# THE ELECTROOPTICAL SHUTTER—ITS THEORY AND TECHNIQUE

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Abstract

This paper records a development in the design of the single cell type of electrooptical shutter together with a theory describing its action. Part I. gives a description of the shutter together with a method of observing simultaneously two stages of the phenomenon being studied. Part II gives the theory of shutter action. A condensed summary of the physical optics is followed by a development of the electric circuit theory which gives the manner of closing of the shutter when the leads to it are fairly short. The theory is illustrated by a series of numerical examples based on typical experimental conditions which show the effect of varying each controlling factor. The times required for closing (90 to 100 percent transmission) are of the order of $4\times10^{-9}$ sec. under favorable conditions. The necessity for a Kerr cell of small electrostatic capacity is shown. Part III gives three groups of experimental data which were obtained with the shutter in a study of *spark breakdown:* one indicates the time resolution attainable; another gives the total time lag in closing; and a third gives the change in this lag with certain conditions. The information of the second group combined with the theory yields the rate of fall of voltage across the controlling spark gap (static breakdown), a typical value being $14\times10^{-9}$ sec. to fall to 20 percent of the initial value (76 cm Hg, 5 mm gap). The theory and experimental data are found in complete agreement. An improved method is given for the determination of the correct damping resistance in the shutter circuit. Part IV gives a description and the factors considered in the design of the Kerr cells developed in the present study, also the essentials of the technique for their proper use.

## Introduction

THIS paper is concerned with the Abraham and Lemoine type of electrooptical shutter, a single cell, "open-to-closed" type of shutter. The two cell, "closed-to-open-to-closed" type of shutter developed by Beams[1] will not be treated here. The principle of the Abraham and Lemoine electrooptical shutter has been known for some thirty years,[2] and use has been made of it by various experimenters when a device was needed to analyze the light of a rapidly varying phenomenon.[3,4,5,6] Thus it has been used to measure the time lag of fluorescence, the difference in times of appearence of the spectrum lines in a spark discharge and for a study of the early stages of spark discharges. The validity of the results obtained with it have been somewhat

[1] Beams, J.O.S.A. **5**, 597 (1926).
[2] Abrahan and Lemoine, C. R. **129**, 206 (1899).
[3] Gottling, Phys. Rev. **22**, 566 (1923).
[4] Beams, Phys. Rev. **28**, 475 (1926).
[5] Locher, J.O.S.A. **17**, 91 (1928).
[6] Lawrence and Dunnington, Phys. Rev. **35**, 396 (1930).

justly criticised.[7] It is in fact a long step from an understanding of its simple principles to its application in obtaining quantitative measurements when the times involved are of the order of $10^{-8}$ to $10^{-9}$ sec.

The information presented in the present paper is the result of a study which was initially undertaken to investigate spark breakdown phenomena. It was necessary to develop an apparatus which would make possible an optical analysis of a process moving from initiation to completion in a time of the order of $10^{-8}$ sec. The electrooptical shutter was chosen for this work as offering the only feasible means. It has been developed to the point of providing a satisfactory method for such a study, with the interpretation of the results reasonably quantitiative and the limitations of the shutter fairly well known.

During the later stages of the present study which has been in progress since the publication of a previous paper,[6] a paper appeared by von Hámos[8] recording the results of a somewhat similiar study. Since however the present study has been developed further, with the experimental method differing in several essentials, and the mathematical treatment considerably less approximate, no essential duplication results.

## Part I. Description of the Electrooptical Shutter

The apparatus constituting the shutter proper is shown diagrammatically in Fig. 1. Light from the phenomenon at $S$ which is to be analyzed is passed

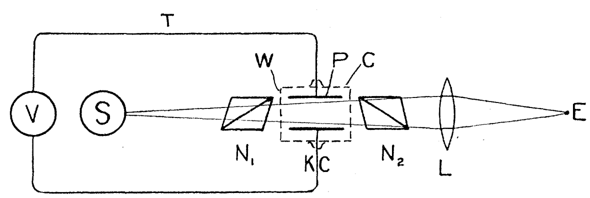


Fig. 1. Schematic representation of the electrooptical shutter.

through the nicol prism $N_1$, between the plates of the Kerr cell $KC$ and then to the second nicol $N_2$ which is crossed with the first (i.e. set for total extinction of the light). A lens $L$ which focuses the light on a photographic plate or an eyepiece at $E$ completes the optical system. The Kerr cell consists of two parallel metal plates $P$ (the size and spacing of which must be computed) placed in a glass cell $C$ containing a liquid which possesses the property of becoming doubly refracting under the influence of an electric field. Electrical connections $T$ are made from these two plates to the voltage $V$ which is used to control the shutter. In the present study the optical source $S$, which was a spark gap, also provided the control voltage $V$ through the voltage across the gap. This combining of $S$ and $V$ is not an unimportant detail as it provides the all important synchronization between the initiation of the phenomenon being studied and the closing of the shutter at some definite stage of the phenomenon.

[7] Gaviola, Phys. Rev. **33**, 1023 (1929).

[8] Von Hámos, Ann. d. Physik (5) **7**, 857 (1930).

The action of the shutter when controlled by a spark gap as shown in Fig. 2 is briefly as follows. Voltage from a direct current source is applied simultaneously to gap and Kerr cell. The existance of an electric field in the liquid of the Kerr cell causes it to become doubly refracting. Hence the plane polarized light incident upon the cell emerges elliptically polarized and a certain fraction of it passes through the second nicol prism. Thus when a potential exists across the plates of the Kerr cell the shutter may be considered open. When the applied voltage has reached a sufficient value the gap "breaks down." Two important characteristics of the breakdown are a rapid collapse of the voltage across the gap and the emission of light in the gap. Following the breakdown an electric wave is propagated out along the wires $T$ toward the Kerr cell, while light is emitted and also travels toward the cell. The electric wave reaches the cell at a time after the beginning of the breakdown equal to the length of wire path between the gap and cell divided by the wave velocity (very nearly the speed of light), while the light reaches there after

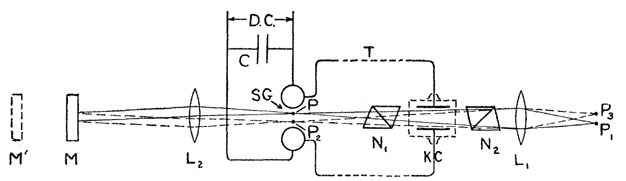


Fig. 2. Experimental arrangement of the shutter which provides a means of simultaneously observing two stages of development.

a time equal to the path traveled divided by the speed of light. Light arriving at the cell before the voltage wave passes on through, that arriving after is rejected due to the collapse of voltage and consequent vanishing of the double refraction.

Measuring times from the beginning of the break in the voltage across the gap, the length of time $t_p$ which the shutter remains completely open after the breakdown is given by

$$t_p = \frac{L_w - L_l}{3 \times 10^{10}} \tag{1}$$

where $L_w$ and $L_l$ are respectively the lengths in centimeters of the wire path and light path from gap to cell. $t_p$ will be referred to as the *path* time of cut-off. A study of the deviation of the actual or *effective* time of cut-off from this path time will constitute a large part of the paper.

It is evident from the expression for $t_p$ that the time of cut-off can be varied by varying the length of either the wire or light paths. Normally the length of light path is fixed and the wire path is varied. However with this arrangement there is a limit to the quickness of cut-off since the wire path is always longer than the light path. To obtain still quicker cut-off times the light path must be lengthened without changing the wire path. This is conveniently accomplished by the second optical system devised in the present study and shown in Fig. 2. The lens $L_2$ produces a parallel beam of light which is reflected back by the mirror $M$ through the same lens and comes to focus at a position $P_2$ slightly to one side of the source $P$ (spark). It then passes on through the shutter system and produces a second image at $P_3$ by the side of the direct one. Since this second image has a longer light path, it has a smaller time of cut-off, and therefore represents an earlier stage of the phenomenon. Indeed if $L_l$ is made larger than $L_w$, negative times result (i.e. the shutter closes ahead of breakdown). This condition has been used frequently. *This double optical system hence makes possible the simultaneous observation of two stages of the phenomenon, the time difference between them, being known as accurately as one desires to measure the length of the additional light path of the second system.*

## Part II. Theory of the Shutter Action

### (A) Physical optics.

A complete treatment of the physical optics of double refraction as applied to the Kerr effect has been summarized by Kingsbury.[9] The relation of use here is that giving the intensity $I$ of light transmitted by the shutter system (neglecting absorption and loss by reflection) as a function of the electric field between the Kerr cell plates.

$$I = I_0 \sin^2 (\pi b L E^2) \tag{2}$$

Where $I_0$ is the incident intensity, $b$ is Kerr constant of liquid used, $L$ is length of plates in direction of optic axis, and $E$ is field strength in e.s. volts /cm. This equation is valid only for the usual condition in which the electric field makes an angle of 45° with the plane of polarization of both nicol prisms.

In general the edge effect of the electric field at the ends of the Kerr cell plates will cause an appreciable error. This may be allowed for by replacing the $L$ of Eq. (2) by the following relation given by Chaumont[10] for the effective length $L'$.

$$L' = L + \frac{a}{\pi}\left[1 + \frac{d}{a}\log\left(1 + \frac{a}{d}\right)\right]. \tag{3}$$

Where $L$ is the length of plates in direction of light, $a$ is their separation and $d$ is their thickness. This relation is accurate only when the length and width of the plates is at least three times their separation, and when there is an appreciable body of liquid around the plates.

Since nitrobenzene has a much larger electrostatic Kerr constant than that of any other known liquid, it should be used. Many values have been published for the Kerr constant of nitrobenzene, the differences possibly being due among other causes to variations in purity, but that given by Möller[11] seems to be reliable for the very pure substance. It is $b = 3.46 \times 10^{-5}$ at 20°C and a wave-length of 5460A. The constant decreases rapidly with increasing temperature.[12]

[9] Kingsbury, Rev. of Sci. Inst. **1,** 22 (1930). For a general discussion of the electrooptical Kerr effect see Handb. d. Physik **21,** 724.

[10] Chaumont, Ann. d. Physik (9) **5,** 31 (1916).

[11] Möller, Phys. Zeits. **30,** 20 (1929).

[12] Hand. d. Physik **21,** 769–780.

Inspection of Eq. (2) shows maximum (i.e. 100 percent) transmission is attained for that value of the field (call it $E_m$) given by

$$E_m = (2bL)^{-1/2}. \tag{4}$$

It will be convenient to rewrite (2) in terms of this value.

$$\frac{I}{I_0} = \sin^2\left[\frac{\pi}{2}\left(\frac{E}{E_m}\right)^2\right]. \tag{5}$$

To give a visual picture of the behavior of intensity with field strength, Eq. (4) is shown graphically in Fig. 3. For small values of $E/E_m$ the sine is approximately equal to the angle, so that the intensity is proportional to the fourth power of the field. As $E$ increases to $E_m$ the power in effect decreases from four to zero. This fact has led previous experimenters to the belief that

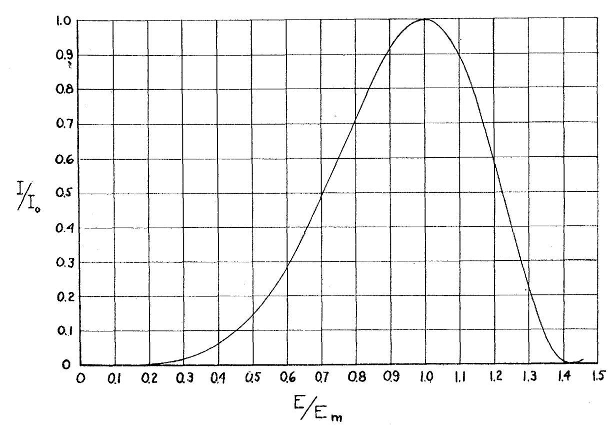


Fig. 3. The fractional intensity transmitted shown as a function of the field.

a "quicker" cut-off is obtained if smaller initial fields are used. On the contrary it will be shown in the following section that *quicker cut-offs are obtained with higher fields.*

**(B) Electric circuit theory.**

With this brief review of the optical behavior of the Kerr cell, let us consider the electric circuit of which it is a part. From this viewpoint, the Kerr cell is simply a small condenser. It is situated at the end of a transmission line (whose length may be a few centimeters or many meters) containing distributed inductance, capacity and resistance, and at the other end of which is a source of voltage $V$ which decreases rapidly when the spark breaks down. We will assume that the drop of this control voltage can be represented by a function of the form

$$V = V_0 e^{-mt}(1 + mt) \tag{6}$$

where $V_0$ is the initial voltage, $t$ the time and $m$ a positive constant. This function was chosen because of the general resemblance of its form to oscillograph curves of static spark breakdown.[13] Its slope at $t=0$ is zero. A nonoscillatory function was chosen since experimentally it is advantageous to have an aperiodic discharge. Further, the time involved in the actual spark breakdown, and hence in the voltage collapse, is small compared to the quarter period of the oscillation normally encountered in condensed discharges, so that any errors caused by erroneous voltage values at relatively late times are of secondary importance in the work at hand. We will also assume that the distributed inductance $l$ and resistance $r$ can be treated as lumped, and that the distributed capacity of the leads is considerably smaller than the capacity of the Kerr cell $c$ so that it can be neglected. Experimentally these circuit assumptions are valid only for short transmission lines, say not more than 0.5 to 2 meters, since the distributed capacity soon approaches that of the Kerr cell. However, the study of the formation stages of spark breakdown requires such short leads so it is of considerable interest to obtain an analysis of the shutter action at these short times.

We have then, in effect (see Fig. 4) a condenser discharging through a symmetrically divided inductance and resistance during the time that the original impressed voltage is falling to zero.* The treatment of the spark gap

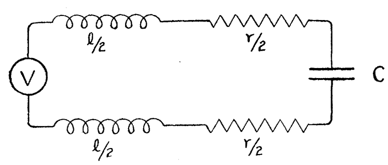

Fig. 4. Simplified shutter circuit.

as a voltage is advantageous not only because it makes possible the utilization of oscillograph data but also because it eliminates the necessity of dealing with the resistance of the spark. In addition it largely takes care of interaction between this shutter circuit and the reservoir condenser-gap circuit because the usual period of the latter is about a hundred times that of the former, and the only appreciable interaction would be the voltage appearing across the gap due to current flowing in the reservoir circuit. Considering, then, the circuit of Fig. 4, the following differential equation may be written:

$$l\frac{di}{dt} + ri + \frac{q}{c} = V_0 e^{-mt}(1 + mt) \tag{7}$$

where $i$ is the current and $q$ the charge on the condenser. With the ordinary circuit notation:

$$b = \frac{r}{2l}, \quad k = (lc)^{-1/2}, \quad i = \frac{dq}{dt}$$

[13] Rogowski and Klemper, Archiv. f. Elektrotechnik **24**, 127 (1930).

* In the treatment by von Hámos,[8] inductance as well as distributed capacity was neglected and the collapse of voltage across the gap was in effect assumed to be instantaneous, so that the problem reduced to the ordinary discharge of a condenser through a fixed resistance. In addition $E_0/E_m$ was assumed to be less than 0.5, so that the transmission was proportional to the fourth power of $E/E_m$.

Eq. (7) may be rewritten:

$$\frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + k^2q = \frac{V_0}{l}e^{-mt}(1 + mt). \tag{8}$$

The solution of this differential equation is

$$\boxed{\frac{q}{q_0} = \frac{E}{E_0} = Ce^{-bt}\cos\left[(k^2 - b^2)^{1/2}t - \theta)\right] + (f + gt)e^{-mt}} \tag{9}$$

where the constants are as follows:

$$f = \frac{k^2[m(3m - 4b) + k^2]}{(m^2 - 2bm + k^2)^2} \tag{10}$$

$$g = \frac{k^2m}{m^2 - 2bm + k^2} \tag{11}$$

$$\theta = \tan^{-1}\left[\frac{mf - g + b(1 - f)}{(k^2 - b^2)^{1/2}(1 - f)}\right] \tag{12}$$

$$C = \frac{1 - f}{\cos\theta}. \tag{13}$$

For convenience in reference, the other symbols will be redefined here in the units in which they will be used: $r$ = resistance in ohms, $c$ = capacity in farads (of Kerr cell), $l$ = inductance in henries, $b = r/2l$, $k = (lc)^{-1/2}$, $m$ = a positive constant, $q/q_0$ = fraction of charge remaining on condenser at time $t$ (in seconds). The origin of time is the beginning of the break in the voltage across the spark gap. $E/E_0$ is the fraction of the initial field existing between the condenser plates at time $t$, $E_0$ being the initial field (volts/cm) as distinguised from $E_m$ which is the field which gives maximum transmission.

The solution of the problem given by Eq. (9) is valid only for an oscillatory discharge but this is the condition of practical interest. The first term in the solution is of the same form as the ordinary solution for instantaneous voltage drop. The second term represents the effect of the gap voltage (see Eq. (6)). The results given by this solution can be best illustrated by a set of numerical examples using as a basis values from a typical experimental set-up. The values that will be used are: $r = 35$ ohms, $l = 0.75\times10^{-6}$ henries, $c = 14.7\times10^{-12}$ farads, $E_0 = E_m$ and $m = 0.21\times10.^9$ The first four are directly measured quantities, the last was obtained indirectly from experimental data as will be explained in Part III. The procedure will be to vary, one at a time, each of the five quantities, holding the other four constant, and to study the effect. Since what is finally desired is not the behavior of $E/E_0$ but of the transmission of the shutter, the values of $E/E_0$ obtained from (9) will be translated into $I/I_0$ by means of (5). The ratio $I/I_0$ will be referred to as the transmission.

Before proceeding to the example, the concept of time must be clearly understood. Since lumped constants were used in obtaining the above solution, no direct account is taken in it of the time $t_w$ required for propagation of the wave from gap to shutter, (i.e. of $L_w/3\times10^{10}$, see Eq. (1)). Mr. H. W. Washburn[14] of this laboratory has made a derivation on the basis of distrib-

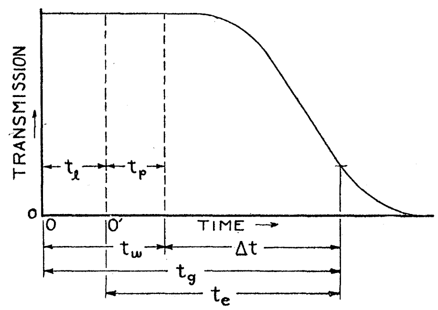


Fig. 5. Relation of various times to each other and to a typical computed transmission curve.

uted constants, which work will be published soon. For short lengths of leads to the Kerr cell, the two solutions give essentially the same results providing the time of propagation is neglected in the author's solutions. In effect then, lumping the constants, automatically allows for the time of propagation to a first approximation. Eq. (9) then gives directly the state of

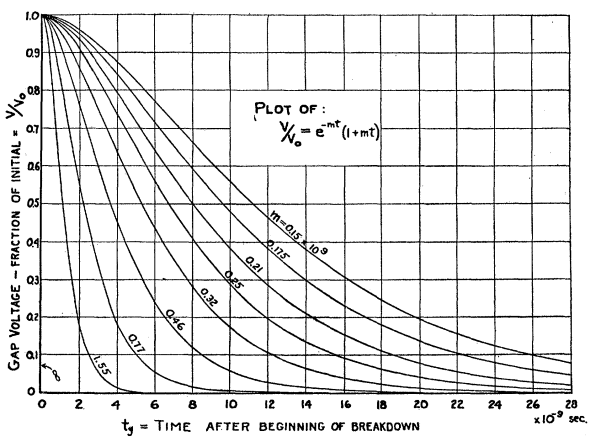


Fig. 6. Various assumed rates of voltage collapse across the spark gap.

transmission of the shutter at various times after breakdown. The $t$ of this solution is the time scale given in all figures in this section and will be referred to *as* the gap time $t_g$. However, *to obtain the effective action of the shutter on the light the time $t_l$ required for the light to travel from gap to shutter must be subtracted from the given times.* In the experimental set-up this amounted to $0.5\times10^{-9}$ sec. This corrected time scale will be referred to as the effective

[14] Washburn, Phys. Rev. **38**, 584 (1931)—a preliminary report.

time $t_e$. Fig. 5 shows a typical transmission curve as obtained from Eq. (6) with the relation between the various times indicated. If the closing of the shutter were instantaneous the transmission curve would be coincident with the right hand dotted line. The interval designated as $\Delta t$ represents the lag in the closing of the shutter to any given state of transmission and is numerically equal to $(t_g - t_w)$ or $(t_e - t_p)$.

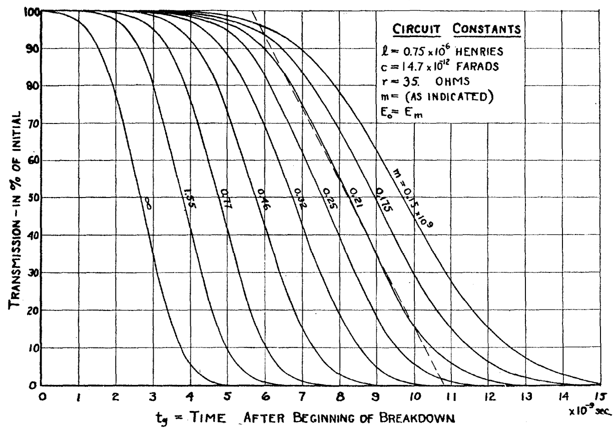


Fig. 7. Effect of varying the rate of collapse of spark gap voltage on the closing of the shutter.

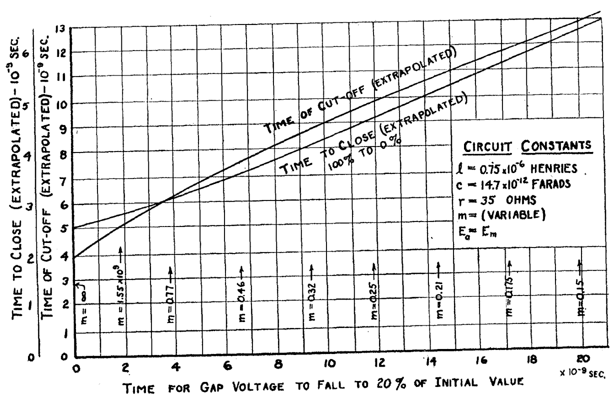


Fig. 8. Relation between rate of voltage fall and (a) time of cut-off of shutter (extrapolated), (b) time for shutter to close (extrapolated) from 100 to 0 percent transmission.

Since the value of $m$, that is of the rate of voltage collapse on the gap, has a large effect on the transmission curves, this will be studied first. Fig. 6 shows a plot of the voltage function (Eq. (6)) for various values of $m$ from $\infty$ to $0.15 \times 10^9$. Thus if $m = 0.21 \times 10^9$, the gap voltage will fall to 20 percent in $14.3 \times 10^{-9}$ sec. Fig. 7 gives the corresponding set of transmission curves. They are seen to be of quite different shape than the exponential curves assumed by previous experimenters. That for $m = 0.21 \times 10^9$ corresponds to the experimental condition. The path time of cut-off (Eq. (1)) for the above example is $t_p = 0.7 \times 10^{-9}$ sec. If a straight line is drawn through the transmission curve for the above $m$, *the extrapolated time of cut-off is* $(10.8-0.5)\times 10^{-9} = 10.3\times 10^{-9}$ sec. (*effective scale*) *as contrasted with the* $t_p$ *of* $0.7\times 10^{-9}$ *sec.* Even with an instantaneous voltage drop ($m=\infty$), the extrapolated cut-off time is $4.0\times 10^{-9}$ sec. In drawing the above straight line, it is tipped slightly to allow for the rounding off of the transmission curve. The amount of tipping necessary was judged from some experimental data in Part III. As would be expected, the more slowly the voltage falls, the later the cut-off occurs. This is shown quantitatively in Fig. 8 which gives directly the relation between the rate of voltage fall as obtained from Fig. 6 and the corresponding extrapolated times of cut-off as obtained from Fig. 7. The times given in this and later tables are gap times. In addition the extrapolated time required for the shutter to close if its action were linear from start to finish instead of from about 90 to 10 percent is shown as a function of the rate of voltage fall. For example for $m=0.21\times 10^{-9}$ sec., the time to close is $(10.8-5.7)\times 10^{-9} = 5.1 \times 10^{-9}$ sec. This time to close is of interest since the time resolution of the shutter depends upon it. The other factor influencing the time resolution is the proportional rate of intensity increase of the light source being studied. This is considered later.

There is another quite important effect to be noted from the variation of the rate of voltage collapse, namely, the pronounced decrease in the oscillations (the damping resistance being held constant) with slower rates of voltage fall. This is shown in Table I in which the peak transmissions of the first and second oscillations are given for a few values of $m$. The first oscillation (i.e. first reversal of voltage) is so suppressed with $m=0.21\times 10^9$ that it is less than the following (positive) oscillation, and with $m=0.15\times 10^9$ it is simply an undulation in the tail of the cut-off curve. A simple mechanical analogy is often helpful in visualizing the behavior of the shutter circuit, namely a simple pendulum suspended in a viscous fluid. The original displacing force can be thought of as being reduced in the same manner as the gap voltage. This force retards the motion of the pendulum toward the equilibrium position, hence the smaller oscillations and their nonsymmetrical character. It is apparent then that *the slower the rate of voltage collapse, the less damping resistance will be required for satisfactory operation.*

TABLE I. *Effect of rate of voltage collapse on magnitude of oscillations.*

| $m$ | Time for voltage to fall to 20 percent $10^{-9}$ sec. | 1st Oscillation | | 2nd Oscillation | |
|---|---|---|---|---|---|
| | | Time of peak $10^{-9}$ sec. | Maximum transmission (percent) | Time of peak $10^{-9}$ sec. | Maximum transmission (percent) |
| $\infty$ | 0 | 10.7 | 66.9 | 21.2 | 30.3 |
| $0.46\times 10^9$ | 6.6 | 14.6 | 17.9 | 25.0 | 7.20 |
| $0.21\times 10^9$ | 14.4 | 18.0 | 0.100 | 27.5 | 0.407 |
| $0.15\times 10^9$ | 19.8 | — | — | 27.8 | 0.0025 |

Before the shutter will function properly it is necessary to adjust the damping resistance to the proper value. The effect of the damping resistance on the transmission curve is shown in Fig. 9. It is seen to be largely one of slowing down of the rate of closing with only a very slight displacement of the curve to later times. This is quantitatively shown by the extrapolated data given in the upper half of Table II. The effect of the oscillations can be judged from the data on the second oscillation given in the same table. The decrease of amplitude with resistance is so rapid that for a resistance

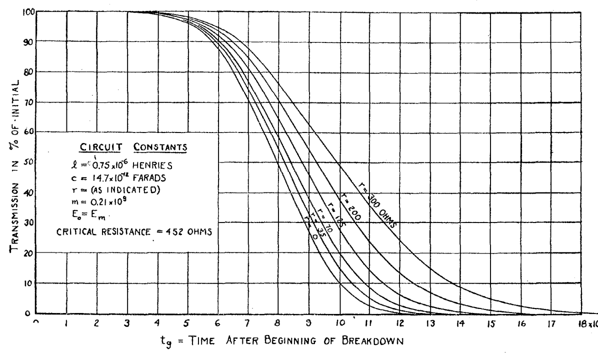


Fig. 9. Effect of varying the resistance in the Kerr cell circuit on the closing of the shutter.

TABLE II. *Data from Fig. 9 and 10 showing effect of varying the damping resistance and the initial field strength.*

| | Variable | Time of cut-off (extrapolated) $10^{-9}$ sec. | Time to close (extrapolated) $10^{-9}$ sec. | 2nd Oscillation: Time of peak $10^{-9}$ sec. | 2nd Oscillation: Maximum transmission (percent) | Initial transmission (percent) |
|---|---|---|---|---|---|---|
| | $r$ | | | | | |
| Fig. 9 | 0 | 10.45 | 4.85 | 27.2 | 3.30 | 100 |
| | 35 | 10.8 | 5.1 | 27.5 | 0.407 | " |
| | 70 | 11.15 | 5.35 | 28.6 | 0.048 | " |
| | 125 | 11.8 | 5.9 | 30.5 | 0.0015 | " |
| | 200 | 12.7 | 6.7 | (30.5) | $(3.8\times10^{-7})$ | " |
| | 300 | 14.0 | 8.0 | (30.5) | $(2.9\times10^{-7})$ | " |
| | $E_0/E_m$ | | | | | |
| Fig. 10 | 0.4 | 10.0 | 6.4 | 27.5 | 0.010 | 6.19 |
| | 0.6 | 10.1 | 6.3 | " | 0.053 | 28.6 |
| | 0.8 | 10.4 | 6.15 | " | 0.235 | 71.0 |
| | 1.0 | 10.8 | 5.1 | " | 0.407 | 100.0 |
| | 1.2 | 11.2 | 3.7 | " | 0.845 | 58.9 |
| | 1.414 | 11.6 | 2.9 | " | 1.63 | 0.0 |

of 200 ohms or greater the second oscillation also becomes merely an undulation in the tail of the transmission curve. The critical resistance (aperiodic discharge) is 452 ohms. With increasing resistance the decrease of extraneous light passed by the oscillation is counteracted by the increase due to the rapid lengthening of the foot or tail of the transmission curve so that an optimum resistance value must be found (see Part III).

As has been mentioned before, the belief that a quicker cut-off is obtained if an initial field strength is used corresponding to a small ratio of $E_0/E_m$ is not founded on fact. The effect of varying the initial field is shown

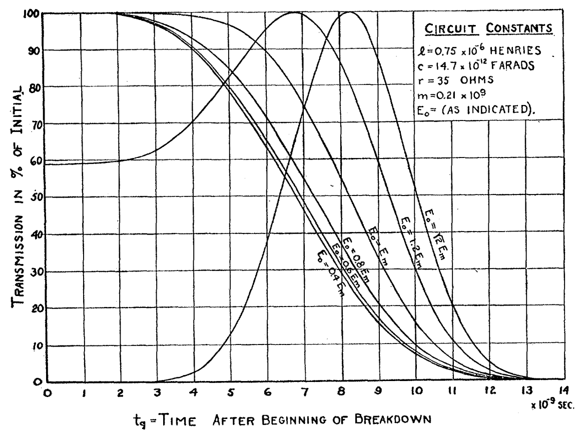


Fig. 10. Effect of varying the initial field (or voltage) in Kerr cell on the closing of the shutter.

in Fig. 10. It is apparent that more rapid cut-offs are obtained with *larger* $E_0/E_m$ ratios, that is, the slope of the transmission curve is greater. This is especially true for values of $E_0 > 0.8\,E_m$. The extrapolated data are given in

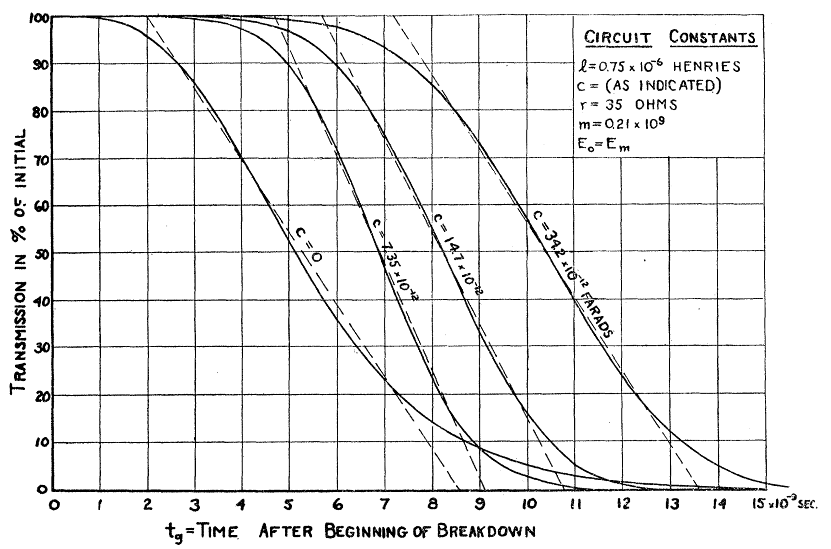


Fig. 11. Effect of varying the capacity of the Kerr cell on the closing of the shutter.

the lower half of Table II, together with that on the second oscillation and the initial transmission. The advantages of higher $E_0/E_m$ ratios are: smaller times to close (which is of the greatest importance when very rapidly changing phenomena are being studied) and far greater light intensities; the disadvantage is the rapid increase of extraneous light due to the increased magnitude of the oscillations. Consideration of the curve $E_0 = 2^{1/2}E_m$ leads to the suggestion that the closed-open-closed action of the Beams[1] type of shutter could be attained with the single cell shutter being considered here. It can, but only with an approximately monochromatic light source, since the Kerr constant changes so rapidly with wave-length.

We now come to the important consideration of the effect of the capacity of the Kerr cell on the shutter action. The manner of closing is shown in Fig. 11 for three capacities, one larger and one smaller than that actually used in the experimental set-up. With increasing capacity there is both a displacement to later times and a decrease in the rate of closing. The former can be allowed for by shorter leads to the shutter but the latter is a disadvantage which can not be corrected, *hence the necessity of using a Kerr cell with as small a capacity as is experimentally feasible.* The extrapolated data are given in the upper half of Table III together with data on all the oscilla-

TABLE III. *Data from Fig. 11 and 12 showing effect of varying the capacity and inductance.*

| | Variable | Time of cut-off (extrapolated) $10^{-9}$ sec. | Time to close (extrapolated) $10^{-9}$ sec. | Oscillation number | Time of peak $10^{-9}$ sec. | Maximum transmission percent | Relative energies passed by oscillations |
|---|---|---|---|---|---|---|---|
| | Capacity $10^{-12}$ farads | | | | | | |
| Fig. 11 | 34.2 | 13.6 | 6.4 | 1 | 24.5 | 2.45 | 8.30 |
| | | | | 2 | 40.2 | 0.87 | |
| | | | | 3 | 56.5 | 0.145 | |
| | | | | 4 | 73.0 | 0.063 | |
| | 14.7 | 10.8 | 5.1 | 1 | 18.0 | 0.100 | 1.00 |
| | | | | 2 | 27.5 | 0.407 | |
| | | | | 3 | 38.5 | 0.103 | |
| | | | | 4 | 48.5 | 0.042 | |
| | | | | 5 | 59.5 | 0.015 | |
| | 7.35 | 9.1 | 4.4 | (1) | (14.2) | 0.003 | 0.49 |
| | | | | 2 | 19.7 | 0.400 | |
| | | | | 3 | 28.0 | 0.016 | |
| | (0) | (8.6) | (6.6) | — | — | — | 0.01 – |
| | Inductance $10^{-9}$ henries | | | | | | |
| Fig. 12 | 1.80 | 14.0 | 6.7 | 1 | 25.5 | 1.18 | 3.19 |
| | | | | 2 | 41.3 | 0.184 | |
| | | | | 3 | 25.5 | 1.18 | |
| | 0.75 | 10.8 | 5.1 | (Same a | s above for | $c = 14.7$) | 1.00 |
| | If $m = \infty$ | | | | | | |
| | 1.80 | 6.4 | 4.3 | | | | |
| | 0.75 | 4.0 | 2.65 | | | | |

tions of any appreciable magnitude. An increase in capacity is seen to cause a large increase in the oscillations, the damping resistance remaining constant. The last column of Table III gives an approximate idea of light energies (i.e. extraneous light) passed by the oscillations. The values are calculated on the basis of a constant intensity source. The extraneous light is seen to decrease rapidly with decreasing capacity. The curve on Fig. 10 marked $c=0$ is the mathematical result obtained if the capacity of the cell is assumed zero. Actually, this is not a valid procedure since the basic assumption that the cell capacity is large with respect to the distributed capacity of the leads is no longer true. The curve, however, is of interest since it represents the manner of closing if the Kerr cell voltage exactly followed the gap voltage. Comparison with the other curves and a study of the behavior of the terms in Eq. (6) show the advantageous effect of the oscillatory condition of the circuit, namely that the oscillatory term *adds* to the upper part of the curve and, with sufficiently small capacities, *subtracts* from the lower part, thus doubly causing a quicker cut-off.

We will consider now the last of the five variables, namely inductance. Experimentally, changing the inductance is normally equivalent to chang-

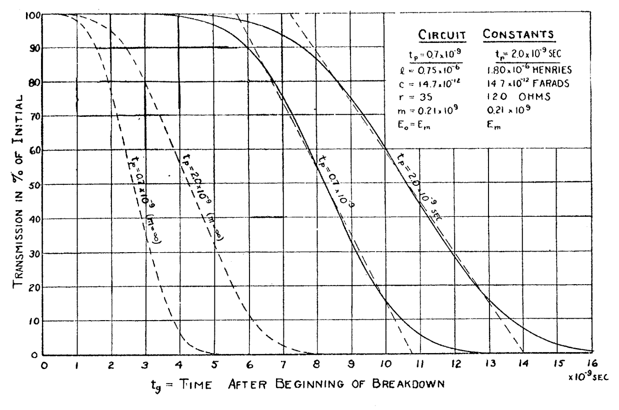


Fig. 12. Manner of shutter closing for two values of inductance (equivalent experimentally to the two path times $t_p=0.7$ and $2.0\times10^{-9}$ sec.) Dotted curves give corresponding manner of closing if voltage drop were instantaneous.

ing the length of leads to the shutter, or in other words to changing the path time $t_p$. It is true that leads of a given length may be geometrically arranged to give different inductances, but if the requirement of low distributed capacity is met, there is little leeway for change in inductance of the short leads being considered in this paper. The previous statement that lumping the constants allows for the time of propagation to a *first approximation* is pertinent at this point. Fig. 12 shows the manner of closing for two inductances corresponding to the path times indicated. Similar to the change with increasing capacity there is to be observed both a displacement of the transmission curve to later times and a decrease in the rate of closing. A displacement equal to the increase in path times, namely (2.0–0.7) or $1.3\times10^{-9}$ sec. is to be expected (see Fig. 5) but the calculated amount judged from the straight line intercepts at 100 percent is $1.6\times10^{-9}$ sec. This, plus the increase in rate of closing results in a retardation in the time of cut-off of $(14-10.8)$ or $3.2\times10^{-9}$ sec. (see lower half of Table III). This serves to emphasize a fundamental defect in the electrooptical shutter, namely, that *the rate of closing decreases as the path time of cut-off is increased*. It is to be noted that only in this last case have two electric constants been varied in the same illustration. The resistance was varied as well as the inductance to correspond to the best experimental conditions, as these curves will be

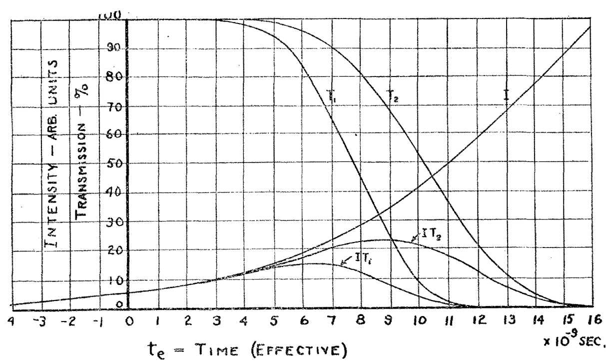


Fig. 13. Effectiveness of shutter closing when the light source is rapidly increasing in intensity. The areas under the $IT$ curves represent the energies transmitted.

checked against experimental data in Part III. From the last column of Table III, it is seen that even under the best experimental conditions the amount of extraneous light passed by oscillations increases with inductance. The dotted curves of Fig. 12 give the manner of closing for the conditions discussed above except with $m = \infty$.

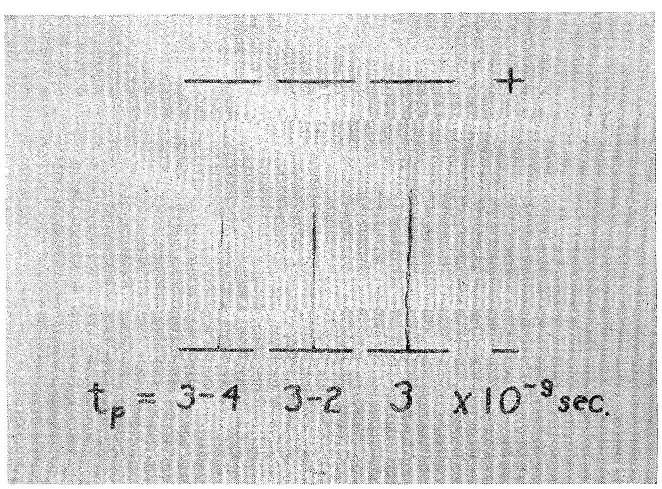


Fig. 14. Change in appearance of a spark in steps of $2\times10^{-9}$ sec. Pressure = 76 cm of Hg, gap length = 5 mm.

The curve representing the light transmitted by the shutter will be similar in form to the transmission curves only if the light source is of constant intensity, a condition not usually found in practice. Fig. 13 illustrates the conditions when the intensity is rapidly increasing. The time scale has been corrected for the light path. The two transmission curves $T_1$ and $T_2$ are the same as those in Fig. 12, while the intensity curve $I$ has been estimated from experimental data and corresponds to the growth of the central bright region of the spark illustrated in Fig. 14. The intensity transmitted is represented for the two cases by the curves $IT_1$ and $IT_2$, and the energies of the transmitted pulses of light are proportional to the areas under the $IT$ curves. It is seen that a majority of the energy is passed in a region near the peak of the $IT$ curves, especially for later times of cut-off. Hence, to the eye or a photographic plate, what happens during a time interval of a few billionths of a second—the interval occurring somewhere along the upper half of the transmission curve—is predominant and what happens before and after is hardly seen. Thus, with a rapidly increasing intensity, the closed-open-closed effect of the Beams[1] type of shutter is approximated. An idea of the time resolution obtained under the foregoing conditions can be gained directly from the ordinates of the $IT$ curves. Considering the $IT_1$ curve the transmitted energy representing the phenomenon at $11\times10^{-9}$ sec. (consider say a time interval of $0.1\times10^{-9}$ sec. duration) is only 17 percent of that at $8\times10^{-9}$ sec., that at $12\times10^{-9}$ sec. is only 4.5 percent of the same. *The theory thus indicates a time resolution of the order of 3 or $4\times10^{-9}$ sec. with a rapidly increasing intensity, and about 5 to $6\times10^{-9}$ sec. with a constant intensity source (see data on "times to close" in Table III for Fig. 12).*

## Part III. Experimental Results Bearing on the Action of the Shutter

By use of the electrooptical shutter to study the early stages of spark breakdown, three classes of experimental data have been obtained which yield information of use in checking the foregoing theory: first, visual studies on the difference in appearance of sparks over intervals of a few billionths of a second, which give data on the time resolution; second, photographic data giving the time of the extrapolated cut-off; and third, measurements giving the change in the time of cut-off with certain shutter conditions. Unless otherwise stated all the experimental data were taken under the following set of conditions: air pressure in gap = 76 cm of Hg; gap length = 5 mm; time for reservoir condenser energy to begin to feed into gap $=5\times10^{-9}$ sec. (determined by distance of $C$, Fig. 2, from gap); capacity of Kerr cell = 14.7 $\times10^{-12}$ farads; for path time $t_p=0.7\times10^{-9}$ sec., inductance of shutter circuit $=0.75\times10^{-6}$ henries and resistance = 35 ohms; for $t_p=2.0\times10^{-9}$ sec., inductance $=1.80\times10^{-6}$ henries and resistance = 120 ohms. In all cases the spark breakdown occurred in completely dried air, the cathode was illuminated with ultraviolet light, and the reservoir condenser with a capacity of $0.008\times10^{-6}$ farads was connected to the gap through a linear distributed resistance of 50 ohms.

### First class.

A typical visual study of the change in appearance of a spark with time is shown in Fig. 14. The path times are given at the bottom, the first number (3) meaning the ordinary path time as computed by Eq. (1), while the subtracted number is the time required to travel the additional light path introduced by the second optical system (see Fig. 2). Thus the development is shown in steps of $2\times10^{-9}$ sec. What is of interest here is not only the change in general appearance—the central region connecting with the cathode—but also the change in detail, small spurs and irregularities appearing both in the newly developed region in front of the cathode and also in the central region itself. A growth merely in length and brilliance of the central region would not necessarily yield any data on the time resolution since the displacement to later times of a very slowly falling transmission curve would allow more energy to pass and thereby increase the brilliance and observed length. *But the observed change in both form and detail certainly requires a time resolution of the order of $4\times10^{-9}$ sec. or less. This agrees with the theoretical estimate of 3 to $4\times10^{-9}$ sec. made from Fig. 13 for somewhat similar conditions.*

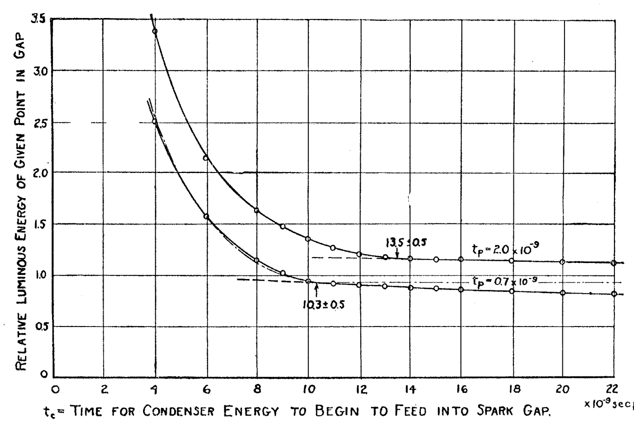


Fig. 15. Effect of variation of the distance of the reservoir condenser from the spark gap on observed luminous energy of the central region of breakdown. Shown for two times of cut-off: $t_p = 0.7$ and $2.0\times10^{-9}$ sec. (Chain curve is computed effect for $t_p = 0.7\times10^{-9}$ sec.)

### Second class.

The results of a photographic study giving directly the times of extrapolated cut-off for two path times are presented in Fig. 15. If the intensity of the central region of a spark such as in Fig. 14 is observed at a given path time it will be found to vary with the distance of the reservoir condenser ($C$, Fig. 2) from the gap. If this condenser is removed entirely the only electrical energy available to maintain a voltage across the gap is the charge stored on the electrodes and on the adjacent connecting wires. During breakdown this will feed into the gap, there resulting a certain rather rapidly falling voltage-time relation and a corresponding intensity-time curve for any given point in the gap. If the reservoir condenser is now placed at a moderate distance from the gap, everything during breakdown proceeds as before up the old time $t_c$ at which the energy from the condenser begins to feed into the gap. Assuming the speed of propagation along the connecting wires is that of light, $t_c$ is given by

$$t_c = \frac{2 \times (\text{length of a connecting wire in cm})}{3 \times 10^{10}} \tag{14}$$

since the wave must go to the condenser and back. From this time on (at least for a certain period) the voltage on the gap is higher than formerly, there being more energy fed in, and as a consequence the light intensity at the given point is also greater. If $t_c$ is greater than the extrapolated time of cut-off (effective time scale), there will still be no observed increase of intensity since the shutter is closed before the increase occurs; but if the condenser is now moved towards the gap a point will be found on the near side of which there is a continuous increase of intensity. This point corresponds to the condition of equality between $t_c$ and the extrapolated cut-off time.

The procedure experimentally was to move the condenser step by step from a point as close as feasible to a distance as great at 10 or 11 meters, taking a composite photograph of the spark at each step. This series of photographs (negatives) was photometered and the density recorded of the given point in the gap (chosen as approximately the center of the central bright region, see Fig. 14). These densities were then translated into relative energies by a density energy curve, the latter being obtained from series of composite pictures taken through diaphragms of various sizes, the number of exposures on each composite being kept constant, to eliminate any intermittency effect. The averaged results of four runs for each $t_p$ (two taken with the condenser moving away from gap, and two vice versa) are shown in Fig. 15. There is seen to be a definite point of inflection which point determines the effective time of cut-off.

That the shape and position of the transmission curve for $t_p = 0.7 \times 10^{-9}$ sec. given in Fig. 12 is substantially correct was proved in the following manner. A rate of voltage fall (i.e. a value of $m$) was chosen for this curve of Fig. 12 so that with the other constants corresponding to the condition $t_p = 0.7$, there resulted a theoretical transmission curve with an extrapolated cut-off time of $10.3 \times 10^{-9}$ sec. (see Fig. 8). Then assuming that the *increase* in instantaneous intensity produced by the condenser energy beginning to feed into the gap is linear with time and is zero at the time $t_c$, a series of such $\Delta I$ curves was drawn, the curves being parallel with feet at times corresponding to various $t_c$ times. The product of ordinates of any one $\Delta I$ curve with the transmission curve gives the transmitted $\Delta I$ curve, and the area under it is proportional to the transmitted increase in energy. The plot of these areas, reduced to a comparable scale, gives the chain curve of Fig. 15, which is seen to be almost coincident with the experimental curve including the location of the inflection point. This is a fairly good check on the steepness of the calculated transmission curve since a steeper cut-off would give a more rapidly rising chain curve and vice versa. The uncertainty in the check is the rate of increase in $\Delta I$. It also constitutes a double check on the rate of fall of voltage chosen in the foregoing manner, since not only does the rate of fall determine the location of the inflection point but also the steepness of the transmission curve (see Fig. 7 and "Time to Close," Fig. 8).

That the extrapolated time of cut-off obtained as previously described by drawing a straight line through the transmission curve at a slight angle to its front (see Fig. 12) corresponds to the effective time of cut-off as given experimentally by the point of inflection of the curves of Fig. 15 is proved as follows: a new computation of the relative energy increase is made exactly as before but using this straight line instead of the computed transmission curve. If the straight line is slightly tipped as shown in Fig. 12, the inflection point of this last energy increase curve can not be distinguished from the former (chain curve). This is to be expected from the fact that the *additional* energy passed by the tail of the transmission curve is proportional to the product of two small quantities and is therefore a quantity of the second order. However, this result is not to be interpreted as meaning that the tail of the transmission curve can be neglected in general, since the *total* energy passed by it is proportional to the product of a small and large quantity (the total intensity) and hence is of the first order, though usually small.

The possible existence of an error in the foregoing method for determining the effective time of cut-off should be mentioned. It is possible that an appreciable time lag exists between the *beginning* of the voltage change on the gap due to the condenser wave and the *beginning* of the intensity change due to it, the lag being due to a time interval between excitation and emission. However, the close agreement between the chain and experimental curves of Fig. 15 seems to indicate that the lag is small, that is, it would be of the order of $10^{-9}$ sec. rather than $10^{-8}$ sec. For example, if the lag amounted to even $5\times10^{-9}$ sec. the time to close (calculated) would be increased to 1.6 times its former value, and the resulting chain curve would be much too straight to fit the experimental curve. It is still possible though that the shape of the actual $\Delta I$ curve would compensate for this. However, the reduced time resolution weighs against this, as does the change in the time of cut-off with inductance considered in the next paragraph. A second possible source of error is from the fact—as pointed out before—that the manner of the latter part of the voltage collapse is effected by the position of the reservoir condenser. The increase in voltage occurring when the condenser energy begins to feed in is probably temporary and followed by a more rapid drop due to the increased conductivity. This drop might even make the voltage less at later times than before, so that the net result on the shutter action is problematical. In any case the experimental data sets an approximate lower limit to the effective time of cut-off.

A further check on the theory is found in the movement of the effective cut-off (Fig. 15) with an increase in inductance, or $t_p$. The effective times of cut-off as obtained from Fig. 15 are $10.3\times10^{-9}$ sec. at $t_p=0.7\times10^{-9}$ sec. and $13.5\times10^{-9}$ sec. at $t_p=2.0\times10^{-9}$ sec. These are to be compared with the extrapolated times of cut-off of the curves of Fig. 12 which were computed for the same conditions. It is to be recalled that the rate of fall of voltage for the given spark condition was determined in Fig. 12 by picking that rate for which the $t_p = 0.7$ curve gave an extrapolated cut-off time in agreement with the above experimental value of $10.3 \times 10^{-9}$ sec. (see Fig. 8). Then using this rate of fall—the rate should be reasonably independent of the shutter adjustment—the curve for $t_p = 2.0 \times 10^{-9}$ sec. was computed and was found to have an extrapolated cut-off time of $(14.0 - 0.5)$ or $13.5 \times 10^{-9}$ sec., in exact agreement with experimental data. This perfect agreement is probably accidental. All experimental results were completed before the theoretical calculations, and the accuracy of neither is sufficient to lead to quite such close agreement. However, the agreement constitutes another good check on the correctness of the theory and the rate of voltage fall used, for faster rates of fall give smaller changes in cut-off time and vice versa for slower rates. For example if the voltage required $28.5 \times 10^{-9}$ sec. to fall to 20 percent, instead of $14.3 \times 10^{-9}$ sec. as above, the *change* in the extrapolated cut-off time would be $3.7 \times 10^{-9}$ sec. Further data from this example are of interest in considering the possible error discussed above. The extrapolated cut-off time for this example is $15.5 \times 10^{-9}$ sec. instead of the former value of $10.3 \times 10^{-9}$ sec. Hence the example corresponds to a possible excitation lag as discussed above of $5.2 \times 10^{-9}$ sec. The extrapolated time to close is $8.4 \times 10^{-9}$ sec. as compared with the former $5.1 \times 10^{-9}$ sec., a 65 percent increase. This would increase the theoretical estimate of the time resolution to about 5 or $6 \times 10^{-9}$ sec., which is to be compared with the experimental data indicating something less than $4 \times 10^{-9}$ sec.

**Third class.**

Another check on the theory was made as follows: a change in the Kerr cell capacity was effected without changing the optical constants of the original cell by connecting a second cell across the first, using very short leads. The second cell thus functioned only as a capacity, providing an increase from $14.7 \times 10^{-12}$ farads to $34.2 \times 10^{-12}$. The best damping resistance for the larger capacity proved to be the same as that for the smaller. The spark breakdown was observed visually first with the smaller capacity and with $t_p = (0.7 - 2.0) \times 10^{-9}$ sec., (i.e. using the longer or mirror optical system—see Fig. 2—with an additional light path of $2 \times 10^{-9}$ sec.). With the larger capacity the light path had to be increased an equivalent of $2.4 \times 10^{-9}$ sec. (average of 10 readings) to obtain the same stage of development as before, which means an increase in the effective time of closing of that amount. The increase measured in this manner is not exactly that of the extrapolated time of cut-off but a sort of mean displacement of the transmission curve (in this connection study the *IT* curves of Fig. 13). The calculated transmission curves for these two capacities are given in Fig. 11. The mean displacement between them is $2.2 \times 10^{-9}$ sec., which agrees well with the measured value of $2.4 \times 10^{-9}$ sec. At larger values of $t_p$, the measured increase in lag is much greater. For example at $t_p = 16 \times 10^{-9}$, it is $9 \times 10^{-9}$ sec.

The best damping resistance for any given shutter arrangement must be determined experimentally. The experimental data on which it was determined for the two conditions $t_p = 0.7$ and $2.0 \times 10^{-9}$ sec. are shown in Figs. 16, 17, and 18. For any given $t_p$, the photographic method is to take a series of composite pictures with various damping resistances. The photometered curves of a few of these at $t_p = 0.7 \times 10^{-9}$ sec. are given in Fig. 16. Visually

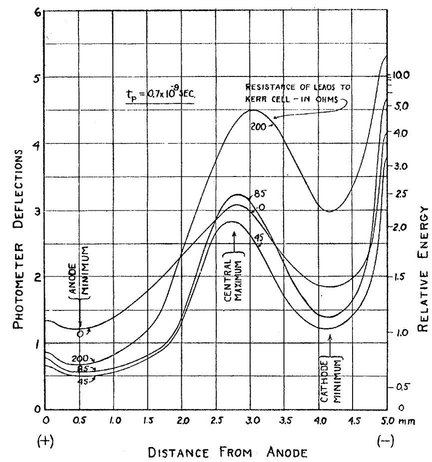


Fig. 16. Photometer curves showing energy distribution observed in gap at $t_p = 0.7 \times 10^{-9}$ sec. for various damping resistances.

this stage of development (with resistance around 35 ohms) appears about the same as the left figure in Fig. 14. It is to be noted that visually the bright regions (center and cathode) appear another order of magnitude brighter,

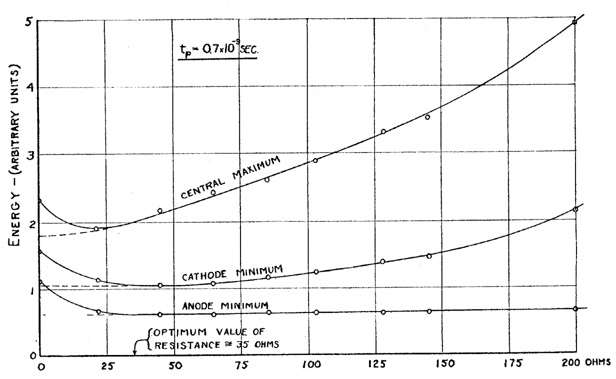


Fig. 17. Effect of varying the damping resistance on the luminous energy at three critical points in the gap (see Fig. 14). $t_p = 0.7 \times 10^{-9}$ sec.

relative to the anode region, than the energies as read from the photometer curves would indicate. Discussion of this with Professor R. S. Minor of the optometry department indicates that it is probably due to an overshooting of the brightness sensation, which phenomenon increases rapidly with energy.[15] Considering the fact that in the development of this type of breakdown the central region first connects with the cathode and much later with the anode, or expressed differently, since the rate of intensity increase is rapid in the cathode minimum region and quite slow in the anode minimum region, it is possible to differentiate in Fig. 16 between variations due to changes in the oscillations and changes in the rate of cut-off. Thus the drop in transmitted energy between zero and 45 ohms resistance is approximately the same at all three critical points (see arrows), namely, about 0.4, and is due to a large decrease in oscillation (see Table II)—the decrease in the speed of cut-off being negligible (see Fig.9)—which oscillations pass light from the completely developed spark. Further increases in resistance cause rapid increases of energy at the cathode minimum but only very small increases at the anode minimum. This is due to the increasing slowness of cut-

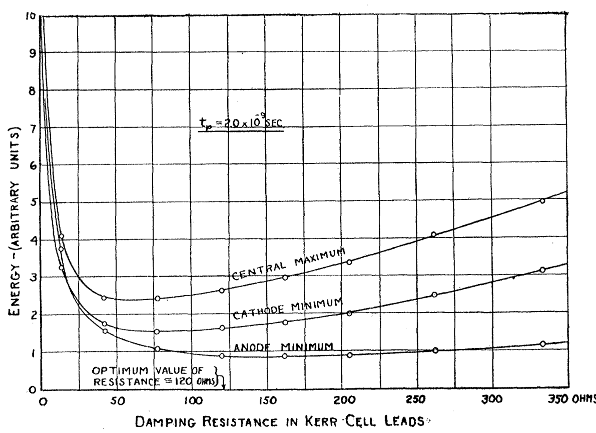


**Fig. 18. Effect of varying the damping resistance on the luminous energy at three critical points in the gap (see Fig. 14). $t_p = 2.0 \times 10^{-9}$ sec.**

off combined with the relative rates of intensity increase, the decrease from oscillations now being negligible. The behavior of the three critical regions is more clearly shown by plots of their ordinates as in Fig. 17 (averaged data from several runs). If the energy passed by oscillations—equal to the difference between the anode minimum curve and its extrapolated straight line (dotted)—is subtracted from the other two curves the results represent quite closely the effect from zero to 200 ohms of the slowing down of the speed of cut-off with resistance. It is apparent that the net result also depends on the rate of intensity increase. The optimum value of resistance was chosen as that at which the anode minimum curve first falls substantially to its minumum, namely 35 ohms. The visual effect of small energy changes in this anode region is more evident than several times the same energy change in the central region, due probably to a saturation effect from the brilliance of the latter.

[15] Broca and Sulzer, Jour. d. Physiol. et d. Path. Gen. No. 4, July 1902 quoted in "Light and Work" by Luckiesh, p. 167.

Fig. 18 shows a similar set of curves for $t_p = 2.0 \times 10^{-9}$ sec., the optimum resistance value was chosen as 120 ohms. This is 17.2 percent of the critical resistance while the 35 ohms at $t_p = 0.7 \times 10^{-9}$ sec. is only 7.7 percent of the corresponding critical resistance. If the period of the shutter circuit were infinitely short, the shutter would exactly follow the control voltage and there would be no oscillations, hence no need of any damping resistance. As the period is lengthened, the tendency to oscillate increases, hence the required percentage of the critical camping resistance $2(l/c)^{1/2}$ also increases.

Data can be obtained from Fig. 17 and 18 to check roughly the assumed intensity curve of Fig. 13. With the optimum damping resistance, the maximum energy of the central region is 2.00 for $t_p = 0.7$ and 2.65 for $t_p = 2.0$, the ratio of the first to the second being 1.33. The ratio of the energies represented by the $IT$ curves of Fig. 13 is 1.64. This means that the intensity

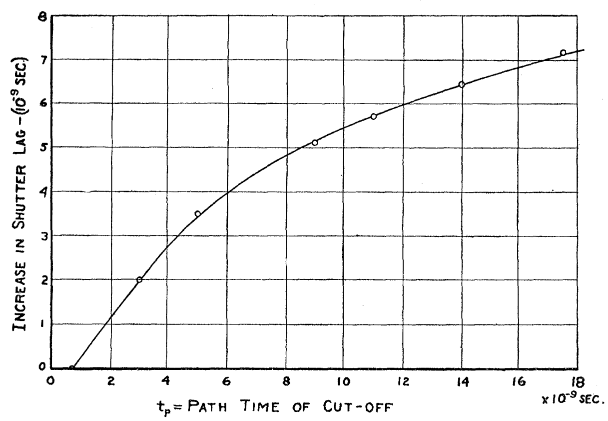


Fig. 19. Increase in shutter lag with path time of cut-off, relative to the lag at $t_p = 0.7 \times 10^{-9}$ sec. Data taken at a gap length of 5 mm and a pressure of 76 cm of Hg.

curve was assumed to rise a little too rapidly beyond say $5 \times 10^{-9}$ sec., or that the part of the curve to the left of $5 \times 10^{-9}$ sec. was assumed too low (i.e. the whole curve might be displaced upward a little).

Since the effective time of cut-off increases more rapidly than the path time of cut-off, a correction curve is needed which can be used to correct the path times for their *increase* in lag relative to an arbitrarily chosen time. Such a curve is given in Fig. 19 for the spark conditions given at the beginning of this section, that is for a pressure of 76 cm Hg and gap length of 5 mm. The curve gives the increase in the *effective* time of closing relative to that at $t_p = 0.7 \times 10^{-9}$ sec. Thus at $t_p = 2 \times 10^{-9}$ sec. the time of closing is $(2 - 0.7 + 1.2)$ or $2.5 \times 10^{-9}$ sec. later than that at $t_p = 0.7 \times 10^{-9}$ sec. It is of interest to note the agreement of this value with that from the two corresponding theoretical curves of Fig. 12. The change in the effective time of closing as judged by their mean time displacement is $2.4 \times 10^{-9}$ sec. The increase in the correction is seen to be rapid at first, then less rapid as the shutter circuit conditions go over into those which must be treated as a transmission line.

The curve was obtained through use of the second or longer optical system (see Fig. 2). For example, if $t_p$ is made $6\times10^{-9}$ sec., an increase of $(6-0.7)$ or $5.3\times10^{-9}$ sec., and an increase in the light path equivalent to $9.3\times10^{-9}$ sec. is required to compensate, then the increased lag at $t_p=6\times10^{-9}$ sec. is $(9.3-5.3)$ or $4.0\times10^{-9}$ sec.

In summary it may be said that the experimental data given in the foregoing provide many interlocking checks on the theory presented in Part II. The most important of these are: the agreement between the theoretically estimated and experimentally determined time resolution; the correctness of the general shape and position of the calculated transmission curves as indicated by the agreement between the chain curve of Fig. 15 (based on one of these calculated transmission curves) and the corresponding experimental curve of the same figure, and also as indicated by the agreement between calculated and experimental data as to the time movement of these curves with changing inductance and capacity; the above agreements also provide a check on the rate of voltage collapse of the spark gap as determined by the method given which involves both experimental data and the theory.

## Part IV. Design and Technique

The general form of the Kerr cell developed in the present study is shown in Fig. 20. The body $T$ was made of Pyrex tubing, the brass plates $P$ were supported by heavy tungsten leads $L$ sealed into the glass. The windows $W$ are described later. The reservoir chamber $R$ provides a means of *completely* filling the cell proper with liquid as well as an air cushion to take care of temperature expansion.

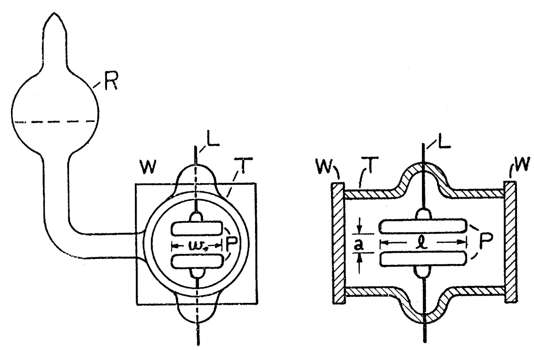


Fig. 20. Form of Kerr cell, showing end view and longitudinal cross section.

The size and spacing of the plates must first be determined. Both theory and experience indicate that the usable voltage range of a cell considering the factors of intensity, quickness of cut-off and extraneous light is that which gives an initial field of from 75 to 100 percent of $E_m$ (i.e. the field which gives maximum transmission)—or certainly not more than 65 to 110 percent $E_m$. Knowing then the maximum voltage $V_0$ (e.s.u.) at which the cell is to be used and assuming a plate spacing $a$ (in cm), Eq. (4) gives the necessary length $l$ (in cm) to a first approximation:

$$l = \frac{a^2}{2bV_0^2}. \tag{15}$$

The width $w$ of the plates together with their separation, determines the aperture for light and the electrostatic capacity. A large aperture is desirable since greater intensity can be attained (providing the cross section of the nicol prisms is not the limiting factor) but this must be made secondary to reduction of the electrostatic capacity to a reasonable minimum, as this last determines the quickness of cut-off (see Table III). Neglecting the edge effect, which may be very large, the capacity mmf is given by

$$C = 0.0885k\frac{wl}{a} \tag{16}$$

where $k$ = dielectric constant (about 37* for nitrobenzene), and all dimensions are in cm. Putting Eq. (15) in this gives

$$C = 0.0885\frac{k}{2bV_0^2}wa. \tag{17}$$

Thus the capacity can be *decreased* by decreasing the separation. This also decreases the length. *The determination of the size and spacing of the plates is therefore a compromise between obtaining the minimum electrostatic capacity and maximum usable aperture, and the limiting conditions are respectively the maximum field which the liquid will stand and the maximum length of cell which can be used without certain optical troubles, especially distortion, becoming pronounced.*

As an example the dimensions will be given of the cell used as an illustration in the present study. They are: plate length 1.0 cm, width 1.2 cm, separation 0.5 cm. This cell had a $V_0$ of approximately 16.4 kv and a measured capacity of 14.7 mmf.

If the mean electric field is in the region of the breakdown value (for nitrobenzene a variable depending on degree of purity but of the order of 45,000 volts/cm or higher) it is essential that the edges of the plates be rounded. However, rounding the edges of the plates destroys the accuracy of Eq. (3) so that $E_0$ can not be calculated.

The windows of the Kerr cell have presented more or less trouble. There are two essential requirements: the surfaces of the windows must have a *very* high degree of planeness and be parallel to the same degree; and second, the glass must be strain free. The first requirement results from the large prismatic action by the nitrobenzene on all irregularities in the outer window surfaces and the consequent blurring of detail in the image; the second requirement is apparent from the location of the cell between crossed nicols. The best solution found is the type of plate glass used in making shatter-proof windshields. It can be obtained in $\frac{1}{8}$ inch sheets, and satisfies the first

* International Critical Tables, Vol. II, p. 89.

requirement perfectly. It is not strain free, but specimens can be found in which the degree of strain is very slight. The only other glass found that was of sufficient planeness was that in a selected lot of plano-ophthalmic lenses. The average plano lens produces a complete blurring of all detail due to the surface irregularities.

The windows are fastened to the body of the cell by water glass (sodium silicate). Before filling with the liquid the cell should be thoroughly baked out (at a temperature below that which will cause dehydration of the sodium silicate) to remove all the moisture possible. The cell should be sealed off after filling. The use of water glass has proved rather unsatisfactory due to its contamination by moisture of the liquid used. A search is being made for a better substance.

The most important part of the Kerr cell is the liquid used in it. Since nitrobenzene has a far greater Kerr constant than any other known substance, it alone will be considered. As purchased commercially, it is in an exceedingly impure state. To obtain a substance pure enough to function at all satisfactorily the liquid should be treated with a weak base such as aluminum or calcium oxide to remove acids and then distilled very slowly in vacuum (retaining the middle third), during which process the water vapor should be frozen out with liquid air. A far superior product results if the distillation is repeated a second time.

Experimentally the effect of impurities in the nitrobenzene are quite evident since they seem to cause a slowing down of the speed of closing of the electrooptical shutter. This is especially evident in the increase of the extraneous light when the early stages of a spark breakdown are being observed. The effect visually is striking. For example, with a gap of 5 mm at atmospheric pressure and $t_p = 1 \times 10^{-9}$ sec. the region consisting of the anode third of the gap appears almost dark in comparison to the bright central region when sufficiently pure nitrobenzene is used. With an impure sample, any degree of luminosity up to about a third or half that of the central region can be obtained depending on the amount of impurity. A very slight trace of moisture destroys the quickness of cut-off, hence the conductivity of a cell seems to be a fairly good indication of the performance that may be expected of it. The conductivity of pure nitrobenzene is given by Möller[11] as $7 \times 10^{-11}$ mhos/cm$^3$. The conductivity of the nitrobenzene used in this study was not measured after its distillation. However, in a cell a year old that was still performing satisfactorily it was $2.5 \times 10^{-10}$ mhos/cm$^3$ and in one passing a large amount of extraneous light it was $1.2 \times 10^{-9}$.

An electrolytic purifying action mentioned by Möller[11] has been noticed throughout the study. Upon the application of a direct current voltage there is a sudden rush of current. This will drop back to a steady value in a few seconds if the liquid is reasonably pure. If, however, it requires a minute to several minutes the cell will be found to have a poor cut-off. It has been found that even with a cell in which the nitrobenzene is reasonably pure there is considerable extraneous light passed if the cell is operated during the two or three seconds required for the electrolytic purification. This peculiarity must be considered in photographic work. Erroneous results are avoided by first holding the potential slightly under the breakdown value for a sufficient time.

A factor of major importance is the adjustment of the damping resistance of the Kerr cell circuit to the optimum value. Since the correct value can not be exactly calculated with our present knowledge, the adjustment must be made experimentally. The adjustment is a compromise between the condition of no resistance which gives the quickest possible cut-off but large amounts of extraneous light and the condition of critical damping which eliminates all oscillations but causes a very slow cut-off. It can be made photographically in the manner described in Part III, or visually with somewhat less accuracy by studying the same intensity changes considered in Fig. 16 and 17.

The correct resistance must be obtained for each Kerr cell and for each length of leads to it, the value for short leads being normally between 5 and 25 percent of the critical resistance. For exact work, the adjustment should be made at the pressure and gap length to be subsequently used as the rate of fall of the spark gap voltage varies with gap length and pressure and hence the necessary damping resistance also varies (see Table II). It is hardly necessary to say that the damping resistances must be distributed linear resistances symmetrically spaced, a half on either side of the cell. An assortment of chromel high resistance wire was used in the present study with resistances as high as seven ohms per cm. At the frequencies encountered in the present work, the radio frequency resistance is almost identical with the direct current resistance.*

There remains to be mentioned one essential experimental condition which has apparently been neglected by all previous experimenters. To provide a path of variable length between spark gap and Kerr cell, a four wire "trolley" with two moveable jumpers is usually used. With such an arrangement there is normally a considerable length of the trolley system which, though not functioning as a part of the path between gap and shutter, is nevertheless electrically connected to the circuit. This unused section of the trolley causes a large delay in the closing of the shutter. The action is probably quite involved but the essential factor is the electrostatic capacity of the unused section. This constitutes in effect a condenser connected across the mid-point of the leads to the Kerr cell and which must be discharged before the shutter can close. An actual example will illustrate the magnitude of its effect. A four wire trolley with wires 4.5 meters long provided a 9 meter path from gap to shutter and hence gave a path time of cut-off $30\times10^{-9}$ sec. (the light path was so small it can be neglected in this discussion). The connection of an additional 8 meter section of trolley onto the above without moving the trolley (i.e. leaving the path still 9 meters) caused a delay in the cut-off of about $12\times10^{-9}$ sec., that is, caused an increase of

* The values of the ratio r.f./d.c. resistance found by von Hamos (reference 8) are apparently somewhat in error since what was actually measured was the r.f. resistance of the leads *plus* the effective spark resistance.

40 percent in the path cut-off time. A 4 meter section gave a retardation of about $6\times10^{-9}$ sec. An obvious experimental way to avoid the difficulty is to divide the trolley into many short insulated sections, and connect with short jumpers as many sections as are needed.

## PART V. DISCUSSION

The nitrobenzene Kerr cell as developed in the present work has several important advantages over the carbon bisulphide cell used in previous work. The most important of these is probably the reduction in capacity with the consequent faster cut-off. Eq. (17) shows that cells designed for the same voltage $V_0$ and having the same aperture for light $wa$ but using various liquids will have capacities proportional to the ratio $k/b$ where $k$ is the dielectric constant and $b$ the Kerr constant. This ratio for nitrobenzene is $1.07\times10^6$ and for carbon bisulphide $7.65\times10^6$, thus the capacity of the later would be approximately seven times that of the former. Further, the length of an equivalent carbon bisulphide cell is approximately 100 times that of a nitrobenzene cell since the Kerr constant of nitrobenzene is 100.7 times greater (see Eq. (15)). Thus the cell equivalent to the one illustrated in this paper would have a length of about 121 cm instead of 1.0 cm (the edge effect being allowed for). In other words, it is not feasible to build a carbon bisulphide cell which will give 100 percent initial transmission for voltages less than say 50,000 volts. If the length of the previous carbon bisulphide cell is reduced to the usual 12 cm, its initial transmission at the rated voltage is reduced by a factor of 1/665.

It should be mentioned that the results of the theoretical section, Part II, are strictly true only for light in a narrow band of wave-lengths near to that for which the Kerr constant was chosen (5460A). As seen from Eq. (4) the value of the field $E_m$ which gives maximum (i.e. 100 percent) transmission is inversely proportional to the square root of the Kerr constant $b$. For longer wave-lengths the Kerr constant decreases, hence $E_m$ would be larger. For example at 6000A, $b$ has decreased to $3.04\times10^{-5}$ and for the same voltage the $E_0/E_m$ ratio becomes 0.94. Conversely for a shorter wave-length, say 4800A, $b$ has increased to $4.17\times10^{-5}$ and $E_0/E_m$ becomes 1.10. The effect on the transmission curves can easily be estimated from Fig. 10. The justification for choosing $b$ at a wave-length of about 5500A is that both the eye and the photographic plates used are most sensitive in the blue region, furthermore it is probable that a majority of the visible spark energy is also in the blue.[6]

It is noted that no account has been taken in this paper of a possible lag in the Kerr effect. The existence of such a lag has never been demonstrated and its magnitude is probably less than any of the time intervals dealt with in this paper.[16]

The complete agreement between the theory here presented as to the manner of closing of the shutter and all obtainable experimental data

[16] Beams and Lawrence, Jour. Frank. Inst. **206**, 169 (1928).

indicates convincingly the general correctness of the former. Further, since the shutter action can now be definitely determined, a tool is available for the *quantitative* analysis of the light from very rapidly varying phenomena. The electrooptical shutter is ideally adapted for the study of spark breakdown since the spark itself controls the shutter, but it will undoubtedly be adapted to other fields of study possibly by the use of impact instead of static breakdown as the source of the controlling voltage.

In conclusion I wish to thank Professor E. O. Lawrence for many helpful discussions and suggestions. I also wish to thank Mr. T. K. Kostko for his help in the somewhat lengthy computations involved in the theoretical section.

*Note added in proof:* Von Hámos[8] has made the ingenious suggestion that two shutters be connected electrically in parallel but arranged optically in series to obtain a faster cut-off. The capacity curves $c = 7.35$ and 14.7 of Fig. 11 provide a means of estimating how this arrangement would function. Assuming two cells with the smaller capacity, $7.35 \times 10^{-12}$ farads, the square of the ordinates of the 14.7 curve would represent the net transmission. The time to close of this squared curve is $4.05 \times 10^{-9}$ sec. as compared with $4.4 \times 10^{-9}$ sec. for a single shutter, so that the improvement is small. However, the light passed by oscillations should be negligible. This last would be a decided improvement when making observations at early times.

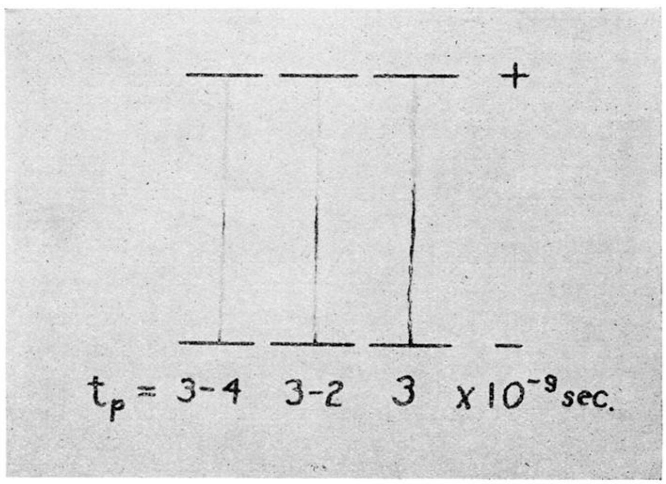


Fig. 14. Change in appearance of a spark in steps of $2\times10^{-9}$ sec. Pressure = 76 cm of Hg, gap length = 5 mm.