THE CATHODE FALL OF AN ARC

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Abstract

The distribution of energy of electrons at the boundary of the cathode fall of an arc is worked out for the thermionic arc and for the high field arc upon both the classical Schottky theory, and the quantum mechanical theory of high field emission. For the same value of cathode fall, the electrons from the thermionic arc have energies several volts greater than those from the high field arc. To give the positive ions necessary for space charge purposes, the cathode fall of the high field arc consequently must be several volts greater than that of the thermionic arc, if the mechanism of production of positive ions is the same in the two arcs. The minimum value of the difference in cathode fall is 4 to 6 volts. Tests are thus suggested for Langmuir's high field theory of cold cathode arcs. Data for the mercury arc indicate that electrons, in sufficient number to produce the necessary positive ions, have energies certainly less than 7 volts at the boundary of the cathode fall. Either the field theory does not apply to the mercury arc, or positive ions are produced by a cumulative process or other complicated means.

IN MANY arcs, current appears to be carried at the cathode primarily by electrons emitted thermionically. To neutralize the electron space charge, and to allow large currents to be drawn with low voltage, some positive ions must exist throughout the discharge, even near the cathode. Since the mobility of the positive ion is so small compared to the electron mobility, only a relatively small positive ion current is needed to provide the positive space charge. Chiefly, the current at the cathode is electron current. To replenish the positive ions, the electrons emitted from the cathode, acquiring sufficient energy in passing through the cathode fall, ionize the molecules of the active gas.

Such a simple theory seems plausible for normal arcs with cathodes heated from an external source, or with cathodes of refractory materials which reach a high temperature. For example, in both tungsten and carbon arcs, the cathode is at a temperature which will give a thermionic current density equal to the observed current density.[1]

When other types of arcs are observed—arcs in which the cathode temperature does not exceed a few hundred degrees—difficulties are encountered at once. Such arcs occur frequently: for example, an arc and its cathode electrode may be moved relative to each other with such velocity that the cathode never reaches a high temperature—the so-called "Stolt arc."[2,3,4] Again, in metals with high vapor pressures at moderate tempera-

[1] K. T. Compton, Trans. A.I.E.E. **46,** 870 (1927).

[2] H. Stolt, Zeits. f. Physik **26,** 95 (1924); Ann. d. Physik **74,** 80 (1924); Inaug. Dissertation, Uppsala, 1925.

[3] Slepian, Trans. A.I.E.E. **48,** 526 (1929).

[4] Seeliger and Wulfhekel, Phys. Zeits. **31,** 691 (1930).

tures, such as cadmium, mercury, etc., it is hard to believe the cathode temperature rises above a few hundred degrees. In many such "cold cathode" arcs, no appreciable purely thermionic emission can occur. How, then, is the current carried at the cathode?

Langmuir[5] first suggested that in the mercury arc electrons may still constitute the major portion of the current at the cathode, but that instead of being emitted thermionically the electrons are pulled out of the relatively cold cathode by a very high field. Later, this theory has been amplified by Compton[6] and MacKeown.[7] Though the potential drop at the cathode is only a few volts, it is conceivable that the potential is concentrated over such a small distance that the gradient may be of the order of millions of volts per cm. Such high fields are known to pull out quite high current densities of electrons.[8]

To test Langmuir's theory experimentally, though highly desirable, seems impossible to do directly. But, indirectly, a test appears possible from measurements on the cathode fall of the arc, in a manner which will be discussed below.

Magnitude of Positive Ion Current Necessary

Whatever the mechanism of freeing electrons from the cathode, a certain minimum positive ion current is necessary to neutralize the negative space charge. For this purpose, as many positive ions are needed if the electrons are pulled out under a high field as if they are emitted thermionically. The actual magnitude of the neutralizing positive ion current need not be very great—less than a thousandth part of the electron current[9] for most ions. So if the electron current density is a few thousand amps./cm^2, a positive ion current of only a few amps./cm^2 is sufficient.

It is possible, however, that many more positive ions than necessary for neutralization of electron space charge will be needed for the high field case. As Slepian and Haverstick[10] have pointed out, the high field postulated at the cathode is produced by the positive ions, and in order to have sufficiently high fields to pull out electrons, the minimum positive ion current must approach the magnitude of thousands of amperes per cm^2. If the electron field current densities are much larger than this, then, as before, the total positive ion current may be determined largely by the amount necessary for neutralization of electron space charge. If, however, the field current densities are moderate, the fraction of current carried by positive ions may be considerably larger than required for neutralization. Evidence deduced by Stern, Gossling, and Fowler[11] from cold cathode currents drawn from small wires,

[5] Langmuir, G. E. Review **26,** 731 (1923).

[6] Compton and Van Voorhis, Proc. Nat. Acad. Sci. **13,** 336 (1927). Compton, Trans. A.I.E.E. **46,** 868 (1927).

[7] MacKeown, Phys. Rev. **34,** 611 (1929).

[8] Millikan and Eyring, Phys. Rev. **27,** 51 (1926). Gossling, Phil. Mag. **1,** 609 (1926).

[9] Compton, Phys. Rev. **21,** 266 (1923); Trans. A.I.E.E. **46,** 873 (1927).

[10] Slepian and Haverstick, Phys. Rev. **33,** 52 (1929).

[11] Stern, Gossling, and Fowler, Proc. Roy. Soc. **124,** 699 (1929).

seems to show that the current densities are extremely high, though the emitting areas are very small. Whether the same conditions result in the case of the arc is, of course, uncertain; even if they do, the emitting spots may be so small, and so scattered about that a very large gross positive ion current will be needed to maintain a high average field over a large area. In any case, then, the total positive ion current required for an arc in which electrons are pulled out by a high field is at least as large as required for an arc in which electrons are emitted thermionically from a cathode maintained at a high temperature; quite possibly, the positive ion current must be much larger for the high field arc. The actual magnitude of the positive ion current density for the high field arc must be at least of the order of a thousand amps./cm^2.

Energy Distribution of Electrons—Classical Theory

The simplest hypothesis as to the production of positive ions is that electrons acquire sufficient energy in traversing the cathode fall to ionize the active gas. Then, electrons, equal in number to the positive ions necessary, must have total energies on emerging from the cathode fall at least equivalent to the ionization potential of the gas from which the positive ions are produced.

Upon the classical theory of an electron gas in a metal, in thermal equilibrium with the metal, the distribution of velocities of the emitted electrons is Maxwellian; experiment confirms the velocity distribution as Maxwellian.[12] The probability of an electron having a total energy between E and $E+dE$ as it emerges from the work function field can be shown easily to be

$$P(E)dE = \frac{e^2}{(kT)^2}\epsilon^{-eE/kT}EdE \tag{1}$$

where E is expressed in volts. The average energy of the electrons is 2 kT, which, at temperatures ordinarily associated with thermionic emission, is equivalent to only a few tenths of a volt. At the boundary of the cathode fall in an arc, practically all the energy of the electrons is the energy acquired from the field through which they have passed. The actual distribution in energy as secured directly from (1) is

$$f(E)dE = \frac{e^2}{(kT)^2}\epsilon^{-e(E-V_c)/kT}(E - V_c)dE \tag{2}$$

where V_c is the cathode fall. The distribution is sketched diagrammatically in Fig. 1. A certain proportion of the electrons must have energies at least as large as the ionizing potential. The shaded area in Fig. 1 is equal to the ratio of positive ion current to electron current, so it is just this fraction which must have energies greater than the ionizing potential, V_i. The diagram makes clear that

[12] Richardson and Brown, Phil. Mag. **16,** 353 (1908); Jones, Proc. Roy. Soc. **A102,** 734 (1923); Germer, Phys. Rev. **25,** 795 (1925).

$$V_c + \alpha = V_i + \beta$$

or

$$V_c = V_i + \beta - \alpha. \quad (3)$$

If a high field is applied to the cathode, the emission of electrons will be increased, according to the Schottky effect.[13] The force on the electron, due

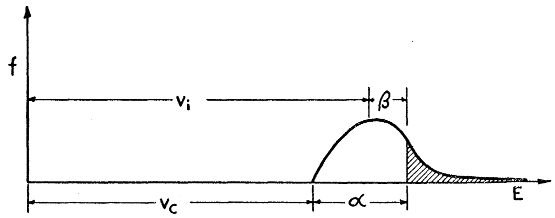

Fig. 1. Energy distribution of electrons at boundary of cathode fall, thermionic case.

to the surface field, is that of the curved line in Fig. 2, following Langmuir.[14] The external applied field is MN, shown constant for simplicity; in the case of the arc, the area OMNP is equal to the cathode fall, V_c'. If particles incident on a region of retarding force have a Maxwellian distribution of velocities, it is a characteristic of such a distribution that those particles which pass through the retarding force will also possess a Maxwellian velocity distribution with same mean energy as the incident particles. This is true regardless of the amount of work done against the field. At the point D in Fig. 2, the

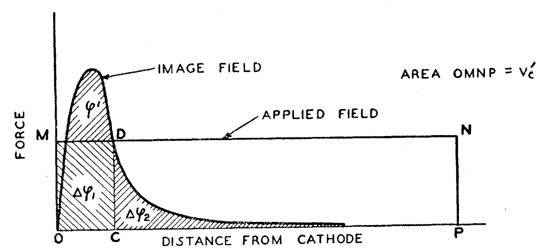

Fig. 2. Force on an electron with a high field applied.

intersection of the surface field and the applied field, the force on an electron is zero; all electrons which reach D from within the metal will escape. At D, then, the probability of an electron having an energy between E and $E+dE$ is exactly that given by Eq. (1). Beyond D, an electron gains an energy $DNPC-\Delta\phi_2$. But $DNPC = V_c' - \Delta\phi_1$, so the gain in energy is $V_c' - \Delta\phi_1 - \Delta\phi_2 = V_c' - \Delta\phi = V_c' - (\phi - \phi')$ where ϕ is the ordinary work function, and ϕ' is the effective work function under the action of the external field. $\Delta\phi$ is exactly the reduction in work function due to the Schottky effect. The distribution in energy of the electrons at the boundary of the cathode fall is changed from Eq. (2) to

[13] Schottky, Phys. Zeits. **15,** 872 (1914); **20,** 220 (1919); Ann. d. Physik **44,** 1011 (1914); Zeits. f. Physik **14,** 63 (1923).

[14] Langmuir, Trans. Am. Electrochem. Soc. **29,** 162 (1916).

$$f_1(E)dE = \frac{e^2}{(kT')^2}\epsilon^{-e(E-V_c+\Delta\phi)/kT'}(E - V_c + \Delta\phi)dE. \tag{4}$$

The function is sketched roughly in Fig. 3. No electrons have an energy less than $V_c' - \Delta\phi$. The area shaded is equal to the area shaded in Fig. 1, since the

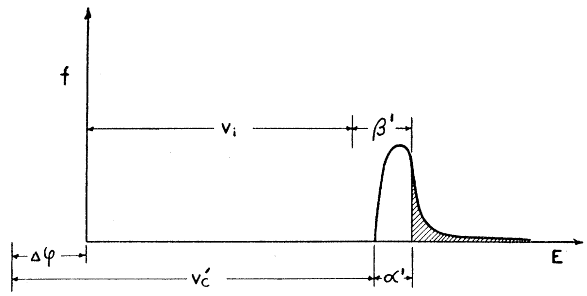

Fig. 3. Energy distribution of electrons at boundary of cathode fall, high field case, classical theory.

fraction of electrons with energy sufficient for ionization must at least be as great as in the thermionic case. As before,

$$\begin{aligned} V_c' - \Delta\phi + \alpha' &= V_i + \beta' \\ \text{or} \qquad V_c' &= V_i + \beta' - \alpha' + \Delta\phi. \end{aligned} \tag{5}$$

Since the arcs to which the high field theory are supposed to apply are those whose cathodes are at temperatures too low for thermionic emission, T' in Eq. (4) is considerably less than T in Eq. (2). So, for the same production of positive ions in the two arcs, α' must be less than α. If more positive ions have to be produced in the high field arc, as seems easily possible, α' will be decreased still further. For the last reason, β' will probably be greater than β. Therefore, V_c' must exceed V_c by at least $\Delta\phi$. The cathode fall of the high field arc must be greater than that of the thermionic arc.

An idea of the magnitude of the difference in cathode falls may be secured as follows: by Schottky's theory, the electron emission in a high field is

$$i = 60T^2\epsilon^{-e(\phi-\Delta\phi)kT}. \tag{6}$$

If $T = 800°K$, $\phi = 4$ volts, and $\Delta\phi = 3.4$, $i = 6400$ amps/cm^2. In order to obtain current densities of a few thousand amps/cm^2, $\Delta\phi$ must approach close to the whole work function.

An estimate of α and α' comes from the probability of an electron having a velocity greater than α: from Eq. (1)

$$\begin{aligned} P(\alpha) &= \int_\alpha^\infty P(E)dE \\ &= \left(\frac{e\alpha}{kT} + 1\right)\epsilon^{-e\alpha/kT}. \end{aligned} \tag{7}$$

In the thermionic arc, take $T=3200°K$, and the following values are obtained

$$\alpha=3 \text{ volts}, \ P(\alpha)=2.25\times10^{-4}$$
$$\alpha=1.5 \text{ volts}, \ P(\alpha)=2.8\times10^{-2}$$

In the high field arc, take $T=800°K$; then

$$\alpha'=0.75 \text{ volts}, \ P(\alpha')=2.25\times10^{-4}$$
$$\alpha'=0.37 \text{ volts}, \ P(\alpha')=2.8\times10^{-2}$$
$$\alpha'=0.25 \text{ volts}, \ P(\alpha')=0.123$$

The ratio of positive ion current to electron current in the thermionic arc is undetermined; it must be at least of the magnitude of the minimum value of $P(\alpha)$, in order to neutralize the space charge. It may be somewhat greater, if a strong positive charge exists in front of the cathode. Likewise, the value of positive ion current for the high field arc is undetermined; for electron current densities of a few thousand, $P(\alpha')$ must have about the maximum value given, in order to provide a sufficiently high field. So, α probably exceeds α' by more than a volt. How much larger β' is than β is also uncertain. It would seem though that V_c' should exceed V_c by five volts or more.

Energy Distribution of Electrons—Quantum Mechanics Theory

Though Langmuir originally offered the ordinary Schottky effect as an explanation of the origin of electrons, it is now known that, at temperatures and fields too low for the Schottky effect to play an appreciable part, quite large current densities of electrons can be secured[8]—the so-called "cold cathode" effect. Too, the classical theory of energy distribution among the electrons in a metal has been abandoned in favor of the more satisfactory theory based on Fermi statistics. Sommerfeld's development of the electron theory of metals, based upon Fermi statistics,[15] shows at temperatures sufficient for thermionic emission those electrons which have enough energy to escape possess a Maxwellian distribution of velocity, though at low temperatures there is nothing resembling a Maxwellian distribution. The theory also gives an increased mean energy to the electrons, but at the same time the work function is increased a like amount. So, for thermionic emission, the classical theory of the energy distribution of electrons still holds.

At low temperatures, where an inappreciable number of electrons have enough kinetic energy to escape through the surface forces of the metal, quantum mechanics shows that under the action of a sufficient high field, electrons will still be able to escape.[16] This can best be understood by referring to Fig. 4, partly following Nordheim.[17] On the right, the solid line represents potential energy plotted against distance from the cathode metal, upon the idealized picture of a potential barrier at the surface of the metal—the "outer" work function of Sommerfeld. The effect of the high field is to re-

[15] Sommerfeld, Zeits. f. Physik **47,** 1 (1928). See also the summary by Darrow, Rev. of Mod. Phys. **1,** 90 (1929).

[16] Fowler and Nordheim, Proc. Roy. Soc. **A119,** 173 (1928). Nordheim, Phys. Zeits. **30,** 177 (1929).

[17] Nordheim, Phys. Zeits. **30,** 180 (1929).

duce the potential beyond the barrier, finally by an amount V_c in the case of the arc. Actually, the surface field produces a gradual rise of potential corresponding to the image field over most of the range, somewhat like the dotted line. On the left of Fig. 4, as abscissa is plotted the distribution function, $N(W)$, for the *normal* components of energy of electrons incident on the metal surface against energy, in volts, as ordinate, following the probability function given by Nordheim[17]

$$N(W) = \frac{4\pi me}{h^3} kT \log (1 + \epsilon^{-e(W-\mu)/kT})dW. \tag{8}$$

For $T=0$, the distribution follow the solid line, no electrons having energy greater than μ, the "inner" work function; at a higher temperature, the dotted line shows the distribution.

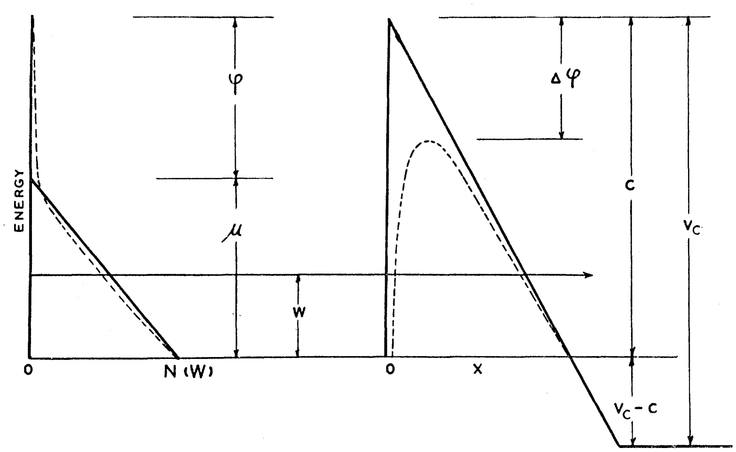

Fig. 4. Energy diagram for high field.

Quantum mechanics show that it is possible for an electron of normal energy W, much less than C, to escape from the metal, if the width of the potential barrier *through* which it must pass is not too great; that is, if the applied field is sufficiently large. Fowler and Nordheim[18] have derived an expression for such an electron emission; at $T=0$, using the idealized form of potential, they find

$$i = \frac{6.2 \times 10^{-6}\mu^{1/2}}{C\phi^{1/2}} F^2\epsilon^{-6.8\times 10^7\phi^{3/2}/F} \tag{9}$$

i is in amps/cm^2; μ, C, and ϕ are in volts; and F is the applied field, in volts per cm. ϕ is the ordinary work function, being equal to $C-\mu$.

Now the diagram in Fig. 4 makes plain that the energy which an electron has upon emerging from the cathode fall is the sum of the normal component of energy, the components of energy in other directions, and the energy gained in the field V_c-C; or

$$E = V_c - C + W + \text{parallel comp. of energy}. \tag{10}$$

For ordinary thermionic emission in weak fields, the normal energy W must be greater than C. So, by Eq. (10), all electrons emitted have energies greater than the cathode fall; the actual distribution has already been given. For zero temperature, it is a characteristic of the Fermi distribution that no electron has a total energy greater than μ. So, in the cold cathode current at $T=0$, the *maximum* value of E is $V_c - C + \mu$ or $V_c - \phi$. For temperatures above zero, but still moderate, such as occur in high field arcs, more and more electrons have energies greater than μ. The distribution of energies can be worked out approximately.

Fowler and Nordheim[16] calculate the probability of an electron with normal energy W passing through the potential barrier under an applied field F to be

$$D(W) = \frac{4[W(C-W)]^{1/2}}{C}\epsilon^{-4K(C-W)^{3/2}/3F} \tag{10}$$

where

$$K = \left(\frac{8\pi^2 m}{h^2}\right)^{1/2}$$

This holds for $W < C$.

The number of electrons in unit volume of the metal with velocity components between u and $u+du$, v and $v+dv$, $w+dw$, is[15]

$$f(u, v, w)dudvdw = \frac{2m^3}{h^3}\frac{dudvdw}{\epsilon^{[m(u^2+v^2+w^2)-2\mu]/2kT}+1}$$

where $\mu = (3n/\pi)^{2/3}h^2/8m$, and n is the number of electrons per unit volume. Take the u velocity direction normal to the surface. Then, the number of electrons incident on unit surface in unit time is

$$\nu_i = f(u, v, w)ududvdw. \tag{11}$$

The number of the above velocity class, which will be emitted is,

$$\nu_e = \nu_i D(\tfrac{1}{2}mu^2). \tag{12}$$

Substitute
$$\begin{aligned} u &= c\cos\theta \\ v &= c\sin\theta\cos\phi \\ w &= c\sin\theta\sin\phi \end{aligned} \tag{13}$$

where c is the magnitude of the electron velocity. Then, Eq. (12) becomes

$$\nu_{e(\theta,\phi,c)} = \frac{2m^3}{h^3}\frac{c^3\sin\theta\cos\theta d\theta d\phi dc}{\epsilon^{(mc^2-2\mu)/2kT}+1} \cdot\frac{4}{C}\left[\tfrac{1}{2}(mc^2\cos^2\theta)(C-\tfrac{1}{2}(mc^2\cos^2\theta))\right]^{1/2}\epsilon^{-4K(C-(mc^2\cos^2\theta)/2)^{3/2}/3F} \tag{14}$$

To obtain the number of incident electrons, which will later be emitted, with velocity between c and $c+dc$, Eq. (14) must be integrated from $\phi=0$ to $\phi=2\pi$; the integration for θ must be carried out in two steps. For $\frac{1}{2}mc^2 < C$, θ must be integrated from 0 to $\pi/2$; for $\frac{1}{2}mc^2 > C$, θ must be integrated from $\cos^{-1}(2C/mc^2)^{1/2}$ to $\pi/2$. Performing the integrations, with close approximations which are valid for strong fields and not too high temperatures, for $\frac{1}{2}mc^2 < C$,

$$\nu_{e(c)} = \frac{4(2)^{1/2}\pi m^{5/2}F}{Kh^3C} \frac{\exp[-4K(C - mc^2/2)^{3/2}/3F]c^2dc}{\exp[mc^2 - 2\mu)/2kT] + 1} \tag{15}$$

For $1/2mc^2 > C$

$$\nu_{e(c)} = \frac{8\pi m^2 F}{Kh^3C^{1/2}} \frac{cdc}{\exp[(mc^2 - 2\mu)/2kT] + 1} . \tag{16}$$

An electron of velocity c incident on the surface will have an energy

$$E = \tfrac{1}{2}mc^2 + V_c - C$$

at the boundary of the cathode fall space. Making this substitution, the number of electrons emerging from the cathode fall with a total energy between E and $E+dE$ is, for $E < V_c$

$$\nu_{e(E)} = n(E)dE = \frac{2(2)^{1/2}m^{1/2}e^{3/2}F}{h^2C} \cdot \frac{\exp\{[8(2)^{1/2}\pi m^{1/2}e^{1/2}(V_c - E)^{3/2}]/3hF\}(E - V_c + C)^{1/2}dE}{\epsilon^{e(E-V_c+\phi)/kT} + 1} \tag{17}$$

and for $E > V_c$

$$n(E)dE = \frac{2(2)^{1/2}m^{1/2}e^{3/2}F}{h^2C^{1/2}} \frac{dE}{\epsilon^{e(E-V_c+\phi)/kT} + 1} \tag{18}$$

where all energies are in volts. The function $n(E)$ is plotted roughly in Fig. 5, for $T=0°$K, and $T=800°$K. Only one value of field is used; the higher the

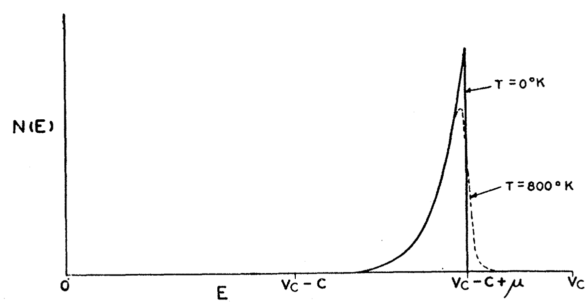

Fig. 5. Energy distribution at boundary of cathode fall for electrons emitted under influence of high field, quantum mechanics theory.

field, the greater $n(E)$, the increase being in an exponential manner. Of course, all electrons have an energy greater than $V_c - C$. At $T=0$, the exponential in the denominator becomes infinitely large for $E > V_c - C + \mu$, that is, $E > V_c - \phi$.

The value of field to be expected in a high field arc can be estimated by means of Eq. (9). Pairs of values of i and F, calculated from that equation are given:

$F = 10^7$ volts/cm	$i = 10^{-16}$ amps/cm^2
$F = 5 \times 10^7$	$i = 3.38 \times 10^4$
$F = 10^8$	$i = 3.25 \times 10^7$

Upon this theory, then, the field is probably of the order of 5×10^7 volts/cm. A field of this magnitude gives a reduction in work function $\Delta\phi = 2.68$; the corresponding Schottky current density will be less than one ampere at low temperatures. The electrons which go over the top of the potential barrier are thus entirely negligible; only the electrons passing through the potential need be considered. Integration of Eq. (18) from V_c to ∞ gives the number of electrons with energies greater than V_c; at $T = 800°\text{K}$, this number also turns out to be entirely negligible. The integration of Eq. (17) from E_1, say, to V_c, gives the effective number of electrons with energies greater than E_1. A rough approximation of this integral shows that electrons in number just beginning to be appreciable have energies greater than $V_c - 3$ volts.

Some objection might be raised to the above calculation, since the transmission coefficient, $D(W)$, developed by Fowler and Nordheim was based upon the idealized potential barrier of Fig. 4. The actual potential curve may change $D(W)$, especially near the peak. Too, the approximations used in determining $D(W)$ break down at $W = C$. Because of the mathematical difficulties, an exact solution for $D(W)$ and determination of the energy distribution appears impossible at present. It does not seem, however, that the conclusions as to the difference in the cathode fall will be altered. A lower limit for the difference can be secured from investigating the distribution in *total* energy of the electrons incident on the inner surface of the metal, which can be calculated precisely. In Eq. (11) for the number of electrons incident on unit area per second, make the substitution of Eq. (13). Then

$$\nu_{i(c,\theta,\phi)} = \frac{2m^3}{h^3} \frac{c^3 \sin\theta \cos\theta \, d\theta d\phi dc}{\epsilon^{(mc^2 - 2\mu)/2kT} + 1} \tag{19}$$

To obtain the number incident with velocity between c and $c + dc$, Eq. (19) must be integrated for ϕ from 0 to 2π, and for θ from 0 to $\pi/2$. Thus

$$\nu_{i(c)} = \frac{2\pi m^3}{h^3} \frac{c^3 dc}{\epsilon^{(mc^2 - 2\mu)/2kT} + 1} \cdot \tag{20}$$

Put $E = \frac{1}{2}mc^2$, and express energies in volts,

$$\nu_{i(E)} = N(E)dE = \frac{4\pi me^2}{h^3} \frac{EdE}{\epsilon^{e(E-\mu)/kT} + 1} \cdot \tag{21}$$

This gives the number of electrons incident per second with energies between E and $E + dE$. The function $N(E)$ is plotted in Fig. 6, for several temperatures.

The number of electrons incident on unit area per second with an energy greater than any amount, say λ, is found by integrating Eq. (21) from λ to ∞. If λ is a little larger than μ, the integration can be done easily, and

$$N(\lambda) = \frac{4\pi mekT}{h^3} \lambda \epsilon^{-e(\lambda-\mu)/kT}. \tag{22}$$

It will be instructive to express the number of electrons as a current. Chang-

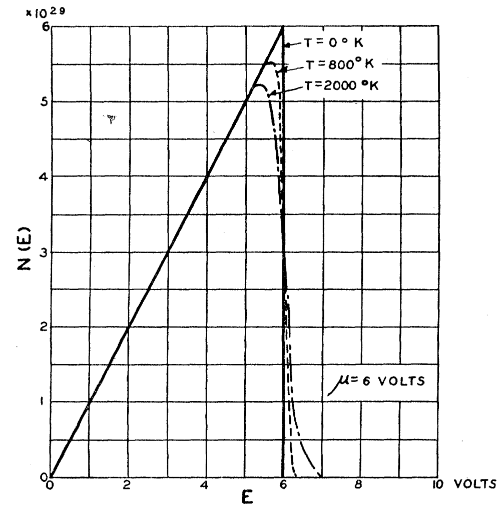

Fig. 6. Distribution in total energy for electrons incident on surface of metal.

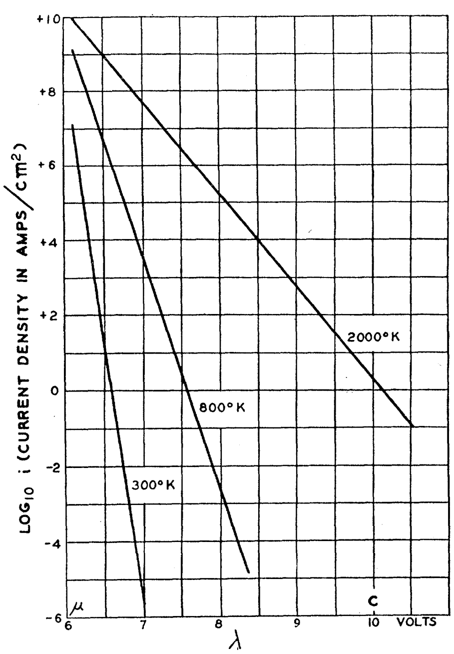

Fig. 7. Current density of electrons incident on surface of metal with total energy greater than λ.

ing to practical units, the current density of electrons incident on the metal surface with an energy exceeding λ is

$$i_\lambda = 1.3 \times 10^6 T\lambda\epsilon^{-11,600(\lambda-6)/T} \text{ amps/cm}^2 \tag{23}$$

Log i_λ vs λ, for different T, is plotted in Fig. 7. In all the above calculations, the values $\mu=6$, $C=10$, $\phi=4$ have been taken. If C and μ are actually higher, the effect to which attention is drawn here will be enhanced.

The emission of electrons depends on the normal component of energy, not on the total energy; so not all of the electrons with total energy λ will be emitted. Those that are emitted will have an energy at the cathode fall boundary

$$E = V_c - C + \lambda.$$

So, only a current *less* than i_λ will have energies exceeding $V_c-(C-\lambda)$. At a few hundred degrees, i_λ is in excess of a few hundred amperes only when $C-\lambda$ is greater than 2.5 to 3 volts. The emitted electron current density with energies large enough for ionization must approach a thousand amperes, in order to provide sufficient positive ions for maintenance of the high field. So, i_λ must be much larger than 1000 amps/cm^2 and hence $C-\lambda$ must be still larger than 3 volts. Since electrons sufficient to provide the necessary positive ions have energies 1.5 to 3 volts greater than the cathode fall in the thermionic arc, while they have energies less than the cathode fall in the high field arc by an amount greater than 2.5 to 3 volts, the cathode fall of the high field arc must be at least 4 to 6 volts greater than that of the thermionic arc.

Most of the above discussion has been based upon the simple theory that an electron emerging from the cathode fall of the arc must have an energy equal to or greater than the ionizing potential in order to produce a positive ion. This assumption is not necessary, however; exactly the same argument could be deduced if electrons of energy less than the ionizing potential could produce ions, as by a cumulative process. *So long as the mechanism of production of positive ions is the same in the two arcs and that mechanism depends upon electron energy, the high field arc must have a cathode fall several volts greater than a thermionic arc.*

Suggested Tests for High Field Theory

In order to determine whether Langmuir's high field theory applies to any particular arc, the cathode fall of the arc should be measured and compared with the cathode fall of a thermionic arc under the same conditions. A way of determining whether or not Langmuir's theory is true in general would be to start an arc, by high voltage breakdown, between refractory electrodes, and observe, probably by means of a cathode-ray oscillograph, if the arc voltage changed as the cathode became heated enough for considerable thermionic emission. An alternative would be to reduce the hearing current of a filament maintaining an arc, and observe if the cathode fall changed.

If in these tests, the cathode fall did not change in the passage from thermionic arc to non-thermionic arc, then it could be concluded that Langmuir's theory did not apply. The converse, however, would not necessarily prove the correctness of the Langmuir theory. Some other mechanism might give an increased cathode fall, as well as the high field mechanism.

The Mercury Arc

None of the above experiments have been carried out yet. The cathode fall of the mercury pool arc, however, has been measured recently by Killian,[18] and by Compton and Lamar,[19] and found to be about 10 volts. From this, what would one conclude about the mechanism of the mercury arc?

The temperature of the cathode spot has been variously estimated from 600°K to 2000°K. From Stark's[20] observation of a continuous spectrum at the cathode, early investigators assumed the cathode temperature to be about 2000°K. As pointed out by Compton,[6] the continuous spectrum may arise from other sources, and the actual temperature may be only 600°K. Seeliger[21] calculated the temperature of the cathode spot, from the rate of vaporization given by Guntherschulze, to be 670°K. The Brown-Boveri Company[22] have measured the temperature of a mercury arc cathode by an optical pyrometer, and found 2360°K. The cathode spot, however, was fixed by means of a tungsten rod, so the temperature may have been different from a mercury pool arc. Too, a green filter was used with the pyrometer, which seems hardly justifiable. Lubcke[23] shows that pyrometric measurements in a discharge are not at all accurate; he concludes the temperature of the cathode is not above 773°K. So, 800°K seems a safe upper limit to take for the cathode temperature.

To produce even a field of 10^6 volts/cm, a positive ion current density of 690 amps/cm^2 is necessary, according to Slepian and Haverstick's calculations.[10] To give a field of 5×10^7 volts/cm, which is necessary for large electron current densities, a positive ion current of 3.5×10^5 amps/cm^2 would be needed. It seems doubtful if surface irregularities can exist to cause a marked local multiplication of the average field. So, the positive ion current must be well above 10^3 amps/cm^2. From Fig. 7, at 800°K, an equal electron current will have energies less than the cathode fall by an amount greater than 3 volts. To produce ions at a single impact, the cathode fall would have to be over 13.4 volts, while the measured fall is only about 10 volts. If the high field does exist, an entirely negligible number of electrons have energies of 10 volts; and certainly less than a thousand amperes have energies even approaching 7 volts. One is forced to one of two conclusions, if the above temperature and cathode fall are correct: (1) the mercury arc is not a high field arc; (2) if the mercury arc is a high field arc, ionization at a single impact does not occur, and a complicated mechanism for the production of positive ions must be devised—such as cumulative ionization, attainment of high random electron velocities through scattering, etc. Since a cathode fall of 5.5 volts has

[18] Killian, Phys. Rev. **31,** 1122 (1928).

[19] Compton and Lamar, Review of paper presented before National Academy of Sciences, Washington, April 1930, printed in Science **71,** 517 (1930). Also Phys. Rev. **37,** 1069 (1931).

[20] Stark, Phys. Zeits. **5,** 51, 750 (1904).

[21] Seeliger, Phys. Zeits. **27,** 37 (1926).

[22] Brown-Boveri Review **16,** 61 (1929).

[23] Lubcke, Zeits. f. tech. Phys. **10,** 598 (1929).

been found for the low voltage mercury arc with thermionic cathode,[24] the last conclusion is not incompatible with the ideas developed in this paper.

In a very recent paper, Compton[25] has also pointed out that electrons emitted as a result of a high field have reduced energies, though he does not consider the distribution in energy in detail; and he has expressed the opinion that in the mercury arc positive ions are produced by a cumulative process.

Perhaps the same remarks concerning the existence of a high field arc may apply to the cadmium arc, and possibly also to the thallium arc, where Nottingham[26] has found the cathode fall to be very close to the ionizing potential of the cathode metal.

Conclusions

An analysis has been given, showing that the kinetic energy of an electron emitted under the action of a high field is several volts less than the energy of an electron emitted thermionically, at the end of the same applied voltage. Consequently, if the same mechanism for the production of positive ions exists in a high field arc as in a thermionic arc, the cathode fall of the high field arc must be several volts greater than that of a thermionic arc. This shows that the Langmuir high field theory of an arc should not be applied arbitrarily without considering in detail the way in which positive ions are produced. Careful study of the cathode falls of arcs, with regard to the electron energies, may afford a better understanding of the phenomena at the arc cathode.

Finally, from the measured cathode fall and cathode temperature of the mercury arc, one must conclude either that the high field theory does not apply to the mercury arc, or that ions are produced by some complicated mechanism such as cumulative ionization.

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[24] Compton, Trans. A.I.E.E. **46,** 868 (1927).
[25] Compton, Phys. Rev. **37,** 1077 (1931).
[26] Nottingham, Journ. Franklin Inst. **206,** 43 (1928).