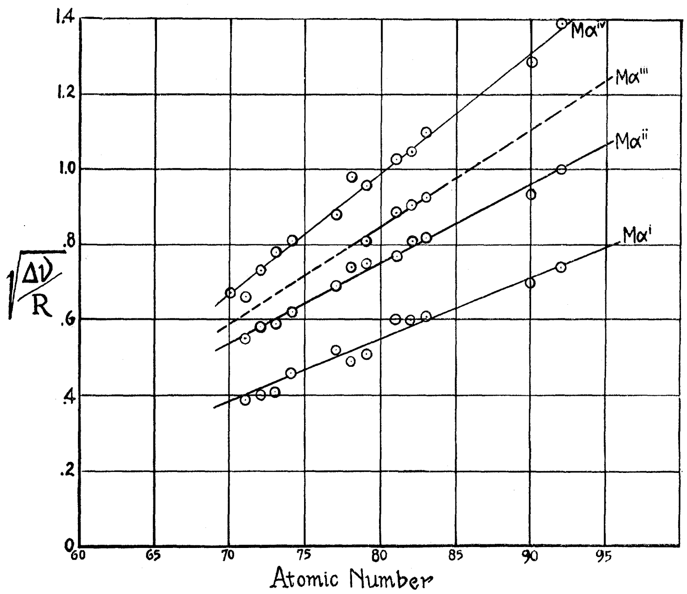

Fig. 3. $M\alpha_1$ satellites.

lines, so as to have satellites. The modified part of the parent line (the satellite) will be much fainter than the parent line and accordingly the satellite will be readily observable only for the stronger diagram lines which for the M-series are $M\alpha_1$ and $M\beta$.

Experimental Results

In Table II is given the results of the study of the $M\alpha_1$ lines and their accompanying satellites. The terminology of F. K. Richtmyer has been used for satellite designation. The satellites have been called $M\alpha^{i}$, $M\alpha^{ii}$, $M\alpha^{iii}$ and $M\alpha^{iv}$, where $M\alpha^{i}$ is the satellite nearest the peak of the parent line, and the others are located successively towards shorter wave-lengths.

In Tables II and III, values of $\Delta\lambda$ are the differences in wave-length between the satellites and the parent lines, ($M\alpha_1$ or $M\beta$), and are determined by direct measurements on the spectrum plates. $\Delta\nu/R$ and $(\Delta\nu/R)^{1/2}$ are then

TABLE II. *Data on satellites of $M\alpha_1$ lines.*

M line	U (92)	Th (90)	Bi (83)	Pb (82)	Tl (81)	Au (79)	Pt (78)	Ir (77)	W (74)	Ta (73)	Hf (72)	Cp (71)	Yb (70)	
$M\alpha_2$	3.9146	4.1416	5.1181	5.2845	5.4543	5.8372	6.0440	6.2581	6.9763	7.2440	7.5308	—	8.1359	λ Wave-length values (angstroms)
$M\alpha_1$	3.9010	4.1290	5.1070	5.2730	5.4430	5.8270	6.0330	6.2480	6.9670	7.2350	7.5230	7.8230	8.1240	
$M\alpha^{i}$	3.8920	4.1198	5.0961	5.2626	5.4319	5.8164	6.0236	6.2364	6.9562	7.2248	7.5128	7.8136	—	
$M\alpha^{ii}$	3.8843	4.1114	5.0878	5.2537	5.4237	5.8055	6.0113	6.2292	6.9469	7.2148	7.5035	7.8029	—	
$M\alpha^{iii}$	—	—	5.0821	5.2486	5.4174	5.8014	—	—	—	—	—	—	—	
$M\alpha^{iv}$	3.8690	4.0890	5.0724	5.2400	5.4083	5.7922	5.9950	6.2153	6.9330	7.1998	7.4897	7.7937	8.0917	
$M\alpha_2$	233.71	220.91	178.76	173.13	167.74	156.74	151.38	146.20	131.14	126.30	121.49	—	112.45	ν/R values
$M\alpha_1$	234.53	221.59	179.16	173.50	168.10	157.04	151.65	146.43	131.32	126.46	121.62	116.95	112.62	
$M\alpha^{i}$	235.08	222.08	179.53	173.86	168.46	157.30	151.89	146.70	131.53	126.63	121.78	117.10	—	
$M\alpha^{ii}$	235.53	222.53	179.83	174.15	168.69	157.60	152.20	146.90	131.70	126.81	121.93	117.25	—	
$M\alpha^{iii}$	—	—	180.03	174.32	168.89	157.70	—	—	—	—	—	—	—	
$M\alpha^{iv}$	236.47	223.26	180.37	174.60	169.17	157.96	152.61	147.20	131.97	127.07	122.16	117.39	113.07	
$M\alpha^{i}$	0.55	0.49	0.37	0.36	0.36	0.26	0.24	0.27	0.21	0.17	0.16	0.15	—	$\Delta\nu/R$ values
$M\alpha^{ii}$	1.00	0.94	0.67	0.65	0.59	0.56	0.55	0.47	0.38	0.35	0.31	0.30	—	
$M\alpha^{iii}$	—	—	0.87	0.82	0.79	0.66	—	—	—	—	—	—	—	
$M\alpha^{iv}$	1.94	1.67	1.21	1.10	1.07	0.92	0.96	0.77	0.65	0.61	0.54	0.44	0.45	
$M\alpha^{i}$	0.74	0.70	0.61	0.60	0.60	0.51	0.49	0.52	0.46	0.41	0.40	0.39	—	$(\Delta\nu/R)^{1/2}$ values
$M\alpha^{ii}$	1.00	0.94	0.82	0.81	0.77	0.75	0.74	0.69	0.62	0.59	0.58	0.55	—	
$M\alpha^{iii}$	—	—	0.93	0.91	0.89	0.81	—	—	—	—	—	—	—	
$M\alpha^{iv}$	1.39	1.29	1.10	1.05	1.03	0.96	0.98	0.88	0.81	0.78	0.73	0.66	0.67	
$M\alpha^{i}$	0.0090	0.0092	0.0109	0.0104	0.0111	0.0106	0.0094	0.0116	0.0108	0.0102	0.0102	0.0094	—	Δλ values (angstroms)
$M\alpha^{ii}$	0.0167	0.0176	0.0192	0.0193	0.0193	0.0215	0.0217	0.0188	0.0201	0.0202	0.0195	0.0201	—	
$M\alpha^{iii}$	—	—	0.0239	0.0244	0.0256	0.0256	—	—	—	—	—	—	—	
$M\alpha^{iv}$	0.0320	0.0310	0.0336	0.0330	0.0347	0.0338	0.0380	0.0327	0.0327	0.0352	0.0333	0.0293	0.0323	

Richtmyer Notation $\left\{\begin{array}{l} M\alpha^{ii}=M\alpha'' \\ M\alpha^{iv}=M\alpha' \end{array}\right\}$ Siegbahn Notation computed from these values of $\Delta\lambda$, taking the values of λ from Lindberg's data.[12,13] The wave-lengths of the several satellites are computed by subtracting from the values of λ* the corresponding values of $\Delta\lambda$.

The semi-Moseley graph for the $M\alpha_1$ satellites is given as Fig. 3. From the graph, it is seen that there are four distinct satellites of $M\alpha_1$. $M\alpha^i$ and $M\alpha^{iv}$ are relatively faint lines, while $M\alpha^{ii}$ is a relatively strong line. $M\alpha^{iii}$ appears resolved from $M\alpha^{ii}$ only for atomic numbers 79 through 83. By comparison of Fig. 3 with Fig. 1 it can be seen that $M\alpha^{ii}$ and $M\alpha^{iv}$ are the satellites $M\alpha''$ and $M\alpha'$ observed by Lindberg.[12,13]

Thus Lindberg's observations are verified, and, in addition, two new satellites are observed, namely $M\alpha^i$ and $M\alpha^{iii}$. Fig. 3 shows a distinct im-

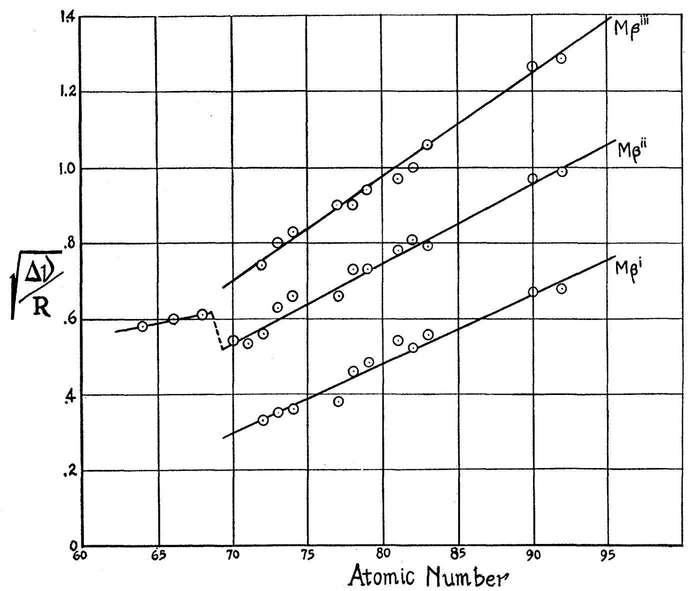

Fig. 4. $M\beta$ satellites.

provement over Fig. 1 in the grouping of the data points about the average straight lines, indicating improved experimental accuracy. The graph shows that $(\Delta\nu/R)^{1/2}$ plotted against Z, atomic number, is a straight line relation within experimental error.

In Table III the results of the studies of $M\beta$ lines are tabulated. The data are again plotted as a semi-Moseley graph in Fig. 4. Three distinct satellites are shown here, and they have been called $M\beta^i$, $M\beta^{ii}$ and $M\beta^{iii}$, using the Richtmyer method of designation. As before, $M\beta^i$ is the satellite closest to the parent line maximum, and $M\beta^{ii}$ and $M\beta^{iii}$ are successively farther from this peak.

Referring back to Fig. 2, one readily sees that $M\beta^{ii}$ is Lindberg's satellite

* It will be noted that the last figure in each parent line wave-length value is given as zero. Lindberg's data does not give this last figure. It was used merely for convenience in computing the data of the present paper.

TABLE III. *Data on satellites of $M\beta$ lines.*

M line	U (92)	Th (90)	Bi (83)	Pb (82)	Tl (81)	Au (79)	Pt (78)	Ir (77)	W (74)	Ta (73)	Hf (72)	Cp (71)	Yb (70)	Er (68)	Dy (66)	Gd (64)	
$M\beta$	3.7080	3.9330	4.8980	5.0640	5.2380	5.6110	5.8150	6.0240	6.7410	7.0060	7.2890	7.5860	7.8920	8.5760	9.3450	10.2340	λ Wave-length values (angstroms)
$M\beta^{i}$	3.7011	3.9254	4.8901	5.0564	5.2294	5.6030	5.8070	6.0179	6.7344	6.9996	7.2828	—	—	—	—	—	
$M\beta^{ii}$	3.6933	3.9172	4.8795	5.0455	5.2197	5.5929	5.7951	6.0064	6.7188	6.9848	7.2711	7.5697	7.8714	8.5484	9.3101	10.1925	
$M\beta^{iii}$	3.6831	3.9100	4.8687	5.0361	5.2099	5.5807	5.7853	5.9914	6.7072	6.9758	7.2569	—	—	—	—	—	
$M\beta$	246.74	232.63	186.79	180.67	174.67	163.06	157.34	151.89	135.73	130.59	125.52	120.60	115.94	106.66	97.91	89.40	ν/R values
$M\beta^{i}$	247.20	233.08	187.10	180.94	174.96	163.29	157.55	152.03	135.86	130.71	125.63	—	—	—	—	—	
$M\beta^{ii}$	247.72	233.57	187.50	181.33	175.28	163.59	157.88	152.32	136.17	130.99	125.83	120.88	116.23	107.03	98.27	89.74	
$M\beta^{iii}$	248.41	233.99	187.92	181.67	175.61	163.94	158.15	152.70	136.41	131.15	126.07	—	—	—	—	—	
$M\beta^{i}$	0.46	0.45	0.31	0.27	0.29	0.23	0.21	0.14	0.13	0.12	0.11	—	—	—	—	—	$\Delta\nu/R$ values
$M\beta^{ii}$	0.98	0.94	0.71	0.66	0.61	0.53	0.54	0.43	0.44	0.40	0.31	0.28	0.29	0.37	0.36	0.34	
$M\beta^{iii}$	1.67	1.36	1.13	1.00	0.94	0.88	0.81	0.81	0.69	0.64	0.55	—	—	—	—	—	
$M\beta^{i}$	0.68	0.67	0.56	0.52	0.54	0.48	0.46	0.38	0.36	0.35	0.33	—	—	—	—	—	$(\Delta\nu/R)^{1/2}$ values
$M\beta^{ii}$	0.99	0.97	0.79	0.81	0.78	0.73	0.73	0.66	0.66	0.63	0.56	0.53	0.54	0.61	0.60	0.58	
$M\beta^{iii}$	1.29	1.17	1.06	1.00	0.97	0.94	0.90	0.90	0.83	0.80	0.74	—	—	—	—	—	
$M\beta^{i}$	0.0069	0.0076	0.0079	0.0076	0.0086	0.0080	0.0080	0.0061	0.0066	0.0064	0.0062	—	—	—	—	—	$\Delta\lambda$ values (angstroms)
$M\beta^{ii}$	0.0147	0.0158	0.0185	0.0185	0.0183	0.0181	0.0199	0.0176	0.0222	0.0212	0.0179	0.0163	0.0206	0.0276	0.0349	0.0415	
$M\beta^{iii}$	0.0249	0.0230	0.0293	0.0279	0.0281	0.0303	0.0297	0.0326	0.0338	0.0302	0.0321	—	—	—	—	—	

Richtmyer Notation, $M\beta^{ii} = M\beta'$, Siegbahn Notation

$M\beta'$. Peculiarly too, the curve for the satellite $M\beta^{ii}$ shows a break at atomic number 70, precisely as is shown in Fig. 2. The satellite $M\beta^{ii}$ is a relatively strong line, while $M\beta^{i}$ and M^{iii} are much less intense. Hence it is not surprising that $M\beta^{ii}$ persists at the lower atomic numbers, where $M\beta^{i}$ and $M\beta^{iii}$ are too faint for detection.

By comparison of Figs. 2 and 4 it can be seen that *$M\beta^{i}$ and $M\beta^{iii}$ are new satellites.*

Obviously now, this question arises: if, according to the double-jump hypothesis, a certain set of semi-optical jumps modify one x-ray diagram line to produce a set of satellites, might not these same jumps modify a second x-ray diagram line and produce another similar set of satellites? To test this suggestion, Figs. 3 and 4 are superimposed in Fig. 5. From Fig. 5 it can be seen that the curves for $M\alpha^{ii}$ and $M\beta^{ii}$ are practically coincident; $M\beta^{iii}$ and $M\alpha^{iv}$ while having different slopes, lie in the same region. The lines for $M\alpha^{i}$ and $M\beta^{i}$ lie within a reasonable distance from each other. On the other hand, $M\alpha^{iii}$ has no counterpart in the $M\beta$ series of satellites. The fact that these pairs of lines are not exactly coincident, may be the result of experimental error. On the other hand, there may be some real difference between the satellites of one diagram line and those of a neighboring line. However, the close proximity of the superimposed satellite curves, for two different diagram lines, strongly supports the idea that the same set of optical jumps modifies both diagram lines, to give rise to the observed satellites.

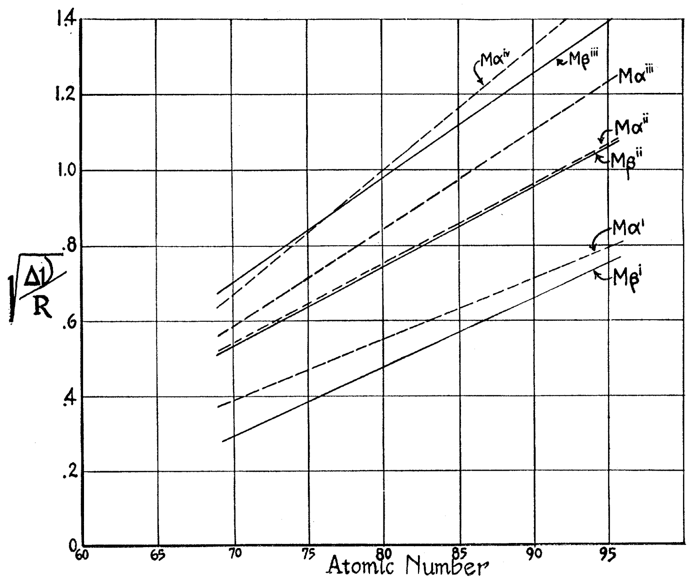

Fig. 5. Sattellites of $M\alpha_1$ and $M\beta$.

In Fig. 4, the semi-Moseley graph for the satellite $M\beta^{ii}$ shows a definite break in the region of atomic number 69 or 70, with a displacement of the former straight line and a change in its slope. It will be remembered that this discontinuity is present in the curve plotted from Lindberg's data (Fig. 2).

Foote's arrangement of electrons in orbits[14] shows that at atomic number 71, the building of the N_{VII} electron shell is just completed. Conversely, at atomic number 70(Yb) the first of the rare earths, the removal of electrons from the N_{VII} shell starts as we go down the atomic number range. It is known also that the Bohr-Coster diagrams, for the M, N and O, x-ray energy-levels, show similar breaks in the region of atomic number 70. Likewise, curves for the screening constant of the M and N electron shells, show breaks at the region of atomic number 70.

The interpretation of these facts, in terms of the double-jump hypothesis, is obvious. In terms of this theory, the semi-Moseley graphs represent a number of radiated semi-optical lines. A graph, showing a break at atomic number 70, means that the optical frequencies must suffer some change at this place in the periodic table. Since these semi-optical lines originate by electron jumps between two atomic energy levels, it at once follows that either one, (or both), of these energy levels must be one of those which show breaks in the Bohr-Coster diagrams, or which show a change in the screening constant in the region of atomic number 70.

It was observed in this research, that the satellites of the diagram line $M\alpha_1$ vanished at Er(68). Here, according to Lindberg,[13] the $M\alpha_1$ line splits into three components. This splitting was observed, too, by the present author. This fact, on the basis of the double-jump hypothesis, affords a possible explanation of the fading out of the satellites of the diagram line $M\alpha_1$. The semi-optical jumps will now modify all three components of the $M\alpha_1$ line, giving rise to three series of satellites, which being correspondingly broad, and therefore faint, will be difficult to observe.

It is significant that the range of values of $\Delta\nu/R$ for the M-satellites, as shown in Tables II and III, and in Figs. 3 and 4, is very close to the corresponding range for K and L satellites as found by other workers.

The spectrum plates of the diagram lines $M\alpha_1$ and $M\beta$ of Th (90), and U (92), show the *satellites of these lines as superimposed on a sort of continuous spectrum*. This fact has already been noticed and explained by F. K. Richtmyer[7] in the case of some satellites at the high atomic number end of the L-series satellite range.

The double-jump hypothesis opens up a new method of attacking the problem of the origin of x-ray satellites. Briefly, it is this. The Richtmyer double-jump equation (reference 7, page 359) is,

$$h\nu_s = h\nu_i + h\nu_0$$

where ν_s is the frequency of the satellite; ν_i that corresponding to the inner jump, and ν_0 that corresponding to the outer jump. From this equation it is clear that if ν_s and ν_i are known, values of ν_0 are predicted. Thus, for a given satellite, there should occur in the spark spectrum of the particular element, a radiated frequency of value ν_0. Perhaps this may be for a forbidden jump, and we may only observe a term-difference in the energy level diagram for the atom,—which term-difference will be of size $h\nu_0$. Thus the double-jump

[14] Foote, Trans. Amer. Inst. Min. Metal. Eng. **63,** 628 (1926).

hypothesis may be experimentally verified by the appearance of the predicted lines. By simple calculations, it can be found from the data in this article, that these values of ν_0 will predict lines in the soft x-ray, ultraviolet, and ordinary optical regions. Remembering that $c=\nu_0\lambda_0$, where c is the velocity of light in vacuo, and λ_0 is the wave-length of the semi-optical line of frequency ν_0, which is represented, according to the double-jump theory, by the values of $\Delta\nu/R$, given in Tables II and III, we have

$$\lambda_0 = \frac{c}{\Delta\nu/R}\,.$$

Using the accepted values for c and R, we find that,

$$\lambda_0 = \frac{911.3 \times 10^{-8}}{\Delta\nu/R}\ \mathrm{cm}.$$

From this last relation, the following values of λ_0 are computed from the values of $\Delta\nu/R$ given in Table II for the four satellites of the diagram line $M\alpha_1$ in the spectrum of Bi (83).

Line	$\Delta\nu/R$	λ_0
$M\alpha^{i}$	0.37	2460A
$M\alpha^{ii}$	0.67	1358
$M\alpha^{iii}$	0.87	1046
$M\alpha^{iv}$	1.21	753

The investigation of the possibility of the occurrence of spark lines, as predicted by x-ray satellites, is under way in this laboratory. Preliminary work seems to show promise, and a detailed report will be made later.

If this connection, as suggested by the double-jump hypothesis, does exist, it means that there is a relation, previously unsuspected, existing between x-ray and optical spectroscopy.

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