# THE AVERAGE LIFE OF THE IONIZED HELIUM ATOM

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#### Abstract

Review of the theory in relationship to ionized helium:—From the results of previous workers it is shown that the theoretical values for the average life of energy levels of ionized helium are 1/16th as long as the life-time of the same levels of hydrogen. A table of average lives for ionized helium is given which shows that the average life becomes progressively greater for higher quantum states.

**Experimental method for measuring the average life**:—Electrons in helium at low pressure were confined into a narrow beam by a longitudinal magnetic field. A transverse electric field drew out the ions formed in the beam. The light was projected on the slit of a spectrograph with the direction of motion of the ions parallel to the slit. Formulas giving the intensity of the displaced spark line as a function of its distance from the center of the electron beam have been derived previously. The average life can be obtained from the measurements of the amount of the displacements of the region of maximum intensity of the spark line from the center of the beam.

Experimental results and comparison with the theory:—(1) A comparison of displacements for the first four lines of the "4686" series of He<sup>+</sup> show that the average life becomes progressively greater for the higher quantum states in agreement with the theory. (2) Results obtained for the line 2733A  $6\rightarrow3$  give an average life of  $1.1\pm0.2 \times 10^{-8}$  sec. for the sixth quantum state in good agreement with  $1.17\times 10^{-8}$  sec. the theoretical value for the mean average life of this level. (3) As the voltage of the electrons in the beam was increased the measured average life became greater which showed the presence of transitions into the sixth state from the higher quantum levels. (4) An alternative method for interpretating the experimental results is given which does not assume a free interchange among the sublevels. The theoretical intensity distribution of each of the five fine structure lines which make up the unresolved line 2733A is calculated. It is found that the resultant intensity distribution is almost exactly the same as the intensity distribution calculated for the case of free interchange or in other words for the case of a single exponential decay law. The experimental intensity curve agrees with the theoretical curve.

Discrepancies existing among authors for values of the matrix amplitude squared for hydrogen:—On comparing the values for the matrix amplitudes squared for hydrogen obtained by different authors, it is found that the values obtained by Slack and Kupper are in disagreement for the following transitions:  $5_1 \rightarrow 4_2$ ,  $6_1 \rightarrow 4_2$ ,  $5_0 \rightarrow 4_1$ ,  $6_0 \rightarrow 4_1$ ,  $6_6 \rightarrow 5_4$ ,  $6_0 \rightarrow 5_1$ ,  $6_3 \rightarrow 5_4$ ,  $6_2 \rightarrow 4_3$ . These discrepancies do not greatly affect the theoretical value for the mean average life of the sixth quantum state, or the alternative method for interpretating the experimental results.

Effect of electron spin:—It is proved that the electron spin corrections do not alter the theoretical interpretation of the experimental results, because it is shown that the average life of the n, l, j, m state is independent of j, and m, and equal to the average life in the nonspin theory of the n, l, m state, independent of m, and also that the mean average life is independent of spin.

O NE of the important quantities which can be calculated from the quantum theory is the average rate of radiation of energy produced by transitions occurring in atoms. Theoretical considerations give directly the probability of transition and consequently the average lifetime in the initial states. It becomes possible, therefore, to test the theory by comparison with experiments which give numerical results for the average lives of excited states. The present work deals with the measurement of the average life for ionized helium. It is possible to obtain a direct comparison between the absolute values of the average life as obtained from the experiment and from the theory.

## PART I. THE THEORY

Since the experimental method used for determining the average life is limited to those emitters which are ions, it is advisable to deal with ionized helium which is the simplest type of ion and one which can be treated theoretically in the same manner as hydrogen.

The average life of an excited state is equal numerically to the reciprocal of the sum of the transition probabilities corresponding to all of the possible transitions which can take place from the level concerned. Expressed analytically, the average life T(n, l, m) of the state n, l, m is given by

$$T(n, l, m) = \frac{1}{\sum_{n', l, n'} A_{n, l, m}^{n', l', m'}}$$

where  $A_{n,l,m}^{n',l',m'}$  is the probability of a transition from state n, l, m to state n', l', m'. This summation is extended over all of the lower states. We see from this equation that in order to obtain a numerical value for the average life a knowledge of the numerical values of all the transition probabilities is essential. Schrödinger<sup>1</sup> quotes a formula for the total intensities of the lines in the Lyman and Balmer series given to him by Pauli, and capable of giving the transition probabilities. Sugiura<sup>2</sup> has calculated the transition probabilities for a number of the lines of the Paschen, Balmer and Lyman series lines of hydrogen. In a later paper<sup>3</sup> he also considered the more general case of the transition probability between two states in a field with a central charge Ze. Slack,<sup>4</sup> using a formula obtained by Sommerfeld and Unsöld, has prepared a table of transition probabilities for a large number of the matrix amplitudes squared for many transitions in hydrogen.

It is well known that from the classical theory the average energy radiated per unit time is given by

$$-\frac{\overline{dE}}{dt} = \frac{64\pi^4\nu^4}{3c^3} |P_1|^2$$

<sup>1</sup> Schrödinger, Ann. d. Physik 80, 437 (1926) for the formula of W. Pauli.

<sup>2</sup> Sugiura, Jour. d. Physique 8, 113 (1927).

<sup>3</sup> Sugiura, Scientific Papers of the Institute of Physical and Chemical Research (Tokyo) No. 193, June 25, 1929.

<sup>4</sup> Slack, Phys. Rev. **31**, 527 (1928).

<sup>5</sup> Kupper, Ann. d. Physik 86, 511 (1928).

where the electrical moment P of the classical vibration is written in the exponential form  $P = P_1 e^{2\pi i \nu t} + P_{-1} e^{-2\pi i \nu t}$ . Utilizing this expression directly in the quantum theory, we have the following for the value of the transition probability,<sup>6</sup>

$$A_{n,l,m}^{n',l',m'} = \frac{64\pi^4 e^{2\nu_3^{n',l',m'}} |J_{n,l,m}^{n',l',m'}|^2}{3c^3h}$$
(1)

where  $J_{n,l,m}^{n',l',m'}$  is the vector matrix amplitude given by

$$|J_{n,l,m}^{n',l',m'}|^{2} = |\mathbf{x}_{n,l,m}^{n',l',m'}|^{2} + |\mathbf{y}_{n,l,m}^{n',l',m'}|^{2} + |\mathbf{z}_{n,l,m}^{n',l',m'}|^{2}$$

with the matrix elements  $(\mathbf{x}, \mathbf{y}, \mathbf{z})_{n,l,m}^{n',l',m'}$  expressed in terms of the normalized wave functions  $\psi_{n,l,m}\psi_{n',l',m'}$  in the following form:

$$\begin{array}{c} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{array} \right)_{n,l,m}^{n',l',m'} = \int_{z}^{x} y \psi_{n,l,m} \ \psi^{*}_{n',l',m'} d\tau \, .$$

Since we are not interested in each matrix element component for different magnetic quantum numbers m the expression  $\sum_{m} \sum_{m'} |J_{n,l,m'}^{n',l',m'}|^2$  is evaluated. From (1) we obtain

$$\sum_{m} \sum_{m'} A_{n,l,m}^{n',l',m'} = \frac{64\pi^4 \nu_{n,l}^{n',l'} e^2 \sum_{m} \sum_{m'} |J_{n,l,m}^{n',l',m'}|^2}{3c^3h}$$

Sugiura defines the quantity  $A_{n,l}^{n',l'}$ , as the mean probability of a transition by the following formula;

$$A_{n,l}^{n',l'} = \frac{\sum_{m} \sum_{m'} A_{n,l,m}^{n',l',m'}}{g_l}$$
(2)

where  $g_l = 2l+1$  is the number of magnetic levels or the statistical weight of the *l*th level. This equation can be thought of as defining the transition probability which determines the rate at which energy is radiated from the magnetic levels of the initial states. It is the reciprocal of this expression which gives the average life, and also the quantity which when multiplied by  $g_l$  will give the total rate at which energy is radiated from all the magnetic levels and consequently give the intensity of the spectrum line. From the summation-rule for the Zeeman components we see that  $A_{n,l}^{n',l'}$  is also equal to  $\sum m' A_{n,l,m'}^{n',l',m'}$  independent of m.

From Eqs. (1) and (2) we have the final expression for the transition probability  $A_{n,l}^{n',l'}$  as defined above,

$$4_{n,l}^{n',l'} = \frac{64\pi^{4}\nu^{3_{n,l}^{n',l'}} \sum_{m} \sum_{m'} |J_{n,l,m}^{n',l',m'}|^{2}}{3c^{3}hg_{1}}.$$
(3)

<sup>&</sup>lt;sup>6</sup> See for instance, Sugiura, reference 2.

Sugiura has evaluated this quantity by obtaining the following expression for the matrix amplitude squared for the case of a central charge Ze and for a transition from state n, l to n', l'

$$\sum_{m} \sum_{m'} |J_{n,l,m}^{n',l',m'}|^{2} = \left(\frac{a_{0}}{4Z}\right)^{2} l'' \frac{(n-l-1)!(n'-l'-1)![(l+l'+3)!]^{2}}{(n+l)!(n'+l')!} + \left(\frac{2n}{n+n'}\right)^{2(l'+2)} \left(\frac{2n'}{n+n'}\right)^{2(l+2)} (C_{n,l}^{n',l'})^{2}$$
(4)

where  $C_{n,l}^{n',l'}$  is the coefficient of  $y_1^{n-l-1}y_2^{n'-l'-1}$  in the expansion of

$$\frac{(1-y_1)^{l'-l+2}(1-y_2)^{l-l'+2}}{\left(1-y_1y_2-\frac{n-n'}{n+n'}y_1+\frac{n-n'}{n+n'}y_2\right)^{l+l'+4}}$$

and l'' is the greater value of l and l',  $a_0 = h^2/4\pi^2 me^2$ . Therefore we see from this equation that the matrix amplitude squared is inversely proportional to  $Z^2$  for any given transition. Taking into account that  $\nu^3$  is directly proportional to  $Z^6$  we see from Eq. (3) that  $A_{n,l}^{n',l'}$  is directly proportional to  $Z^4$ .<sup>7</sup> Thus, in the case of ionized helium the values of the transition probabilities are 16 times greater than the values for the same transitions in hydrogen. This means that the average life of the levels in ionized helium will be 1/16th as long as the average life-time of the same levels in hydrogen. Consequently, for the purpose of calculating the average life for ionized helium we can use directly numerical values for the transition probabilities for hydrogen.

# The mean average life.

The substates corresponding to different l values for a given value of n have for the relativistic case, energies which differ from each other only by a very small amount, so that it is impractical to separate the different fine structure lines in the measurements of the average life.

Sugiura has calculated a mean average life  $T(n) = 1/\sum_{n'} A_n^{n'}$  which can be used for comparison with experiments provided that there exists free interchange or statistical equilibrium among the various sublevels of the *n*th state.  $A_n^{n'}$  is the average rate of transition from the state *n* to *n'* and is given by

$$A_{n}^{n'} = \frac{\sum_{l,l'} g_{l} A_{n,l}^{n',l'}}{\sum_{l} g_{l}}$$

<sup>7</sup> See also Sommerfeld, Atombau and Spektrallinen (Wellenmechanischer Ergänzungsfand) p. 96.

and is called by Slack the total probability of line emission from n to n'. The intensity of the unresolved line will diminish exponentially\* with the time, and the decay time T(n) will be given by

$$T(n) = \frac{\sum_{l} g_{l}}{\sum_{l} \sum_{n',l'} g_{l} A_{n,l}^{n',l'}}$$
 (5)

\* Although it is perhaps obvious that the unresolved line will diminish exponentially with the time on account of averaging the rates of transitions, however it can be shown analytically to be true in the following manner:

Let  $n_{n,l}(t)$  be the number of electrons in the n, l, state. We shall assume that

$$n_{n,l}(t) = f_n(t)g_l \tag{A}$$

where as before  $g_l = 2l+1$  (the number of magnetic levels) and  $f_n(t)$  a factor of proportionality which is independent of l. Therefore,  $N_n(t)$  the total number of electrons in the nth state is given by

$$N_n(t) = \sum_{l} n_{n,l}(t) = f_n(t) \sum_{l} g_l.$$
 (B)

. . . .

The rate at which electrons leave the n, lth state to go to lower states n', l' is given by

$$\frac{dn_{n,l}(t)}{dt} = -\sum_{n',t'} A_{n,l}^{n',t'} n_{n,l}(t) .$$
  
$$\therefore \frac{dN_n(t)}{dt} = \sum_l \frac{dn_{n,l}(t)}{dt} = -\sum_l \sum_{n',t'} A_{n,l}^{n',t'} n_{n,l}(t)$$

By virtue of (A) we see that

$$\frac{dN_n(t)}{dt} = -\frac{\left(f_n(t)\sum_l g_l\right)\sum_l \sum_{n',l'} A_{n,l}^{n',l'} g_l}{\sum_l g_l}.$$

Utilizing Eq. (B), leaves

$$\frac{dN_n(t)}{dt} = -\frac{N_n(t)\sum_l\sum_{n',l'}A_{n,l}^{n',l'}g_l}{\sum_l g_l}.$$

Therefore, we have on integrating, the following

$$N_n(t) = N_n(0)e \exp -\left(\frac{\sum_l \sum_{n',l'} A_{n,l}^{n',l'} g_l}{\sum_l g_l}\right)t.$$

From Eqs. (A), (B), and (C) we find that

$$n_{nl}(t) = \frac{N_n(0)g_l}{\sum_l g_l} e \exp \left(\frac{\sum_l \sum_{n',l'} A_{n,l}^{n,l'l'}}{\sum_l g_l}\right) t.$$

Therefore, the intensity of the spectrum line corresponding to a transition from state n to n'is given by - n'.l'

$$I_{n}^{n}(t) = \sum_{l} \sum_{l'} I_{n,l}^{n',l'}(t) = \left[ \sum_{l} \sum_{l'} \frac{g_{l}A_{n,l}^{n',l} h_{n',l}^{n',l'} N_{n}(0)}{\sum g_{l}} \right] e \exp \left[ - \left[ \frac{\sum_{l} \sum_{n,l'} A_{n,l}^{n',l} g_{l}}{\sum_{l} g_{l}} \right] t \right]$$

We see, from this equation, that the intensity of the unresolved line will diminish exponentially with the time, having a decay time the same as that given above for T(n).

# Effect of electron spin.

A similar expression for the mean average life T'(n) can be written at once taking into account electron spin.

$$T'(n) = \frac{\sum_{l,j} g_{l,j}}{\sum_{l,j} \sum_{n',l',j'} g_{l,j} A_{n,l,j}^{n',l',j'}}$$
(6)

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To determine the effect of electron spin on the values of the average lives, let us consider the general proof that the spin corrections will not effect the intensity.<sup>8</sup> From the general theorem of spectroscopic stability<sup>9</sup>

$$\sum_{j,m,j'm'} \left| J_{n,l,j,m}^{n',l',j'm'} \right|^2 = \sum_{m_s,m_l,m_{s'},m_{l'}} \left| J_{n,l,m_s,m_{l}}^{n',l',ms',m_{l'}} \right|^2.$$

The left hand side has the usual spin coupling while the amplitudes on the right hand side result when the spin is completely decoupled and has separate spacial quantization. The matrix elements on the right side are completely independent of  $m_s$  i.e.

$$X_{n,l,m_s,m_l}^{n',l',m_s'm_{l'}} = \delta_{m_s}^{m_s'} X_{n,l,m_l}^{n',l',m_{l'}},$$

similarly for the y and z components, where

$$\left| X_{n,l,j,m}^{n'l'j'm'} \right|^{2} + \left| Y_{n,l,j,m}^{n',l',j'm'} \right|^{2} + \left| Z_{n,l,j,m}^{n'l'j'm'} \right|^{2} = \left| J_{n,l,j,m}^{n',l',j'm'} \right|^{2}$$

The sum-rule assures that  $\sum_{j'm'} |J_{n,l,j,m}^{n',l',j',m'}|^2$  is independent of j, m, and  $\sum_{m' i} J_{n,l,m_1}^{n',l',j',m'}$  of  $m_l$ . Since there are (2L+1) (2S+1) values of j, m, or  $m_l, m_s$  we have

$$(2S+1)(2L+1)\sum_{j'm'}|J_{n,l,j,m}^{n',l',j',m'}|^{2} = (2S+1)(2L+1)\sum_{ml'}|J_{n,l,ml}^{n',l'ml'}|^{2}.$$

On cancelling out the factor (2L+1) (2S+1) we have the desired result. In addition, if we sum over n' and l' then it is proved at once that the average life of the n, l, j, m state is independent of j, and m, and equal to the average life in the nonspin theory of the n, l, m state independent of m.\*\*

By virtue of the above theorem we can write at once that

$$A_{n,l}^{n',l'} = \frac{\sum_{j,j'} g_{l,j} A_{n,l,j}^{n',l',j'}}{\sum_{j} g_{l,j}}$$
$$g_{l} = \frac{1}{2} \sum_{j} g_{l,j}.$$

also

<sup>8</sup> The writer is indebted to Professor J. H. Van Vleck for this proof.

<sup>9</sup> Cf. for instance Van Vleck, Phys. Rev. 29, 740 (1927).

\*\* In this connection Sugiura has previously mentioned that the decay constants or average lives for the n, l terms are necessarily independent of j as long as the sum-rule holds. By use of these equations it is seen at once from (5) and (6) that T(n) = T'(n). Therefore, the mean average life is independent of the spin corrections.

In Table I are given theoretical values for the average lives and the mean average lives for levels up to and including the seventh state of ionized

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Level	Average life	Mean average life
nı	T(n, l), (sec.)	T(n), (sec.)
1.0		
20 21	1.02 ×10 <sup>-10</sup>	1.36 ×10 <sup>-10</sup>
$3_0$ $3_1$ $3_2$	$\begin{array}{cccc} 1.01 & \times 10^{-8} \\ 3.38 & \times 10^{-10} \\ 9.93 & \times 10^{-10} \end{array}$	6.4 ×10 <sup>-10</sup>
$\begin{array}{c} 4_0\\ 4_1\\ 4_2\\ 4_3\end{array}$	$\begin{array}{c} 1.456 \times 10^{-8} \\ 7.89 \times 10^{-10} \\ 2.320 \times 10^{-9} \\ 4.66 \times 10^{-9} \end{array}$	2.127×10 <sup>-9</sup>
$5_0$ $5_1$ $5_2$ $5_3$ $5_4$	$ \begin{array}{r} 1.82 \times 10^{-8} \\ 1.49 \times 10^{-9} \\ 4.33 \times 10^{-9} \\ 8.73 \times 10^{-9} \\ 1.45 \times 10^{-8} \end{array} $	5.38 ×10 <sup>-9</sup>
$ \begin{array}{c} 6_{0}\\ 6_{1}\\ 6_{2}\\ 6_{3}\\ 6_{4}\\ 6_{5} \end{array} $	$\begin{array}{c} 2.576 \times 10^{-8} \\ 2.443 \times 10^{-9} \\ 7.196 \times 10^{-9} \\ 1.471 \times 10^{-8} \\ 2.682 \times 10^{-8} \\ 3.906 \times 10^{-8} \end{array}$	1.178×10 <sup>-8</sup>
$ \begin{array}{c} 7_{0} \\ 7_{1} \\ 7_{2} \\ 7_{3} \\ 7_{4} \\ 7_{5} \\ 7_{6} \\ \end{array} $	$\begin{array}{c} 3.286 \times 10^{-8} \\ 3.647 \times 10^{-9} \\ 1.175 \times 10^{-8} \\ 2.141 \times 10^{-8} \\ 3.373 \times 10^{-8} \\ 6.449 \times 10^{-8} \\ 8.802 \times 10^{-8} \end{array}$	2.237×10 <sup>-8</sup>

TABLE I. Theoretical values of the average life of ionized helium.

helium. These values for levels up to and including the fourth state were taken from a similar table prepared by Sugiura for hydrogen. The present values for ionized helium differ by a factor of 16 from those for hydrogen for the reason given above. Since Sugiura's table does not include levels higher than the fourth, the values of the average lives for the fifth, sixth and seventh states were obtained from Slack's table of transitior, probabilities for hydrogen. These transition probabilities as obtained by Slack were multiplied by 16 to give the average life for ionized helium as listed in the table.

# Discrepancies existing among the values of the matrix amplitude squared for hydrogen obtained by different authors.

A comparison has been made between the values of the matrix amplitude squared as obtained by Sugiura,<sup>2</sup> Slack<sup>4</sup> and Kupper.<sup>5</sup> For all the transitions

from the second, third and fourth quantum states, the values obtained by the different authors are in good agreement.

For the case of the transitions from the fifth state, Slack's and Kupper's values are in agreement with the exception of the transitions  $5_1 \rightarrow 4_2$  and  $5_0 \rightarrow 4_1$ . For the transitions from the sixth quantum state, Slack's and Kupper's results disagree for the following transitions:  $6_1 \rightarrow 4_2$ ,  $6_0 \rightarrow 4_1$ ,  $6_5 \rightarrow 5_4$ ,  $6_0 \rightarrow 5_1$ ,  $6_3 \rightarrow 5_4$ , and  $6_2 \rightarrow 4_3$ . Table II has been prepared showing the values of the matrix amplitudes squared for the above transitions.

In the third column are given values calculated from Kupper's "Spezielle Serien" formulae. For example, Kupper gives for the  $n_0 \rightarrow 4_1$ . (4P-nS) transition the following formula:

$$\frac{2^{21}n^9(n-4)^{2n-10}(57n^4-38n^2+80)^2}{3(n+4)^{2n+10}}\cdot$$

If we use the more general formula given by Kupper, called the "Allgemeine Formeln der Seriengruppen n-l-1=D' (n'>n)" we obtain for the  $n_0 \rightarrow 4_1$  transition a different quantity:

$$\frac{2^{21}n^9(n-4)^{2n-10}(57n^4-608n^2+1280)^2}{5\times 3(n+4)^{2n+10}}$$

which is evaluated numerically in the fourth column in Table II. For the  $6_0 \rightarrow 4_1$  transition this formula gives 1.59 which is the same value that is obtained from Sugiura's formula. Also it agrees well with Slack's values of 1.88 but is in disagreement with 14.8 the value obtained by Kupper's special series formula. For the  $5_0 \rightarrow 4_1$  transition the agreement is not as good, as illustrated in Table II.

Transition	Kupper's table	Calculated from Kupper's special series formula	Calculated from Kupper's general formula	Calculated from Sugiura's formula	Calculated from Slack's table
$5_1 \rightarrow 4_2$ $6_1 \rightarrow 4_2$ $5_0 \rightarrow 4_1$ $6_0 \rightarrow 4_1$ $6_5 \rightarrow 5_4$ $6_0 \rightarrow 5_1$ $6_3 \rightarrow 5_4$ $6_2 \rightarrow 4_3$	$9.1 \\ 1.21 \\ 105. \\ 14.8 \\ 201. \\ 247. \\ 8.2 \\ 0.48$	1.26 135. 14.8 1851.	10.58 1.59	1.59 1851.	$\begin{array}{r} 0.094\\ 0.013\\ 21.0\\ 1.88\\ 1822\\ 77.60\\ 12.3\\ 0.284 \end{array}$

TABLE II. Comparison of values for the matrix amplitude squared for hydrogen.<sup>10</sup>

These discrepancies do not appreciably effect the theoretical value for the mean average life of the sixth quantum state.

# PART II. THE EXPERIMENT

The experimental method used in the present work for measuring the average life is similar to that previously described by the writer<sup>11</sup> for the case

<sup>10</sup> The values here are  $\frac{1}{2}$  the values given by Sugiura's formula of Eq. (4).

<sup>&</sup>lt;sup>11</sup> Maxwell, Phys. Rev. 32, 721 (1928).

of an election beam of nonuniform current density. It is necessary to modify slightly the derivation used previously by taking into account the more general case where there are a large number of levels involved instead of the simple case of two levels as treated before.

On referring to the previous work we see that if  $\delta n$  is the number of excited ions per second leaving the volume element dv lying between  $\xi$  and  $\xi + d\xi$  then  $\delta n e^{-\beta t(x, \xi)}$  is the number of these which will arrive at a distance x from the lower edge of the beam without becoming deexcited, where  $t(x, \xi)$  $= [2m(x-\xi)/Xe]^{1/2}$  is the time required for the ion to go from  $\xi$  to x.  $\beta$  is the reciprocal of the average life of the level  $n_i$  given by

$$1/\beta = T(n, l) = 1/\sum_{n', l', l'} A_{n, l}^{n'l'}$$

Now the number of transitions which take place from state n, l to n', l' in volume element dA lying between x and x+dx in time dt while the ions move the distance dx is

$$- d_{n,l}^{n',l'}(\delta n e^{-\beta(n,l)t(x,\xi)}) = A_{n,l}^{n',l'} \delta n e^{-\beta(n,l)t(x,\xi)} \frac{\partial t}{\partial x} dx.$$

This corresponds to a similar expression for the number of transitions occurring per second in dA obtained previously in which  $\beta$  has now been replaced by  $A_{n,l}^{n',l'}$ . The subsequent integration is unchanged so that the intensity formulas remain the same, except that they must now be multiplied by  $g_l A_{n,l}^{n',l'}/\beta(n, l)$ . For the previous case, this factor was obviously equal to unity. The introduction of the statistical weight  $g_l$  takes into account the case in which all the magnetic levels for a given n term have the same probability of excitation. The equation locating the maximum of the intensity curve is given by

$$2e^{-[(1/T(n,l)][2mx'/Xe]^{1/2}} - e^{-[1/T(n,l)][2m(a+x')/Xe]^{1/2}} - 1 = 0$$
(7)

This is the equation from which it is possible to determine the average life T(n, l) after measuring experimentally the displacement x' of the maximum of the intensity curve.

## Experimental arrangement.

The experimental arrangement used, showing the electrodes, and the electrical diagram are given in Fig. 1. The electrodes were mounted in a tube through which helium was circulated at a low pressure. The parts were made of molybdenum because it has nonmagnetic properties and because it does not give out occluded gases. The source of electrons was a 7 mil diameter tungsten filament whose cross section is indicated at F in the figure. Placed directly in front of the filament was the circular electrode 1 with a rectangular opening at its center of dimensions 0.6 mm by 4 mm. The filament 18 mm long, was in alignment with this opening, so that only electrons produced over 4 mm of its central portion were capable of being accelerated through the electrode 2 by potential  $V_1$ . The opening in electrode 2 was 2.75 mm by

6.0 mm. The electrons did not touch the edges of this opening because they were collimated into a narrow beam by an external magnetic field H of 2520 gauss applied in the direction of the accelerating electric field. The electrons passed between the electrodes 3 and 4 between which was applied an electric field which served to draw the ions out of the beam. The ions moved in a direction perpendicular to the direction of the electron beam. The magnetic field maintained the electron beam in spite of the cross electric field. The electrons then passed through electrodes 5 and were finally collected by 6 after receiving an additional amount of energy from the battery D. This procedure prevented secondary electrons from returning into the region between 3 and 4.



Fig. 1. Arrangement of electrodes and electric circuit.

The electrical circuit shows how the various potentials were applied. Battery *B* with its resistances produced the accelerating potentials while *C* served to maintain the transverse electric field. The potentials  $V_3$  and  $V_4$  could be arranged so that the space between 3 and 4 occupied by the electron beam was kept at any potential desired relative to the potential of 2. The filament was heated by a 60 cycle alternating current, so that it was not distorted by the force exerted on it by the magnetic field.

The light emitted from the region between 3 and 4 was projected on to the slit of the spectrograph with the image of the beam perpendicular to the slit. The direction of motion of the ions was then parallel to the slit. The distribution of density of the lines of ionized helium would be displaced in the direction of the motion of the emitters. The arc lines on the other hand would be undisplaced and furnished a reference point locating the center of the beam.

Fig. 2 shows the arrangement of the parts in the tube relative to the position of the spectrograph. The filament F was held firmly in position by large clamps made of tantalum. This prevented any large vibrations of the filament due to interaction of the alternating heating current and the external magnetic field. The shape of the electron beam is determined largely

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by the size of the filament. The actual beam was 4 mm deep and approximately 0.4 mm wide. The light emitted from the region between 3 and 4 passed through the quartz window Q and was projected by the quartz lens L on to the slit S of the Hilger  $E_4$  quartz spectrograph, in the manner described above. The lens was focused with respect to the center of the electron beam. This produced, with a magnification of 1.26 times, an image of depth 6.32 mm whose center lay in the plane of S. The spectrograph used was capable of bringing to focus the parts of the image of the beam outside of this plane. The magnification of the spectrograph for the wave-length 2733A for example, was practically unity, so that the total magnification in this case was 1.26 times.

The helium was purified by a glow discharge between iron and misch metal electrodes. It was then drawn through an adjustable leak and a liquid



Fig. 2. A cross section showing arrangement of electrodes with respect to the position of the spectrograph.

air trap into the experimental tube and pumped finally back again into the purifier. The pressure in the experimental tube was generally maintained at 0.01 mm of Hg which means that the mean free path at this pressure was 2 cm for the atoms and about 10 cm for the electrons. This pressure was sufficient to produce enough excitation for simultaneous ionization and excitation. The mean free path of the helium ions was great enough so that they suffered practically no collisions before radiating. Thus de-excitation by collisions was prevented.

#### **RESULTS AND DISCUSSION**

#### The dependence of the average life upon the state of excitation.

Fig. 3 shows results for the first four lines of the "4686" series of ionized helium obtained in some preliminary experiments in which a different type of filament and slit system were used than were illustrated in Fig. 1. Fig. 3 represents curves made from densitometer records taken along the lengths of the lines. With each of these lines there is an arc line which gives the posi-

tion of the electron beam. All the lines were obtained on one plate at a single exposure. The pressure of helium in the tube was  $1.2 \times 10^{-2}$  mm of Hg, the electron voltage was 500 volts and the transverse electric field was 305 volts/cm, while the magnetic field was 2520 gauss. The electron current collected to 6 was 0.5 milliamperes and the positive ion current to electrode 3 was 0.025 milliampere.

The shape of the electron beam in this case was so irregular that it was impossible to obtain values for the average life, but it sufficed for the purpose of comparing the average lives for the different lines. From Fig. 3 we see that for the lines 4686A ( $4\rightarrow3$ ) and 3203A ( $5\rightarrow3$ ) the displacements in the direction of motion of the helium ions are small. However, the line 2733A ( $6\rightarrow3$ ) shows clearly a displacement in the direction in which the positive ions are moving. The next line of this series 2511A, ( $7\rightarrow3$ ) exhibits a greater shift than the previous line 2733A. From Eq. (7) we see that the lines which



Fig. 3. Experimental results obtained illustrating the relative displacements of the first four lines of the "4686" series of He<sup>+</sup>.

have the greatest displacements must originate from levels which have the longest mean lives. Therefore, these experimental results show that the average life becomes progressively greater for levels with larger total quantum numbers. This is in agreement with the theoretical values for the average life as shown in Table I.

#### Simultaneous ionization and excitation by electron impact.

In the derivation of the intensity formulas, it was assumed that at the time of collision the atom is ionized and excited. Compton and Boyce<sup>12</sup> have shown that the spark lines of helium can be formed only by single electron impacts when the electron current densities and gas pressures are low. This was proven by the fact that electrons with 65 volts energy excited only the arc lines of helium whereas the spark spectrum was fully developed at 85 volts. The spark spectrum would have appeared at 55 volts if it had been caused by cumulative action. This has been later confirmed by Elenbass<sup>13</sup>

<sup>&</sup>lt;sup>12</sup> Compton and Boyce, Jour. Frank. Inst. 205, 497 (1928). See also Compton and Lilly, Astrophys. J. 52, 1 (1920).

<sup>&</sup>lt;sup>13</sup> Elenbass, Zeits. f. Physik 59, 289 (1930).

who found that the 4686A line of ionized helium did not appear until about 80 volts.

It was found in the present experiment that with the cross field applied, the line 2733A became very weak as the voltage was decreased until at about 80 volts it remained only strong enough to be measured. In order more accurately to determine the initial potential for the appearance of this line, the transverse electric field was removed so that the electron velocities would be more uniform. It was found that the line was entirely absent at 65 volts whereas it was visible at 82 volts. This confirms the results obtained by the above workers.

We see from the previous work that the intensity of the displaced spark line at any given position along its length is proportional to the electron cur-



Fig. 4. Intensity distribution of line 2733A without the transverse field. The circle represents the cross section of the filament drawn to scale illustrating the size of the beam in comparison with the diameter of the filament.

rent if the excitation is produced at a single collision, whereas if it is due to a double collision the intensity will be proportional to the square of the electron current. Measurements of the relative intensity of both the 4686A and 2733A lines have been made as a function of the electron current with the transverse field applied and the intensity was found always to be directly proportional to the electron current showing therefore again, that the excitation occurs at a single collision.

## Shape of the electron beam.

In the derivation of the intensity formulas it was assumed that the intensity of the electrons in the beam increased uniformly up to the center of the beam and then decreased uniformly to the upper edge of the beam. Fig. 4 shows how the intensity of the line 2733A varies along its length for the case of

no transverse field. This illustrates that the distribution of electrons is nearly the same as the assumed distribution. The dotted lines forming a triangle represent the distribution of electrons which give a width of the beam equal to 0.041 cm. The circle in the upper part of the figure represents the cross section of the 7 mil diameter tungsten filament drawn to scale with respect to the rest of the figure.

When the electric field is applied this distribution is not appreciably changed as long as the velocity of the exciting electrons is greater than the minimum speed for ionization and excitation. This fact is demonstrated by measuring the intensity distribution of a suitable arc line of helium. A line must be selected which is not lengthened by absorption and reemission of the light in leaving the tube. This line must also be excited by approximately the same speed electrons which are capable of ionizing and exciting the helium atom. The helium arc line 4437A was selected because for the case of no transverse field it had practically the same distribution of intensity as the helium spark lines. When the field was applied the shape of the line indicated that the distribution in intensity was practically unaltered for the beam used here. The assumption of the distribution of intensity for the subsequent calculation of the mean life is thereby justified.

## Average life for sixth quantum state.

Fig. 5 shows enlargements made from the actual negatives obtained with the experimental arrangement as given in Fig. 1. Negatives are reproduced here for cases with and without the transverse electric field. The line 2733A of ionized helium is drawn up in the direction of motion of the positive ions when





the field is applied. The lines 2763A and 2723A are two principal series lines of the orthohelium system. They are unaffected by the electric field and their densest portion locates the center of the electron beam. In exposure 2 the electrons had approximately 210 volts velocity, while the cross electric field was 740 volts/cm. The currents and pressure were the same as for the pre-



Fig. 6. Microphotometer records of line 2733A for (1) without cross field, and (2) with cross field. Vertical line marks center of electron beam located by helium arc lines 2723A and 2763A.

vious experiment described above. The magnetic field was 2520 gauss. The spectrum was photographed on an Eastman speedway plate with an exposure of twelve hours. The experimental conditions for exposure 1 were similar to 2 except that the cross field was zero and the voltage of the electrons was 350 volts.

Fig. 6 shows two records made by a Goos and Koch microphotometer taken along the length of the line 2733A, (1) with and (2) without the transverse electric field. The vertical line locates the center of the beam as determined by the center of the two adjoining arc lines shown in Fig. 5. This is accomplished by making microphotometer records for these two arc lines which lie on opposite sides of the line 2733A. By this means it is possible to correct for error produced by improper alignment of the direction of motion of the spectrum plate in the microphotometer. From Fig. 6 we see that for record (2) the line is displaced in the direction of motion of the helium ions. Record (1) is made from the plate shown in exposure 1 of Fig. 5. (2) is taken from an exposure in which the accelerating potential was about 120 volts, the cross field 650 volts/cm, the other conditions were the same as given above.

In Table III are given the experimental values obtained for the average life determined for different energies of the exciting electrons. It is seen that

TABLE III.

Energy of electrons in beam (volts)	Average life experimental values for sixth quantum state (sec.)	Mean average life theoretical value sixth quantum state	
260	$1.6 \pm 0.2 \times 10^{-8}$		
120	$1.5\pm0.2 imes10^{-8}$		
112	$1.3 \pm 0.2  imes 10^{-8}$		
100	$1.1 \pm 0.2  imes 10^{-8}$		
80	$1.1 \pm 0.2  imes 10^{-8}$	$1.17 \times 10^{-8}$ sec.	

the experimental values for the average life decrease as the energy of the electrons is diminished, and approaches the theoretical value for the mean average life. As the energy of the exciting electrons is decreased there will be less excitation to the levels higher than the sixth. The number of transitions downward into the sixth state will thus decrease and cause the average life to become shorter. This conclusion is substantiated by the fact that as the voltage is lowered the intensity of the line 2511A coming from the seventh state diminishes faster than does the intensity of the line 2733A coming from the sixth state. Thus the ratio of the number of electrons in the sixth state to the number in the seventh becomes greater as the voltage is decreased. This will produce fewer transitions into the sixth state and consequently the average life of the sixth state will become shorter. Because of the small energy differences between the sixth and seventh levels it is practically impossible to excite to the sixth state with enough intensity to produce lines which can be photographed, while at the same time produce no excitations to the seventh state.

The values obtained at 100 volts and 80 volts differ from the theoretical value for the mean average life by less than  $0.10 \times 10^{-8}$  sec. which is consider-

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ably less than the experimental error. The agreement obtained here is better than is found by other workers for the case of hydrogen. For example, the theoretical values for the mean average life of the 2nd, 3rd, and 4th quantum states of hydrogen are  $2.18 \times 10^{-9}$  sec.,  $1.03 \times 10^{-8}$  sec. and  $3.4 \times 10^{-8}$  sec. respectively. Wien's<sup>14</sup> canal-ray measurements on  $H_a$  of the Lyman series give  $6.7 \times 10^{-9}$  sec. for the average life of the second quantum state. Slack<sup>15</sup> obtains experimentally the value of  $1.2 \times 10^{-8}$  sec. for this state. For the third quantum state Wien again obtains  $6.7 \times 10^{-9}$  sec. from measurements on  $H_b$  of the Lyman series while Wien and Kerschbaum<sup>14</sup> obtain  $1.85 \times 10^{-8}$  sec. from  $H_a$ of the Balmer series. Their results for the fourth state as determined by  $H_\beta$ give the same value of  $1.85 \times 10^{-8}$  sec.

Curve 1 in Fig. 7 gives the intensity distribution of the line 2733A with respect to the center of the beam, while curve 2 represented by the dotted line gives the theroretical intensity distribution calculated by the use of the



Fig. 7. Intensity distribution of line 2733A found experimentally, given by curve 1 while curve 2 shows a similar distribution calculated from intensity formulas using the theoretical value for mean average life.

intensity formulas (7) and (8)<sup>16</sup> for the average life of  $1.17 \times 10^{-8}$  sec., the value given in Table III for the mean average life. The two curves show approximately the same distribution of intensity, which gives an agreement between the experiment and the theory. It is noticed that the width of the beam is 0.048 cm as compared to 0.041 cm as is given in Fig. 4. This slight increase

<sup>14</sup> Wien, Ann. d. Physik 83, 1 (1927); See also table of values for average lives from canal ray experiments of Wien and Kerschbaum, prepared by Kerschbaum, Ann. d. Physik 83, 294 (1927).

<sup>15</sup> Slack, Phys. Rev. 28, 1 (1926).

<sup>16</sup> These numbers refer to equations on page 725, Phys. Rev. 32, (1928).

in the width of the beam is probably caused by the action of the transverse electric field.

In order to compare the average life measured experimentally with the theoretical value for the mean average life, there must be a continuous interchange between the sublevels so that the number of electrons in any sublevel is proportional to its quantum weight. Sugiura has pointed out in connection with the interpretation of Wien's canal-ray experiments, that the perturba-



Fig. 8. Theoretical intensity distribution for the five fine structure lines of the line 2733A. Curve A represented by the circles is the resultant intensity curve plotted on a reduced scale. Curve  $6\rightarrow 3$  indicated by full dots gives the intensity distribution for the case of free interchange among the *l* values (or for the case of a single exponential decay law). The values for the average life used were taken from Table I, and 658 volts/cm was the strength of field used.

tions to which the hydrogen atoms were subjected were large compared with the component separations due to relativity, so that free interchange existed among the various sub-divisions of a term. In the present work the helium ions radiate in the presence of crossed electric and magnetic fields which further complicates the perturbations of the quantum states. However Elenbass<sup>17</sup> has found that the line 4686A of ionized helium is polarized when ex-

<sup>17</sup> Reference 13.

cited by unidirectional electrons. To interpret these results one must assume that free-interchange did not take place, otherwise the light could not be polarized.

In the present work we cannot prove the existence of statistical equilibrium so that it is considered a postulate for comparing the results of experiments with the mean average life.

## Alternative interpretation of the experimental results.

It is, however, possible to obtain a comparison with the theory by considering the decay time of each of the fine structure lines which make up the single line 2733A, without the assumption of statistical equilibrium. A theoretical intensity curve is obtained for each line individually and then a resultant intensity curve is plotted and compared with the theoretical intensity curve obtained above for the case of free interchange. This method requires a knowledge of the relative population of the substates at the time of excitation.

The results of these computations are illustrated in Fig. 8. The theoretical intensity distribution for the five fine structure lines is plotted with respect to the distance from the center of the beam. The relative intensities of these curves for a given position depends upon the transition probability  $A_{n,l}^{n',l}$ the quantum weight  $g_l$ , and the average life  $1/\beta(n,l)$  as given in the formulas for the intensity distribution.<sup>18</sup> It is noticed that the two lines  $6_3 \rightarrow 3_2$  and  $6_2 \rightarrow 3_2$ 31 predominate and determine approximately the intensity distribution of the resultant intensity curve A which is plotted on a reduced scale. Thus the curve A represented by the circles gives the theoretical intensity distribution for the case of *no interchange* between the *l* values. There is also plotted in this figure the values of the intensity for the case of *free interchange* or (in other words for the case of a single exponential decay law), taken from curve 2 in Fig. 7 and is represented by  $6 \rightarrow 3$  located by the full dots. It is seen that these two sets of points lie almost identically on the same curve which shows that the theoretical distribution is the same for the two cases. Thus we have shown that we cannot distinguish between the two methods of interpreting the experimental results. Both are in good agreement with the experimental intensity distribution obtained.

If we take into account electron spin correction we find that each of the above five lines split up into groups of lines. Since we have proved above that the average life of the n,l,j,th state is equal to the average life of the n,l,j,th state, therefore, these lines resulting from spin will have the same average life as the line from which they originated. This means that in this case we shall have five groups of lines with average lives corresponding to the average lives of the five lines given above. Also the sum of the intensity of the lines in any particular group is proportional to the intensity of the parent relativistic line independent of l.<sup>19</sup> Therefore, the relative intensity intensity intensities of the lines in a group will be the same as the relative intensities.

<sup>19</sup> This has been proved above. See also Sommerfeld and Unsöld, Zeits. f. Physik **38**, 237 (1926).

<sup>&</sup>lt;sup>18</sup> The values for  $A_{n,l}^{n,l'}$  and the average life used here are obtained from Slack's table although Kupper's values could be used without greatly changing the results obtained.

sity of the five single lines for the relativistic case. Thus we see that the distribution of intensity of the resultant line calculated by considering all the additional fine-structure lines due to spin will be exactly the same as for the relativistic case.

#### Errors

# (1) Approximate solution.

The most fundamental error is caused by the fact that the shape of the electron beam is never actually the ideal case in which the distribution is triangular as assumed in the calculation of the intensity of the displaced lines. A more exact treatment could be obtained by a numerical integration performed by introducing experimental values for the electron distribution func-



Fig. 9. Relationship between the displacements and the average life for a field of 658 volts/cm, and beam width 0.048 cm.

tion  $N(\xi)$ . However, Fig. 4 shows that the distribution obtained is approximately triangular so that the results are sufficiently accurate to compare satisfactorily with the theory.

# (2) Center and shape of the electron beam.

In the present experimental conditions there exists a cross field of approximately 660 volts/cm. Since the width of the beam is approximately 0.04 cm there is a drop of potential of about 26 volts across the electron beam itself. This means that there will be a distribution of velocities across the beam within this amount. This will alter the effective distribution curve and will be more pronounced in the case where the speed of the exciting electrons is only a small amount in excess of the minimum velocity required for excitation. This error will become practically negligible for higher velocities of the order of 150 volts or more. It is very difficult to determine the amount of this

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error for the lower velocities without knowing the probability of excitation function for both the arc lines and the spark lines.

# (3) Measurements of displacements.

Fig. 9 shows the relationship between the average life and the displacement of the maximum of the shifted spark line for a cross field of 658 volts/cm. On the ordinate axis there is plotted at the left of the figure the displacements, while at the right is plotted the corresponding distances which are actually measured on the microphotometer record. On the abscissae is plotted the average life for a beam of width 0.048 cm. These are the constants used in obtaining the results given in Table III. The error in reading the displacements on the microphotometer record was not greater than 0.2 mm. Therefore we see from Fig. 9 that the corresponding error for the average life is  $0.20 \times 10^{-8}$  sec. This is the error given in Table III.

# (4) Effect of the thermal velocities of the helium ions.

In the above calculations of the average life it is assumed that when the excited ion is formed it is initially at rest. This, of course, is never actually the case on account of the thermal velocities possessed by the ions. A helium atom or ion at a temperature of 20°C has a root mean square velocity of  $1.38 \times 10^5$  cm/sec. Under the action of a transverse electric field of 732 volt/ cm the ion, initially at rest, would obtain this velocity of  $1.38 \times 10^5$  cm/sec. after being accelerated through a distance of  $5.33 \times 10^{-5}$  cm. This distance is small compared to the distances considered in the present experiment. Thus the initial velocity due to thermal agitation of the ion can be neglected.

## (5) Space charge.

To reduce the effect of space charge in these experiments, the electron current in the beam was decreased until it required about 12 hours exposure to photograph the line 2733A. The electron currents in the beam were not greater than 0.5 milliampere and the positive ion current was less than 0.06 milliampere. It was found that the displacements of the spark line 2733A were not appreciably altered when the currents were reduced to about one-half the above values. This indicates that the distribution of space charge in the presence of the strong transverse field does not disturb appreciably the velocities of the helium ions.

## (6) Shape of emitting portion of the filament.

Although the displacements measured are of the order of 0.002 cm it is unnecessary for the filament to be straight and in proper alignment to less than this amount. The filament however, was not actually straight for its central portion sagged about 0.002 cm. This part of the filament would be out of alignment with the other portions by an amount comparable to the experimentally measured displacements. However, the intensity photographed by the spectrograph is the resultant of the radiation coming from different depths of the beam. Although each radiating segment is not in alignment with the optic axis of the spectrograph and the projecting lens, nevertheless the displacement recorded by the photographic plate will be

exactly the same as if the segments were in very accurate alignment. This is true provided that all segments radiate with the same intensity, which is practically the case. Therefore, any sagging or improper alignment of the filament will produce no error in the measured displacements, but it will obviously produce an error in the value for the width of the beam and in the intensity distribution curves. These errors of 0.002 cm are so small that they are unimportant in determining the width of the beam, however they became more important in the measurement of the intensity distribution.

## (7) Vibrations of the filament.

The maximum amplitude of vibration of the filament due to the interaction between the alternating heating current and the magnetic field was about 0.001 cm which is small compared to the width of the beam so that it caused no appreciable errors.

In conclusion the writer desires to express his thanks to Dr. A. Bramley for helpful discussions of the work.

## Note added in proof, October 2, 1931:

In view of the discrepancies given above for the matrix amplitude squared, Professor Slack has recalculated his values for the intensities and transition probabilities for hydrogen and has kindly furnished the data for Table IV.

With Slack's corrected values for the transition probabilities we find that the average life of the  $5_0$ ,  $6_0$ ,  $7_0$ ,  $7_3$  and  $7_4$  states are changed to  $2.2 \times 10^{-8}$ 

TABLE IV. Summary of corrected intensities in the 4th, 5th, and 6th series of hydrogen (Prepared by Slack).

Component	Previously Intensity ergs/sec.	published* Probability sec. <sup>-1</sup>	Corre Intensity ergs/sec.	ected Probability sec. <sup>-1</sup>	Matrix amplitude squared	Kupper's matrix amplitude squared
$\begin{array}{c} 5_4 - 4_3 \\ 5_3 - 4_2 \\ 5_2 - 4_1 \\ 5_2 - 4_3 \\ 5_1 - 4_0 \\ 5_1 - 4_2 \\ 5_0 - 4_1 \\ 6_4 - 4_3 \\ 6_3 - 4_2 \\ 6_2 - 4_1 \\ 6_2 - 4_3 \\ 6_1 - 4_0 \\ 6_1 - 4_2 \end{array}$	$\begin{array}{c} 20.8 \times 10^{-7} \\ 12.6 \\ 7.26 \\ 0.246 \\ 3.60 \\ 0.0092 \\ 6.09 \\ 9.94 \times 10^{-7} \\ 9.85 \\ 6.58 \\ 0.094 \\ 3.88 \\ 0.0075 \end{array}$	$\begin{array}{c} 43.0 \times 10^{5} \\ 26.1 \\ 15.0 \\ 0.507 \\ 7.43 \\ 0.019 \\ 12.54 \end{array}$	$\begin{array}{c} 20.8 \times 10^{-7} \\ 12.6 \\ 7.26 \\ 0.246 \\ 3.60 \\ 0.92 \\ 3.15 \\ 9.94 \times 10^{-7} \\ 9.85 \\ 6.58 \\ 0.161 \\ 3.88 \\ 0.75 \end{array}$	$\begin{array}{r} 43.0 \times 10^{5} \\ 26.1 \\ 15.0 \\ 0.507 \\ 7.43 \\ 1.9 \\ 6.49 \\ 7 \\ 13.3 \times 10^{5} \\ 13.2 \\ 8.81 \\ 0.215 \\ 5.19 \\ 1.0 \end{array}$	$\begin{array}{c} 628\\ 297\\ 122\\ 4.13\\ 36.3\\ 9.3\\ 10.58\\ 53.3\\ 41.1\\ 19.6\\ 0.48\\ 6.88\\ 1.30\\ \end{array}$	$\begin{array}{c} 630\\ 291\\ 125\\ 4.4\\ 36\\ 9.1\\ 105\\ 55\\ 40\\ 22.5\\ 0.48\\ 6.0\\ 1.21\\ \end{array}$
$\begin{array}{c} 6_{0} - 4_{1} \\ 7_{4} - 4_{3} \\ 7_{3} - 4_{2} \\ 7_{2} - 4_{1} \\ 7_{2} - 4_{3} \\ 7_{1} - 4_{0} \\ 7_{1} - 4_{2} \\ 7_{0} - 4_{1} \end{array}$	$\begin{array}{c} 3.11 \\ 7.94 \times 10^{-7} \\ 8.65 \\ 4.28 \\ 0.057 \\ 2.38 \\ 0.0041 \\ 1.98 \end{array}$	$\begin{array}{c} 4.16 \\ 7 & 8.75 \times 10^5 \\ 9.55 \\ 4.73 \\ 0.063 \\ 2.63 \\ 0.0045 \\ 2.19 \end{array}$	$2.44$ $3.97 \times 10^{-3}$ $4.57$ $4.28$ $0.057$ $2.38$ $0.41$ $1.77$	$\begin{array}{c} 3.25 \\ 7 & 4.37 \times 10^5 \\ 5.04 \\ 4.73 \\ 0.063 \\ 2.63 \\ 0.45 \\ 1.95 \end{array}$	$1.45 \\ 14.6 \\ 12.9 \\ 8.6 \\ 0.115 \\ 2.88 \\ 0.49 \\ 0.71 \\$	$14.8 \\ 14.7 \\ 12.9 \\ 6.7 \\ 0.137 \\ 2.15 \\ 0.41 \\ 4.2 \\ $

\* Slack, reference 4.

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	Previously p	oublished	Corre	cted	Matrix	Kupper's
Component	Intensity ergs/sec.	Probability sec. <sup>-1</sup>	Intensity ergs/sec.	Probability sec. <sup>-1</sup>	amplitude squared	matrix amplitude squared
$\begin{array}{c} & & \\ & & 6_5-5_4 \\ & 6_4-5_3 \\ & 6_3-5_4 \\ & 6_2-5_1 \\ & 6_2-5_3 \\ & 6_1-5_0 \\ & 6_1-5_2 \\ & 6_0-5_1 \\ & 7_5-5_4 \end{array}$	$\begin{array}{r} 4.20 \times 10^{-7} \\ 2.63 \\ 1.97 \\ 0.045 \\ 1.14 \\ \hline 0.617 \\ 1.97 \\ 2.40 \times 10^{-7} \end{array}$	$ \begin{array}{r} 16.0 \times 10^{5} \\ 10.0 \\ 7.50 \\ 0.17 \\ 4.32 \\ \hline 2.35 \\ 7.50 \\ 5.69 \times 10^{5} \\ \end{array} $	$\begin{array}{c} 4.33 \times 10^{-3} \\ 2.63 \\ 1.97 \\ 0.030 \\ 1.14 \\ 0.078 \\ 0.617 \\ 0.268 \\ 0.69 \\ 2.40 \times 10^{-3} \end{array}$	$\begin{array}{c} 7 & 16.4 \times 10^5 \\ 10.0 & 7.50 \\ 0.114 & 4.32 \\ 0.295 & 2.35 \\ 1.02 & 2.63 \\ 7 & 5.69 \times 10^5 \end{array}$	1852. 921. 538. 8.15 226. 15.1 72.3 31.3 26.9 154.	201. 1020. 520. 8.2 203. 20. 67. 15.3 247. 139.
$7_{4} - 5_{3}$ $7_{3} - 5_{2}$ $7_{3} - 5_{4}$ $7_{2} - 5_{1}$ $7_{2} - 5_{3}$ $7_{1} - 5_{0}$ $7_{1} - 5_{2}$ $7_{0} - 5_{1}$	2.46 1.97 0.0093 1.27 1.01 1.10	5.82 4.66 0.022 3.01 $2.402.60$	$\begin{array}{c} 2.46 \\ 1.97 \\ 0.20 \\ 1.27 \\ 0.808 \\ 1.01 \\ 0.258 \\ 0.583 \end{array}$	5.82 4.66 0.48 3.01 1.91 2.40 0.66 1.38	$128. \\80. \\0.82 \\37. \\2.35 \\17.6 \\4.8 \\3.4$	$122. \\ 75. \\ 0.81 \\ 36. \\ 2.37 \\ 20.7 \\ 3.9 \\ 35. $
$76 - 6_5$ $7_5 - 6_4$ $7_4 - 6_3$ $7_4 - 6_5$ $7_3 - 6_2$ $7_3 - 6_4$ $7_2 - 6_1$ $7_2 - 6_3$ $7_1 - 6_0$ $7_1 - 6_2$ $7_0 - 6_1$	$\begin{array}{c} 1.13 \times 10^{-7} \\ 0.635 \\ 0.617 \\ 0.0113 \\ 0.421 \\ \hline \\ 0.235 \\ \hline \\ 0.051 \\ \hline \\ 1.04 \end{array}$	$7.10 \times 10^{5}$ $4.00$ $3.89$ $0.071$ $2.65$ $$ $1.48$ $0.32$ $6.53$	$\begin{array}{c} 1.20 \times 10^{-1} \\ 0.635 \\ 0.545 \\ 0.0057 \\ 0.421 \\ 0.016 \\ 0.246 \\ 0.030 \\ 0.159 \\ 0.098 \\ 0.20 \end{array}$	$\begin{array}{c} 7 & 7.5 \times 10^5 \\ 4.0 \\ 3.43 \\ 0.036 \\ 2.65 \\ 0.10 \\ 1.55 \\ 0.19 \\ 1.0 \\ 0.62 \\ 1.26 \end{array}$	$\begin{array}{c} 4500.\\ 2020.\\ 1420.\\ 15.0\\ 850.\\ 32.4\\ 356.\\ 43.7\\ 138.\\ 85.\\ 58. \end{array}$	4497. 2734. 1582. 14.1 848. 36.3 58.9

TABLE IV. (continued)

sec.,  $3.3 \times 10^{-8}$  sec.,  $5.0 \times 10^{-8}$  sec.,  $2.4 \times 10^{-8}$  sec., and  $4.5 \times 10^{-8}$  sec., respectively. The values for the average life of the other states and the mean average lives remain practically the same. The only lines illustrated in Fig. 8 affected by these corrections is the transition  $6_0 \rightarrow 3_1$ . Since this component is weak in comparison with the other transitions, the resultant intensity curve A is not appreciably changed.



Fig. 5. Enlargements (magnification, five times) made from original negatives obtained for (1) without transverse field and (2) without transverse field showing effect of motion of helium ions for line 2733A. The adjoining arc lines located the center of the electron beam.



Fig. 6. Microphotometer records of line 2733A for (1) without cross field, and (2) with cross field. Vertical line marks center of electron beam located by helium arc lines 2723A and 2763A.