THE TEMPERATURE OF THE LOWER ATMOSPHERE OF THE EARTH*

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Abstract

From the known amounts of the various gases of the atmosphere from sea level to about 20 km, from the observed light absorption coefficients of the gases and from the albedo of the earth's surface the temperature of the atmosphere in radiative equilibrium is calculated on the assumption that the sunlight is the only source of energy. The calculation is perhaps more rigorous than has hitherto been attempted, although it contains a number of approximations. The sea level temperature comes out to be about 19° above the observed world-wide average value 287°K, and the temperature above about 3 km falls many degrees below the observed temperatures. The temperature gradient in levels from 3 to 6 km is greater than that of convective equilibrium and hence the atmosphere would not be dynamically stable if radiation equilibrium prevailed. Therefore air currents take place to bring about convective equilibrium. Continuing the calculation it is found that only when the convective region extends to about 12 km (as is observed), with radiative equilibrium above 12 km (as is observed), does the atmosphere satisfy the conditions of dynamic stability and thermal equilibrium with the received solar energy. For this case the calculated sea level temperature is 290°K in good agreement with the observed value 287°K. Calculation shows that doubling or tripling the amount of the carbon dioxide of the atmosphere increases the average sea level temperature by about 4° and 7°K, respectively; halving or reducing to zero the carbon dioxide decreases the temperature by similar amounts. Such changes in temperature are about the same as those which occur when the earth passes from an ice age to a warm age, or *vice versa*. Thus the calculation indicates that the carbon dioxide theory of the ice ages, originally proposed by Tyndall, is a possible theory.

Introduction

1. IT IS generally accepted that the most important source of heat energy on the surface of the earth is the light of the sun. The only other known source of possible importance, namely, the leakage of heat to the surface by conduction from the interior of the earth, is relatively small, being less than one ten-thousandth of the energy of sunlight. It should therefore be possible to calculate the surface temperature of the earth and the temperature of the atmosphere to various heights above sea level on the assumption that the earth and the atmosphere radiate away to space an amount of energy equal to that received from the sun. Such a calculation requires a knowledge of the amounts of the gases of the atmosphere at all levels from sea level upward, the absorption coefficients of the gases throughout the optical spectrum, the reflecting power of the surface of the earth, the mixing action of winds, and other quantities. The problem has never been worked out completely. In the present paper an attempt at a fairly rigorous solution is presented, although

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it contains a number of approximations, and fair agreement is found with the observed temperatures of the earth and the atmosphere. In the process of working out the problem one is led to an understanding of the effects of radiation and convection on the values of the temperatures. In important papers on the subject Gold[1] discussed types of atmospheres which could be in convective and radiative equilibrium and Maris[2] derived conclusions about the temperatures, mainly of the high atmosphere, from calculations of the rates of energy absorption and radiation in various levels of the atmosphere.

THE OBSERVED TEMPERATURES OF THE ATMOSPHERE

2. The sea level temperature averaged over the earth is 287° Kelvin, or 14° Centigrade. The average temperatures t°K of the atmosphere are plotted as abscissas against the height z above sea level in the dotted curves 1′ and 1″, Fig. 1, which refer to summer and winter conditions, respectively. The

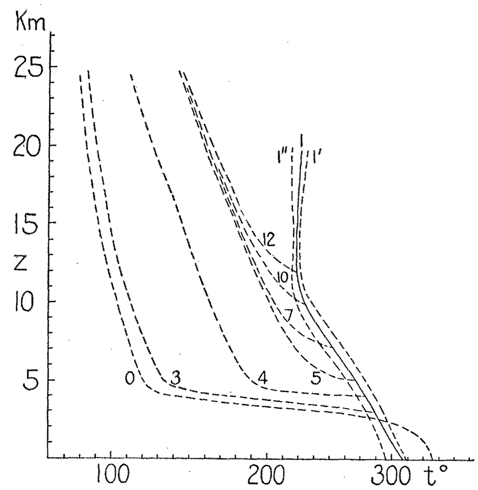

Fig. 1. Curves 1′ and 1″ give the observed temperatures t of the atmosphere at various heights z above sea level, for summer and winter, respectively, curve 1 is the average of 1′ and 1″ and the other curves are theoretical.

data[3] were obtained from 416 souding balloon flights in the years 1900 to 1912 in Europe and hence refer to a north latitude of about 50°. Curve 1, Fig. 1, is the average of the abscissas of curves 1′ and 1″ with the exception of the point at sea level which was plotted at 287°. We assume that curve 1 represents the world-wide average temperatures of the atmosphere, a questionable assumption, but perhaps not far wrong.

It is seen from curve 1, Fig. 1, that t falls off rapidly to about 220°K at $z=10$ or 12 km and is roughly constant from $z=12$ to 20 km. From $z=0$ to about 12 km the winds are relatively swift; this is known as the "convective

[1] Gold, Proc. Roy. Soc. **82,** 43 (1909); later discussed by Emden, Sitz. d. K. Akad. Wiss. zu München, page 55 (1913), and by Milne, Phil. Mag. **44,** 872 (1922).

[2] Maris, Terr. Mag. and Atmos. Elec. **33,** 233 (1928); **34,** 45 (1929).

[3] Humphreys, "Physics of the Air," 2nd Ed., 55 (1929).

region" (or the "troposphere"). From $z=12$ to 20 km, spoken of as the "isothermal region" (or the "stratosphere"), the winds are relatively slow. The temperature of the isothermal region over the equator is usually around 200° K and over polar regions is around 230°K. The height where the isothermal region sets in is about 15 km at the equator and decreases with increasing latitude. At any locality it may vary up and down 5 km or so with weather or other conditions, in polar regions it may descend to the surface of the earth.

In the convective region the atmosphere is in adiabatic equilibrium, but only approximately so. The theoretical adiabatic temperature gradient[4] is about 10 degrees km^{-1}, which gives a temperature of 187° at 10 km for a sea level temperature 287°. The observed value at 10 km is about 220° so that the atmosphere is warmer than it would be if adiabatic equilibrium prevailed. There are a number of factors which together are probably adequate to account for this,[2] such as the heat carried upward by water vapor, the effect of winds blowing from warm to cold regions, the direct absorption of the incoming sunlight by the atmosphere, etc. Hereafter we shall use the portion of curve 1, Fig. 1, from $z=0$ to 12 km as the curve of adiabatic or convective equilibrium. In so doing we take into account after a fashion the transport of heat energy by moisture and the other factors.

The present paper grew out of an attempt to find the answer to three inter-related questions, (1) why is the average sea level temperature about 287°K, (2) why does convection hold sway to about 10 or 12 km and not above, and (3) why is the temperature constant with height from about 12 to 20 km? In their pioneer papers Humphreys[5] and Gold[1] from considerations of radiation equilibrium showed that the average-in-height temperature of the absorbing layers of the atmosphere would not be below 212°K. Gold went a step farther and showed that if the atmosphere consists of two shells, the inner in the adiabatic and the outer in the isothermal state, the inner must extend to a level above 0.5 km but not greatly above 1 km. No detailed answers to the three questions were presented; the answers arrived at in the present paper are given in sections 11 and 12.

The Light Absorption Coefficients of the Gases of the Atmosphere

3. The absorption coefficient α of a gas for light of wave-length λ is defined by

$$I = I_0 \epsilon^{-\alpha x},$$

where I_0 is the original intensity of the light and I is the intensity after passing through x cm of the gas at standard conditions, i.e. 0°C and atmospheric pressure. In the present calculations we are interested primarily in the infrared region of the spectrum and the only gases of the atmosphere which are known to have important absorption bands in this region are water vapor, carbon dioxide and ozone. The values of α for these gases, taken from the tabulations of Maris[2], are plotted in Fig. 2 as ordinates against λ in μ as abscissas. Maris obtained the values of α for water vapor and carbon dioxide by

[4] See, for example, Jeans "The Dynamical Theory of Gases," page 336 (1925).

[5] Humphreys, Astrophys. J. **29**, 14 (1909).

calculation from the tables of Gold[1] who summarized the measurements of Paschen, Rubens and Aschkinass, Angström, Langley and others, made over 30 years ago. In the case of carbon dioxide only the average values of α across the various absorption bands are known, hence the flat tops of the carbon dioxide curve of Fig. 2. The shape of the absorption curve across several of the carbon dioxide bands has been observed, for example the 12.5 to 16μ band is given[6] roughly by the dotted curve a, Fig. 2, but the data are not sufficient to enable one to calculate with any certainty the exact values of α across the bands. Maris calculated the values of α for ozone given in Fig. 2 from rather

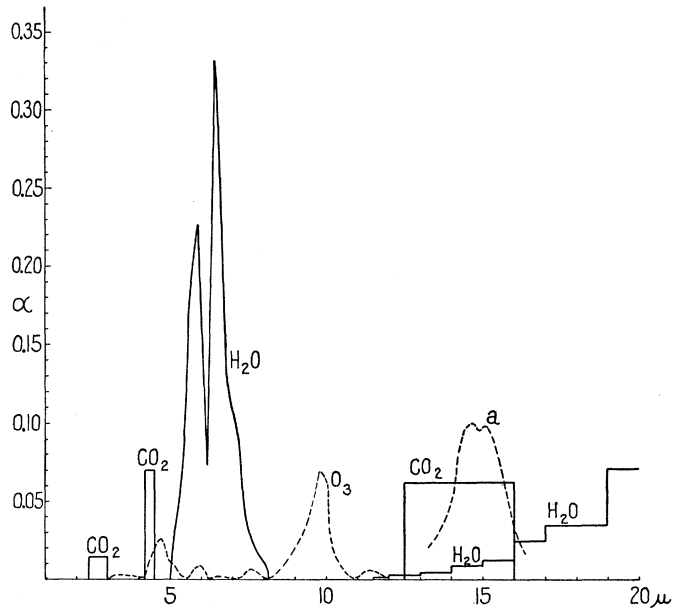

Fig. 2. The light absorption coeffieicnt α of carbon dioxide, water vapor and ozone; for ozone the scale of ordinates must be multiplied by 10.

conflicting measurements of Fabry[7] and of Ladenburg and Lehmann.[8] In general recent investigators have been interested in determining the fine structure, etc., of the bands and not in getting accurate absorption coefficients. It may be hoped that more exact values of α will be obtained with modern infrared spectrographic technique. This would enable the calculations of this paper to be improved, not however very easily. Several approximations, perhaps permissible under the present circumstances, would no longer be so, and to carry out the mathematical analysis without the approximations will be a matter of some labor.

The Distribution of Carbon Dioxide, Water Vapor, and Ozone Up Through the Atmosphere

4. Denote by x cm the equivalent thickness, reduced to standard condi-

[6] Burmeister, Verh. d. Deutsch. Phys. Gesell. **15**, 589 (1913).
[7] Fabry, Proc. Phys. Soc. London **39**, 1 (1926).
[8] Ladenburg and Lehmann **21**, 305 (1906).

tions, of a gas in the atmosphere from sea level to a height z. Let x_b cm be the thickness from sea level to infinity, i.e. outside of the atmosphere; x_b may be spoken of as the thickness to the "top" of the atmosphere. Then $x_b - x$ is the thickness of the gas from any level to infinity. The values of x and $x_b - x$ for carbon dioxide and water vapor are given in Table I. The values for carbon dioxide were calculated from standard tables, e.g. those of Maris[2], of the partial pressures of the gases of the atmosphere; the tables were of course based on direct observations. The values for water vapor were calculated from the pressures of saturated water vapor at the temperatures of the atmosphere given by curve 1, Fig. 1. $x_b = 3300$ cm, which amounts to 2.56 grams of water in a 1 cm vertical column of the atmosphere; this is in agreement with the value 2.6 grams deducible from Arrhenius' values of average humidity.[9] It is assumed that the values of Table I represent world-wide averages. This is satisfactory for carbon dioxide, which is very constant over the earth, but is perhaps questionable for water vapor, which varies tremendously with the weather, etc.

TABLE I.

z	Carbon dioxide x	Carbon dioxide $x_b - x$	Water vapor x	Water vapor $x_b - x$	Average $\alpha(x_b - x)$
0 km	0 cm	252 cm	0 cm	3300 cm	145
3	102	150	2590	410	60
5	142	110	3080	220	5.2
7	164	88	3250	50	3.8
10	192	60	3276	24	2.3
12	207	45	3285	15	1.67
15	225	27	3291	9	1.00
20	239	13	3295	5	0.44
∞	252	0	3300	0	0

For ozone x_b is about 0.3 cm. Ozone exists in the levels above about 50 km[7]. Its distribution with height is not known, but we do not need to know the distribution, for in the present paper we are content to deal with the effects of ozone in a very simple way, as described in section 13.

THE TEMPERATURE OF THE SURFACE OF THE EARTH WARMED BY THE SUN AND ASSUMED TO HAVE NO ABSORBING ATMOSPHERE

5. The solar constant S, i.e. the energy of the sunlight falling on the outside of the atmosphere in the spectrum region from 0.3 to about 3μ, is 1.35×10^6 erg cm^{-2} sec.$^{-1}$. Of this energy about 30 percent is absorbed in the atmosphere[10], mainly in the levels below 10 km, so that only about 70 percent reaches sea level. However, for reasons given in section 10, we assume that the atmosphere is perfectly transparent to the incoming solar radiation and thus treat the 30 percent on the same basis as the 70 percent. The clouds and surface of the earth reflect away to space about 32 percent of the sunlight (reference 3, page 84). Therefore a fraction 68 percent, denoted by A, of the solar energy is absorbed by the earth.

[9] Arrhenius, Phil. Mag. **41**, 264 (1896).

[10] Abbott and Fowle, Annals Astrophys. Obs., Smithsonian Inst., **2**, 173 (1908).

The Stefan-Boltzmann law for the energy E erg cm^{-2} sec.$^{-1}$ radiated by a black body at a temperature t°K is

$$E = \sigma t^4, \tag{1}$$

where $\sigma = 5.71 \times 10^{-5}$ erg deg.$^{-4}$. If the earth had no atmosphere and were in thermal equilibrium with the sun's radiation we have, since the earth receives the sunlight as a disk and radiates as a sphere,

$$4\pi r^2 \sigma t_0^4 = \pi r^2 SA, \tag{2}$$

where r is the radius of the earth, t_0 the temperature of the surface and A is 68 percent. This gives

$$t_0^4 = SA/4\sigma, \tag{3}$$

or

$$t_0 = 252°.$$

t_0 is the black-body temperature. The true temperature t_0' of the earth's surface is given by

$$a t_0'^4 = t_0^4, \tag{4}$$

where a is the fraction of the radiation of a black body at temperature t_0 absorbed by the surface (a is also known as the "emissive power"). If $a = 1$ the surface is a black body and $t_0' = t_0$; if $a < 1$ the surface is not a black body and $t'_0 > t_0$. Although a for the surface of the earth, including the clouds, is about 0.68 for radiation from 0.2 to 3μ, the value of a for radiation of longer wave-lengths is not known. The longer wave-lengths are of importance because t_0 is actually about 287°K and a black body at this temperature radiates mainly in the range from 3 to 30μ (see Fig. 4). We shall assume that $a = 1$, so that $t_0' = t_0$. The assumption is probably not far from the truth because 7/8 of the earth's surface is water and water is absorbent to the longer infrared radiations. The variation of a for the land with changes in climate may be important (see section 14).

The Temperature of the Atmosphere and of the Surface of the Earth in Radiative Equilibrium

6. It is assumed that heat energy is transferred through the atmosphere only by radiative processes. This amounts to assuming that convection is absent, for conduction is on the whole negligibly small compared to radiation or convection. Take the pressures of the atmosphere and the amounts of the component gases at various heights to be those observed, i.e., as given in Table I. Assume, as in section 5, that the atmosphere is transparent to the incoming solar radiation of which 32 percent is reflected out to space by the surface of the earth and the clouds. Thus on the average each cm^2 of the earth's surface absorbs $SA/4$ erg sec.$^{-1}$, where A is 68 percent. The surface is assumed to radiate this energy in the form of black-body radiation (see section 5) upward to the atmosphere which transmits some of the radiation and absorbs some of it. The atmosphere is heated by the radiation which it absorbs and reradiates the energy upward and downward, the downward radia-

tion warming the earth. Thus each layer of the atmosphere is subjected to an upward and a downward stream of radiation. The condition of radiative equilibrium is that the temperature distribution out through the atmosphere be such that each level of the atmosphere radiate as much energy as it absorb and that the energy leaving the top of the atmosphere be equal to $SA/4$.

The first step in expressing the foregoing ideas in mathematical terms is to make an approximation by replacing the complicated absorption bands of Fig. 2 by a single equivalent absorption band of average absorption coefficient α across the band. This simplifies the equations very much. The conclusions which we reach are not appreciably troubled by the approximation. A more exact treatment is hardly justified at the present time in view of the uncertainties in the values of the absorption coefficients and other quantities.

Let the energy emitted from the surface of the earth be E erg cm^{-2} sec.$^{-1}$. Let ϕ be a fraction such that ϕE be the portion of E comprised in those wave-lengths of the spectrum to which the atmosphere is perfectly transparent. Then $(1-\phi)$ E is the energy in the wave-lengths absorbed by the atmosphere. The quanities are shown diagrammatically in Fig. 3a, in which the area under the black body curve is E and the shaded portion is $(1-\phi)E$. The earth receives and absorbs $SA/4$ erg cm^{-2} sec.$^{-1}$ from the sun and a certain amount of energy sent down by the atmosphere, and radiates ϕE directly to space and $(1-\phi)E$ which is absorbed by the atmosphere (see Fig. 3).

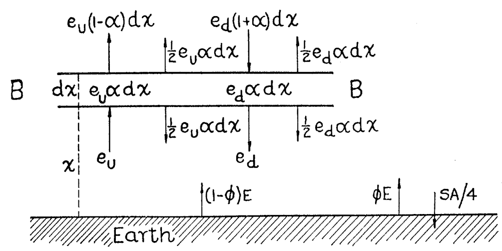

Fig. 3. Diagram showing energy absorbed, transmitted and emitted by an elementary layer BB of the atmosphere.

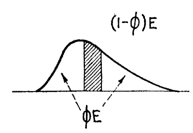

Fig. 3a.

Consider a horizontal layer of the atmosphere at a height x or z cm above sea level, shown by BB, Fig. 3. Let the thickness of the layer be dx. At the layer there is an upward stream of radiation e_u erg cm^{-2} sec.$^{-1}$ and a downward stream e_d, the streams of radiation comprising only the wave-lengths in the single equivalent absorption band of the atmosphere. Of e_u an amount $e_u\alpha dx$ is absorbed by the layer, and an amount $e_u(1-\alpha dx)$ passes through. The absorbed energy is reradiated, $e_u\alpha dx/2$ upward and an equal amount downward (see Fig. 3); similarly for e_d. For radiation equilibrium the temperature t of the layer is expressed by the relation, obtained from the Stefan laws of radiation embodied in (1) and (4),

$$\tfrac{1}{2}(e_u + e_d)\alpha dx = f\sigma t^4\alpha dx, \tag{5}$$

or

$$e_u + e_d = 2f\sigma t^4,$$

where f is the fraction of the energy of a black body at temperature t in the region of the absorbed wave-lengths. In general f, e_u and e_d are functions of x.

Since the atmosphere can radiate only in the spectrum region of the absorption bands, the difference between the upward and downward radiation streams is constant at all heights. Hence

$$e_u - e_d = \text{constant} = SA/4 - \phi E. \tag{6}$$

Denote by de_u the change in e_u in the distance dx measured positively upward. Then de_u is equal to the upward radiation leaving the top side of the layer minus the upward radiation entering the lower side of the layer. Therefore, referring to Fig. 3,

$$de_u = \tfrac{1}{2}e_u\alpha dx + e_u(1 - \alpha dx) + \tfrac{1}{2}e_d\alpha dx - e_u,$$

$$= -\tfrac{1}{2}\alpha(e_u - e_d)dx.$$

Substituting (6)

$$= -\tfrac{1}{2}\alpha(SA/4 - \phi E)dx.$$

Integrating

$$e_u = -\tfrac{1}{2}\alpha(SA/4 - \phi E)x + C, \tag{7}$$

where C is a constant of integration. At the top of the atmosphere $x = x_b$ and $e_u = SA/4 - \phi E$. Hence $C = (1 + \alpha x_b/2)(SA/4 - \phi E)$ and (7) becomes

$$e_u = (SA/4 - \phi E)[1 + \alpha(x_b - x)/2]. \tag{8}$$

Either by a similar derivation, or directly from (6) and (8), we find

$$e_d = (SA/4 - \phi E)[\alpha(x_b - x)/2]. \tag{9}$$

From (5), (8) and (9)

$$2f\sigma t^4 = (SA/4 - \phi E)[1 + \alpha(x_b - x)]. \tag{10}$$

Equation (10) gives t in terms of quantities all of which are known except E. To find E we make use of the fact that at sea level $x = 0$ and $e_u = (1 - \phi) E$. Substituting these values into (8) gives

$$E = \frac{SA}{4} \frac{1 + \alpha x_b/2}{1 + \phi\alpha x_b/2}. \tag{11}$$

From (1) $E = \sigma t_0^4$, where t_0 is the temperature of the surface of the earth, and (11) becomes

$$t_0^4 = \frac{SA}{4\sigma} \frac{1 + \alpha x_b/2}{1 + \phi\alpha x_b/2}. \tag{12}$$

Introducing (11) into (10) yields

$$t^4 = \frac{SA}{8\sigma f} \frac{1 - \phi}{1 + \phi\alpha x_b/2}[1 + \alpha(x_b - x)]. \tag{13}$$

Equations (13) and (12) are the desired relations which give the temperatures of the atmosphere and of the surface of the earth for the condition of radiation equilibrium. In the foregoing derivation we have neglected the curvature of the earth, which introduces no appreciable error, and have regarded the atmosphere as sufficiently homogeneous so that reflection or scattering of the radiation does not occur, which is probably permissible above the cloud region. Further, we have dealt with the streams of radiation as though they flowed only in the vertical direction. Actually, the radiation is everywhere diffuse radiation, but the case of diffuse radiation is very closely given by the vertical radiation derivation for relatively great absorption such as occurs in the infrared bands of the water vapor and the carbon dioxide of the atmosphere. Maris (reference 2, page 245) proved this by a calculation using Gold's evaluation of a certain integral (reference 1, page 62).

7. We must distinguish between the temperatures of the bottom of the atmosphere and the surface of the earth; these of course are not necessarily the same when the energy exchanges occur only by radiation as is assumed in the present case. The temperature t_0 of the surface of the earth is given by (12), which, when the numerical values are substituted for S and A, reduces to

$$t_0 = 252\left(\frac{1 + \alpha x_b/2}{1 + \phi\alpha x_b/2}\right)^{1/4}. \qquad (14)$$

The temperature of the bottom of the atmosphere, denoted by t_0'', is found by putting $x=0$ and $f=1-\phi$ in (13). This gives

$$t_0'' = 252\left(\frac{1}{2}\,\frac{1 + \alpha x_b}{1 + \phi\alpha x_b/2}\right)^{1/4}. \qquad (15)$$

For an atmosphere which does not absorb at all, this being the same as the case of no atmosphere as far as radiation processes are concerned, $\phi=1$ and from (14) $t_0=252°$K. This agrees, as it should, with (3). For a weakly absorbing atmosphere αx_b and $\phi\alpha x_b$ are small compared to unity and (14) and (15) yield approximately

$$t_0 = t_0''(2)^{1/4}, \qquad (16)$$

which is the well-known Schwartzchild[11] relation. For an atmosphere which absorbs strongly in its bands, as is the case of the water vapor and the carbon dioxide of the atmosphere, αx_b and $\phi\alpha x_b$ are large compared to unity, being of the order 10^2, and (14) and (15) give approximately

$$t_0 = t_0'' = 252(1/\phi)^{1/4}. \qquad (17)$$

8. To determine ϕ and f of (12) and (13) the black-body energy curves for a number of temperatures were plotted in Fig. 4. Referring to Fig. 2 we see that there are three important regions of absorption, the first from about 5 to 8μ due to water vapor, the second from about 12 to 16μ due both to carbon dioxide and water vapor, and the third from about 16 to 20μ due to water vapor. In the curves of Fig. 4 these regions are shaded. The ratio of the shaded areas to the total area under the black-body curve, denoted by f_1, f_2 and f_3, respectively, are plotted as ordinates in Fig. 5 against the temperature t as

[11] Schwartzchild, Gött, Nach. page 41 (1906).

abscissa. The three absorption areas together make up the assumed equivalent single absorption band and hence from the definition of f (section 6) we have $f=f_1+f_2+f_3$. f is also plotted in Fig. 5. ϕ is one minus the value of f at sea level.

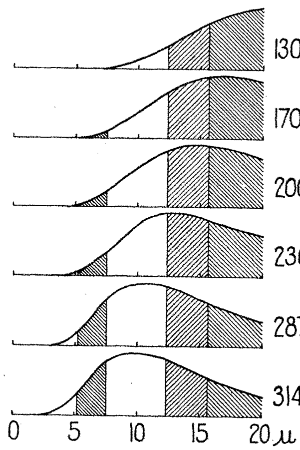

Fig. 4. Spectral energy curves of a black body at various temperatures.

We have left out of account the region of wave-lengths beyond 20μ simply because there are no observations of the absorption coefficients for this range of the spectrum. For example, it is not known whether water vapor is opaque or transparent to these wave-lengths. The method of calculating f

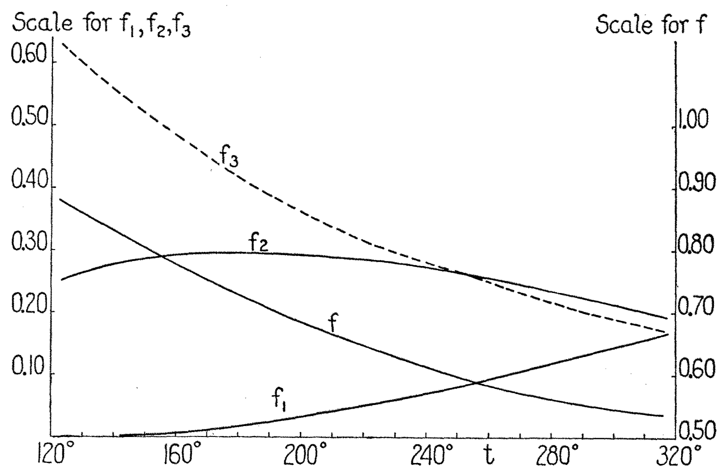

Fig. 5. Values of f_1, f_2, f_3 and f as functions of the temperature.

from the curves up to 20μ amounts to assuming that the atmospheric gases absorb in the region above 20μ to about the same extent as they do below 20μ; it is a sort of compromise between assuming that the atmosphere is either completely transparent or completely opaque to the longer wave-lengths.

The average values of the quantity $\alpha(x_b-x)$ which occurs in (13) were calculated approximately at each height z by combining the values for water vapor and carbon dioxide of Fig. 2. and Table I, weighing them according to the relative amounts of the two gases, the widths of the absorption bands and the average values of α across the bands. The values of $\alpha(x_b-x)$ are given in the last column of Table I. The "top" of the atmosphere is the region above which there is not enough atmosphere to cause appreciable absorption. From the values of α (x_b-x) it is seen that light in the wavelengths absorbed by the single equivalent absorption band is reduced to about 1/5, 1/2.7 and 1/1.5 of its original intensity in getting out to space from 12, 15 and 20 km levels, respectively. Therefore the "top" of the atmosphere is around 25 km.

From the values of $\alpha(x_b-x)$ and from (12) and (13) t and t_0 were calculated in the following way: t_0 was guessed and $1-\phi$ corresponding to t_0 was read off the f curve of Fig. 5. The values of t_0 and ϕ were substituted in (12) (or in (14) or (15)) and if the equation was satisfied t_0 was correct. If not, t_0 was guessed again and the procedure repeated until the correct values of t_0 and ϕ were obtained. These were put into (13) and the same method of successive approximations was used to determine t and f for each height z. The values of t are plotted in curve 0, Fig. 1, which gives the temperatures which would exist in the atmosphere if radiation equilibrium prevailed. The curve does not agree with the observed temperatures of curve 1, Fig. 1, giving a sea level temperature of 306° which is 19° above the observed value, and at levels above 5 km giving temperatures many degrees below those observed.

9. We digress for the moment to consider the effect on t_0 of changes in the carbon dioxide content of the atmosphere. From (14) it is found that doubling, or tripling, or reducing to zero the carbon dioxide of the atmosphere changes t_0 by less than 1° Centigrade. This same conlusion was reached by Angström,[12] Abbott,[13] Humphreys,[3] and others, as the result of qualitative discussions of terrestrial radiation which were perfectly correct as far as they went. However, the assumption of radiation equilibrium has led to atmospheric temperatures at variance with those observed. And we conclude, what was perhaps obvious from the start, that the physics of the atmosphere is not governed by radiation processes alone. So that any calculation of the possible effects on climate of changes in the carbon dioxide of the atmosphere must be left to a later section (section 14) after convection has been considered.

The Temperature of the Atmosphere Taking Into Account Convection and Radiation

10. If the temperature of the atmosphere were that of the radiation equilibrium curve 0, Fig. 1, the atmosphere would be dynamically unstable. The air above 3 km would be so dense, because of its low temperature, that it would descend rapidly; convective equilibrium would be brought about and the t, z curve would approach curve 1, Fig. 1. In general, if the t, z curve at

[12] Angström, Ann. d. Phys. **6**, 173 (1901).

[13] Abbott, reference 10, page 173.

any point is less steep than the convective equilibrium curve at that point, the atmosphere is dynamically unstable and a flow of air occurs in such a direction as to bring about convective equilibrium more or less completely. The t, z curve may of course be steeper than the adiabatic curve in a dynamically stable atmosphere. Convection is of course, also promoted by horizontal winds blowing from night to day areas, from warm to cold regions, etc.

Therefore curve 0, Fig. 1, shows that radiation equilibrium cannot exist in the atmosphere below about 5 km; it might, however, exist at greater heights, for above 5 km curve 0 is steeper than curve 1, Fig. 1. Assume that below a level, denoted by z_1, convective equilibrium exists, as given by curve 1, Fig. 1, and that above z_1 radiation equilibrium prevails. The two states will merge into each other through a region of transition, but for simplicity we regard the transition as abrupt. The assumption means that the energy leaving the earth advances upward through the atmosphere by the process of convection until a level z_1 is reached above which it advances by the process of radiation. We take various values of z_1 and investigate the stability of the atmosphere by calculating t_0 and the t, z curve above z_1 for each value.

Let the equivalent thickness of the absorbing gases, reduced to standard conditions, from z_1 to a higher level z be x_1 and to infinity be x_{b1}. At z_1 the temperature is t_1 as given by curve 1, Fig. 1, and the upward energy stream is e_{u1} where

$$e_{u1} = f\sigma t_1^4. \tag{18}$$

In the region above z_1 we have corresponding to (8) and (9), putting in (1),

$$e_u = (SA/4 - \phi\sigma t_0^4)[1 + \alpha(x_{b1} - x_1)/2], \tag{19}$$

and

$$e_d = (SA/4 - \phi\sigma t_0^4)[\alpha(x_{b1} - x_1)/2]. \tag{20}$$

Substituting $x_1 = 0$ and (18) into (19) and solving for t_0 gives

$$\phi t_0^4 = \frac{SA}{4\sigma} - \frac{f\sigma t_1^4}{1 + \alpha x_{b1}/2}. \tag{21}$$

From (5), (19), (20) and (21) we obtain

$$2f\sigma t^4 = (SA/4 - \phi\sigma t_0^4)[1 + \alpha(x_{b1} - x_1)]. \tag{22}$$

$x_{b1} - x_1$ is the same as $x_b - x$ so that the last column of Table I can be used in calculations from (22). In passing it may be noted that the temperature of the bottom of the part of the atmosphere in radiation equilibrium, obtained by putting $x_1 = 0$ in (22) and using (21), is not exactly equal to t_1 the temperature of the top of the part of the atmosphere in convective equilibrium. This is not an inconsistency, but merely results from the assumption that the convective region merges abruptly into the radiative region. A similar case was discussed in section 7.

For the case $z_1 = 3$ km, t_1 from curve 1, Fig. 1, is 269° and from (21) t_0 comes out to be 305°. With this value for t_0, t was calculated from (22) for z

above 3 km and is plotted in curve 3, Fig. 1. Just as in the case for curve 0, the atmosphere is dynamically unstable for the lower part of curve 3. Further, the value $t_0 = 305°$ is inconsistent with the assumption of $z_1 = 3$ km. For the assumption that t below 3 km be given by curve 1, Fig. 1, requires that t_0 be about 287°. Whereas the radiation equilibrium requirements laid down in (21) call for $t_0 = 305°$. Thus the assumption that $z_1 = 3$ km leads to an atmosphere dynamically unstable and one for which the radiation of energy to space is less than the energy received from the sun.

In a similar manner curves 4, 5, 7, 10 and 12, Fig. 1, were obtained corresponding to the cases $z_1 = 4$, 5, 7, 10 and 12 km, respectively. The respective values of t_0 were 301, 294, 293, 291 and 290°. For $z_1 = 15$ km t_0 was 296°. It is seen that only for z_1 between 7 and 14 km, and better still for z_1 between 9 and 12 km, does the atmosphere satisfy the conditions of dynamic stability and equilibrium with solar radiation. Such an atmosphere agrees closely with the observed atmosphere, for example, the calculated and observed values of t_0 are 290° and 287°, respectively. The agreement is no doubt better than is warranted by the accuracy of the data on which the calculations are based. Apparently the uncertainties and omissions have conspired to counteract each other to some extent.

The reason for treating the 30 percent of the incoming solar radiation, which is mostly absorbed in the levels below 10 km, (see section 5) as though it actually reached sea level is now clear. Since the thermal energy received from the sun by the surface of the earth and the atmospheric levels to about 12 km is spread around and transferred upward by convection, it makes little difference whether the energy is supplied to the earth's surface or to some level in the convective region.

Summary

11. We may summarize the conclusions thus far reached and in so doing give the answers to the first two questions in section 2. If the atmosphere were in radiative equilibrium at all heights the average temperature at sea level would be 306°K, or about 19° hotter, and at levels above 3 km more than 100° colder than it is. Such an atmosphere would be dynamically unstable and vertical convection currents would be set up. These would stir up the atmosphere in the lower levels to give the temperatures of convective equilibrium, thereby cooling the atmosphere at sea level and warming it above. If the convective region extended only to, say, 5 km we find that the atmosphere again is dynamically unstable and in addition that the total radiation emitted from the earth and the atmosphere is less than the energy received from the sun. When the convective region extends to 10 or 12 km (as is observed) the atmosphere is found to be stable, the calculated sea level temperature is about 290° (close to the observed value 287°) and the total radiation emitted from the earth and the atmosphere is equal to the received solar energy. If the convective region extended to a level greater than 12 or 15 km the outgoing radiation is less than the incoming solar radiation. Thus we have proved that the only type of atmosphere, of the types considered, which satisfies the

conditions of dynamic stability and thermal permanence is the type of atmosphere which is observed.

12. Nothing in the foregoing analysis has led to the answer to the third question of section 2, namely, why is the temperature of the atmosphere approximately constant with height from 12 to 20 km? True, it has been shown that, in agreement with the observed lack of winds, the conditions in this region are those of radiative and not of convective equilibrium. But equilibrium with outgoing terrestrial radiation calls for a decreasing temperature with increasing height. The constancy of t with z therefore requires some additional hypothesis. The obvious hypothesis, emphasized by Maris,[2] is that the incoming solar radiation is absorbed in the isothermal region in such a way that, together with the absorbed outgoing terrestrial radiation, t is constant with z. The constancy of t with z instead of being commonplace is in reality strange and unexpected and probably to be explained as the result of the combined action of two or more causes.

The Effect of Ozone

13. A simple calculation shows that the ozone, which exists in levels above about 50 km, has a slight effect on the temperature of the earth; it cools the earth about 1°. Ozone has two general regions of absorption, one in the ultraviolet extending from about $\lambda 2300$ to 2900A and one in the infrared of which the most important portion is the band from about $\lambda 8.5$ to 10.5μ, see Fig. 2. The solar energy curve in the ultraviolet below 2900A is of course not known, but if it be extrapolated into the ultraviolet on the assumption that the sun radiates as a black body at 6000°K the energy from 2300 to 2900A is about 4 percent of the solar constant S. The 3 mm of ozone is sufficiently thick to absorb completely this ultraviolet energy. If the ozone were not present and the ultraviolet energy came through to the earth S would be increased by 4 percent and t_0, instead of being 287°, would be $287 \times (1.04)^{1/4} = 289.8°$.

If the ozone is present in the atmosphere it absorbs the 4 percent ultraviolet energy and reradiates it equally upward and downward. Because the ozone is not very hot, probably not above 500°K, the reradiation is almost entirely in the infrared and mainly in the 8.5 to 9.5μ band. This band happens to be in a region which is little absorbed by water vapor and carbon dioxide, see Fig. 2. Therefore the ozone energy either reaches the surface of the earth or at least enters well into the convective region of the atmosphere, and adds 2 percent to S. The earth radiation at 287° passes out to the ozone where 2 percent is absorbed. The 2 percent is got by calculation from the values of α of Fig. 2 (or see Fig. 4, ref. 2). Of this 1 percent, which amounts to 0.3 percent of S, is reradiated downward to the earth. Thus the radiation received by the earth is equal to S increased by 2.3 percent, and t_0 becomes $287 \times (1.023)^{1/4} = 288.6°$, which is 1.2° less than 289.8°. Therefore the earth is about 1° cooler, *ceteris paribus*, than it would be if the ozone were removed from the atmosphere.

The Carbon Dioxide Theory of the Ice Ages

14. The sea level temperature t_0 was calculated in section 10 to be 290° by means of (21) with $z_1 = 12$ km, $t_1 = 219°$ and $\alpha x_{b1} = 1.67$ from Table I. To determine the changes in t_0 with changes in the amount of carbon dioxide in the atmosphere we may, to a first approximation, assume that αx_{b1} varies, the other quantities remaining constant. The value $\alpha x_{b1} = 1.67$ was the arithmetical average of $\alpha x_{b1} = 2.83$ for the 12.5 to 16μ band of carbon dioxide and $\alpha x_{b1} = 0.52$ for the 14 to 20μ band of water vapor, the 5 to 8μ band of water vapor being of no importance for 219° radiation (see the 200° curve of Fig. 4). If the carbon dioxide of the atmosphere were doubled $\alpha x_{b1} = (2 \times 2.83 + 0.52)/2 = 2.59$; if the carbon dixide were halved, $\alpha x_{b1} = (2.83/2 + 0.52)/2 = 0.92$. With these values in (21) t_0 comes out 294° and 286°, respectively. Thus doubling or halving the carbon dioxide in the atmosphere changes t_0 by 4°. Similarly, tripling or reducing to zero the carbon dioxide increases or decreases t_0 by 7°. Such changes as these in the world-wide average surface temperature of the earth are of about the same amount as occurred during the ice ages.[14] The calculation is thus in support of the carbon dioxide theory of the ice ages advanced long ago by Tyndall.[15] It is perhaps well to mention the essence of the physics underlying the calculation. In the convective region of the atmosphere the outgoing thermal energy is passed upward by convection to the radiative region above 12 km. So that the radiative region controls to a considerable extent the temperatures in the lower lying convective region. And the radiative region owes its optical properties more to the carbon dioxide than to the water vapor there.

Further, an increase or decrease in the world-wide average atmospheric temperatures of a few degrees would give rise to other changes. The water vapor in the atmosphere would be increased or decreased, this would accentuate the temperature changes. At the same time changes in the areas covered by vegetation and snow fields would take place, thus changing the optical properties, that is, the emissive power a, section 5, of the surface of the earth. Whether a would increase or decrease can not be said at the present time, for the reflecting powers, etc., of snow, sand, vegetation, etc. in the infrared to 40μ are not known. In general so many other changes may be thought of, such as a change in the carbon dioxide content of the sea, etc., that speculation as to the exact consequences of an increase or decrease in the average world-wide temperatures of a few degrees seems fruitless.

About all we can conclude is that the carbon dioxide theory of the ice ages is at least a possible one, and that objections which have been raised against it by some physicists[3,12,13] are not valid. We can not conclude that objections may not crop up in the future. Geologists[14] apparently have come upon no evidence, and have no unanimous opinion to offer, as to the fundamental causes of the ice ages. And we let it go at that.

[14] See any treatise on geology, for example, Coleman, "Ice Ages, Recent and Ancient," MacMillan (1926), or the excellent article on glacial periods by Howe, Encyclopaedia Brittanica, 13th ed., 12, 56 (1926).

[15] Tyndall, Phil. Mag. **22**, 277 (1861).