THE EFFECT OF HYPERFINE STRUCTURE ON THE POLARIZATION OF CADMIUM RESONANCE RADIATION

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Abstract

The polarization of cadmium resonance radiation (λ3261) has been calculated assuming that cadmium is composed of two kinds of isotopes having a nuclear moment $i=0$ and $i=1/2$ respectively. The ratio of the isotopes of nuclear moment $i=1/2$ to $i=0$ was taken from the hyperfine structure data on cadmium as measured by Schüler and Keyston. The percentage polarization to be expected for "broad" and "narrow" line excitation and for different orientations of electric vector and magnetic field are given and comparison is made with Soleillet's experiments.

That hyperfine structure can affect the polarization of resonance radiation was shown by MacNair and Ellet[1] in the case of the 2537A line of mercury. In this way they were able to explain the fact that mercury resonance radiation, in the absence of a magnetic field, is only 90 percent polarized instead of 100 percent which one would expect from the Zeeman levels of a line $(2^3P_1-1^1S_0)$ showing no hyperfine structure. More recently Ellet[2] has calculated theoretically the polarization to be expected for certain thallium lines taking into account the known hyperfine structure exhibited by these lines.

Schüler and Keyston[3] measured the hyperfine structure of the sharp triplet in cadmium $(2^3S_1-2^3P_{0,1,2})$. They found that their results could be explained on the assumption that of the various isotopes of cadmium those of even atomic weight have no nuclear moment $(i=0)$ and those of odd atomic weight have a nuclear moment $i=1/2$. The isotopes having no nuclear mo ment give rise to an unshifted hyperfine-structure component while those having a nuclear moment show shifted components. By measuring the relative intensities of the several components, they were able to obtain the abundance ratio of the isotopes of even to those of odd atomic weight.

Applying these considerations to the resonance line of cadmium $(2^3P_1-1^1S_0)$ at λ3261, one would expect to obtain three hyperfine-structure components—$(2^3P_{f=1}\rightarrow 1^1S_{f=0})$ for the isotopes with $i=0$; $(2^3P_{f=3/2}\rightarrow 1^1S_{f=1/2})$ and $(2^3P_{f=1/2}\rightarrow 1^1S_{f=1/2})$ for the isotopes of $i=1/2$. Here, as is customary, $f=i+j$. The hyperfine structure of this line has been measured by Wood[4] and Schrammen,[5] who found only two components. Since with the sources they used it was hard to obtain the 3261A line entirely free from self reversal, it is possible that one of the components may have been missed.

In order to calculate the polarization of cadmium resonance radiation the Zeeman diagrams for the three hyperfine-structure components must be

[1] W. MacNair and A. Ellett, Phys. Rev. **31**, 180 (1928).
[2] A. Ellett, Phys. Rev. **35**, 588 (1930).
[3] Schüler and Keyston, Zeits. f. Physik **67**, 433 (1931).
[4] R. W. Wood, Phil. Mag. **2**, 611 (1926).
[5] A. Schrammen, Ann. d. Physik **83**, 1161 (1927).

drawn. For low magnetic fields when the Zeeman separation is small compared to the hyperfine structure separation, the three components may be treated as three separate lines. The levels for the three components in question are given in Fig. 1. The figures under each Zeeman component give the intensity of each component, while the greek letters and the figures directly under them represent the transition probabilities. The component A, due to isotopes with $i=0$ shows the usual cadmium-like Zeeman pattern, while the components a and b due to isotopes with $i=1/2$ are sodium-like. The *a priori* weights of a:b are as 2:1 which follows from the sum rule and has been shown experimentally by Schüler and Keyston[3] for the line 4678A $(2^3S_1-2^3P_0)$ which should have the same structure as 3261. The *a priori* weight of the component A is taken as 3 on the same scale and the intensities of each component are so chosen that the chance of leaving any given magnetic level shall

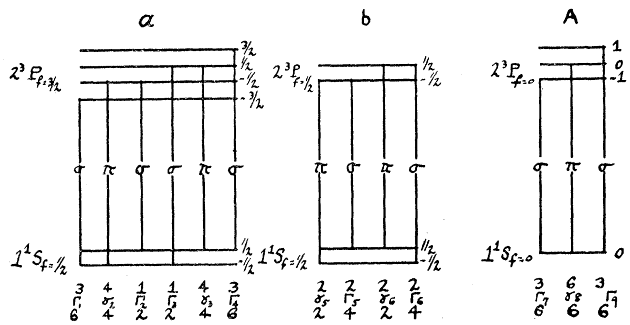

Fig. 1.

be the same for all levels. This is equivalent to the assumption that all the upper magnetic levels of all the hyperfine structure components have the same mean life.

In calculating the polarization, let I_a, I_b, I_A be the intensity of the hyperfine structure component a, b, and A in the source, and N_1, N_2 the relative numbers of isotopes having $i=1/2$ and $i=0$ respectively. Suppose the exciting light beam approaches the resonance tube from the Y direction and observations of the resonance radiation are made along Z. Let the electric vector of the incident light wave make an angle θ with a magnetic field applied to the resonance tube anywhere in the $X-Y$ plane. Let ξ and η be the intensities of the components of the resonance radiation along and perpendicular to the magnetic field respectively. Van Vleck[6] has given formulae for ξ and η in the case of single lines under various excitation conditions. The formulae for a single hyperfine structure component are

$$\xi = CI \sum_i \frac{\gamma_i}{\gamma_i + \Gamma_i}\{\gamma_i \cos^2\theta + \tfrac{1}{2}\Gamma_i \sin^2\theta\} \qquad (1)$$

$$\eta = \tfrac{1}{2}CI \sum_i \frac{\Gamma_i}{\gamma_i + \Gamma_i}\{\gamma_i \cos^2\theta + \tfrac{1}{2}\Gamma_i \sin^2\theta\} \qquad (2)$$

where C is a constant and I the intensity of the exciting light beam.

[6] J. H. Van Vleck, Proc. Nat. Acad. Sci. 11, 612 (1925).

For the case of hyperfine structure under consideration the intensity of the resonance radiation is proportional to the relative intensity of the hyperfine structure components in the source and to the relative number of isotopes of the two kinds present in the resonance tube. Eqs. (1) and (2) will now become

$$\xi = k \sum_j N_j I_j \sum_i \frac{\gamma_i}{\gamma_i + \Gamma_i} \{\gamma_i \cos^2 \theta + \tfrac{1}{2}\Gamma_i \sin^2 \theta\} \tag{3}$$

$$\eta = \tfrac{1}{2} k \sum_j N_j I_j \sum_i \frac{\Gamma_i}{\gamma_i + \Gamma_i} \{\gamma_i \cos^2 \theta + \tfrac{1}{2}\Gamma_i \sin^2 \theta\} \tag{4}$$

where $\sum_i$ is to be taken over all the Zeeman transition probabilities of a given component and $\sum_j$ means

$$N_1 I_a \sum_i (\quad)_a + N_1 I_b \sum_i (\quad)_b + N_2 I_A \sum_i (\quad)_A$$

Now the polarization P is defined by

$$P = \frac{\xi - \eta}{\xi + \eta}.$$

On substituting the values of γ_1, Γ_1 from Fig. 1 in (3) and (4) and carrying out the indicated summation one finds

$$P = \frac{(2I_a N_1 + 3I_A N_2)(\cos^2 \theta - \frac{1}{2} \sin^2 \theta)}{2I_a N_1(5/3 \cos^2 \theta + 7/6 \sin^2 \theta) + I_b N_1 4/3 + 3I_A N_2(\cos^2 \theta + \frac{1}{2} \sin^2 \theta)} \tag{5}$$

Schüler and Keyston measured the relative intensities of the hyperfine structure components for the 4675A line of cadmium and found

$$\frac{I_a + I_b}{I_a + I_b + I_A} = 0.23 \tag{6}$$

from which it follows, since $I_a = N_1$, $I_b = N_1$, $I_A = 3N_2$, that

$$N_2 = 3.34 N_1. \tag{7}$$

Several interesting cases arise for computation. The source may be one which gives broad lines, due to high temperatures, such that $I_a = I_b = I_A$; or it may be a low temperature source giving hyperfine structure lines with relative intensities as given by (6). The incident beam may be polarized in the X direction and the resonance lines may be in a zero magnetic field. This case, due to spectroscopic stability, is the same as if the resonance tube were in a weak magnetic field parallel to X so that $\theta = 0$. The incident light may be unpolarized and the resonance tube in a weak magnetic field parallel to Y so that $\theta = \pi/2$. The results for the various cases are given in Table I.

TABLE I.

	$\theta=0$	$\theta=\pi/2$
Broad Lines	81.9%	69.4%
Narrow Lines	96.4%	93%
Experiment (λ3261)	73%^{A}	85^B%

Comparison with Experiment

The polarization of the resonance line λ3261 in cadmium has been measured by MacNair[7] and Soleillet.[8] The experiments of the former were made with rather high vapor pressures of cadmium so that some depolarizing effect had undoubtedly set in. Soleillet measured the polarization in a zero magnetic field with exciting light polarized parallel to X, and used low vapor pressures. At temperatures of 170°C and 115°C he found the polarization to be constant and equal to 73 percent. Since the 2^3P_1 state of cadmium has a long mean life the polarization of the resonance radiation is greatly affected by weak magnetic fields, of the order of 0.01 gauss. As it is difficult to make the resonance tube free from magnetic fields to this extent he also repeated the experiment in a magnetic field parallel to the incident light beam, and also in a zero field. In these experiments, the exciting light was unpolarized. In both cases he found 85 percent polarization. The experimental conditions for the second set of experiments seems to have been much better than in the first, the resonance tube always being attached to the pumps instead of sealed off as in the former case.

In a zero magnetic field it is clear that the Zeeman separation is small compared to that of the hyperfine structure. For the case of a magnetic field parallel to Y of the order of 50 gauss (Soleillet's field was probably about this strong), the Zeeman separation is of the order of 2×10^{-3} cm^{-1} while that of the hyperfine structure is about $(50\text{–}100)\times10^{-3}$ cm^{-1} so that the assumptions used in the present calculation probably hold.

The ratio of the hyperfine-structure components in the source is, of course, not known, and the measurements are difficult to carry out so that no exact agreement can be expected. If, as one might expect, a source which will excite resonance radiation is more nearly a "narrow line" than a "broad line" source, the agreement with Soleillet's second experiment is quite satisfactory.[9] The fact that the results of Soleillet's second experiment lie between the theoretical values for "broad" and "narrow" lines while those of his first experiment are always too low, is probably due to having less gas in the resonance tube, so that less depolarization by collision occurred, and to better neutralization of stray fields.

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[7] MacNair, Phys. Rev. **29**, 766 (1927).

[8] P. Soleillet; A. Compt. Rend. **185**, 198 (1927); B, ibid. **187**, 212 (1928).

[9] The effect of nuclear spin, of course, is to decrease the value of the percentage polarization below that of 100 percent to be expected for cadmium resonance radiation if there were no nuclear spin. The fact that the percentage agreement between the 85 percent (experiment) and 93 percent (theory) seems good and the difference between 100 percent and the above values gives a large percentage error between theory and experiment does not seem significant, on account of the uncertainties of the experiments as pointed out above. What is of interest, however, is that the observed polarization lies between the theoretical values for the two types of sources and that its value is certainly not as great as 100 percent.