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THE SCATTERING OF X-RAYS FROM PARAFFIN, ALUMINUM, COPPER, AND LEAD

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Abstract

The radiation from a tungsten-target x-ray tube operated at 80 kv was filtered through 0.244 cm of aluminum and the intensities of the scattered radiations from paraffin, aluminum, copper, and lead were observed by the ionization method. The scattered intensities at angles in the range of 30° to 120° with the forward direction of the primary beam were compared with the scattered intensities at 90°. Values of the intensity at 90° for paraffin were compared with the intensity at 90° for the other materials. The scattering from paraffin and aluminum was at an effective wave-length of 0.32A; from copper 0.26A, and from lead 0.27A. The Dirac value of the scattering from paraffin at 90° was used as the basis for calculating the absolute values of the scattering per gram and the scattering per electron.

Introduction

THE newer conceptions of the relations between waves and quanta have led to considerable development of the theory of the intensity of scattered x-rays. The scattering from monatomic gases has been successfully predicted and some progress has been made in developing a theory of the intensities scattered from solid materials. Further progress may be made possible by the accumulation of data on the scattering from solids at different wavelengths. The experiment described in this paper was started with the hope of obtaining more accurate relative values of the angular distribution of scattered x-rays from some of the heavier elements. After collecting part of the data it became apparent that the scattering values could be put upon an absolute basis by comparison with the value of the scattering from paraffin at 90°. Hence great care was taken to compare the intensities from each of the metals used as scatterers in the experiment with the intensity from paraffin under the same experimental conditions. It is hoped that the values of the scattering per electron obtained for these metals may be useful in future theoretical developments.

Apparatus and Procedure

Fig. 1 shows the general arrangement of the apparatus used to measure the intensity of the radiation scattered from blocks of paraffin, aluminum, copper, and lead. A 30 m.a., radiator type, tungsten target, Coolidge x-ray tube was the source of the primary radiation. A General Electric x-ray transformer supplied current to the tube at a potential of 80 kv. The primary rays were limited to a beam of small cross section by the circular apertures at S_1 and S_2. A cylindrical ionization chamber having inside dimensions of 8 by 42 cm, with brass walls approximately 0.6 cm thick was mounted on an arm arranged to rotate about an axis at R. The chamber was filled with ethyl bromide vapor and an excess of the liquid was kept in the bottom of the chamber at all times to insure saturation. The potential across the ionization chamber electrodes was 135 V. Currents in the chamber were measured by a Compton electrometer. Lead shielding around the x-ray tube and around the primary and scattered beams prevented all radiation except that scattered from the scattering block at R from reaching the chamber. Stray radiation was also kept from reaching the electrometer system by lead shields. The longer wave-lengths of the continuous spectrum emitted by the tungsten target were filtered out by 2.44 mm of aluminum placed in the primary beam at F_p. The wave-lengths having intensities greater than 0.1 of the maximum in the filtered beam occupied a band about an octave and a half broad. An

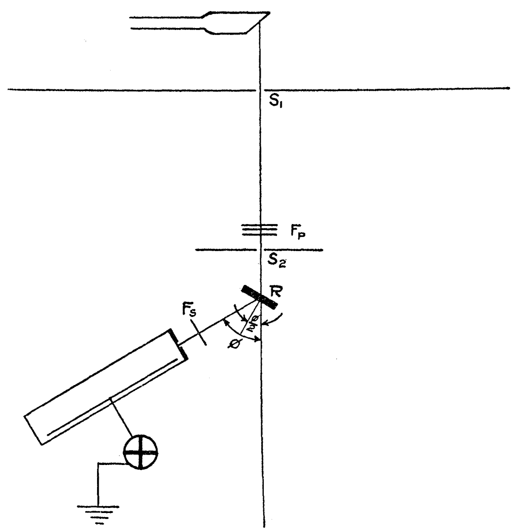

Fig. 1. Arrangement of apparatus.

aluminum sheet 0.812 mm thick was used as a filter in the scattered beam at F_s. This was found to be sufficient to remove the characteristic radiation excited in the scattering materials. The window of the ionization chamber was of 0.255 mm aluminum.

Ratios for each material, of the scattered intensities at angles between 30° and 120° inclusive, compared with the intensities at 90° were first determined. For paraffin and aluminum the scatterers were thin plates of the material, so oriented that the normal to the face of the plate from which the primary beam emerged after passing through the plate made an angle of $\phi/2$ with the forward direction of the primary beam. Thus the scattered rays penetrated the same thickness of the plate as did the primary rays. The copper and lead plates used as scatterers were so thick that none of the primary radiation penetrated them. They were so oriented that rays scattered from the incident face entered the ionization chamber and the glancing angle of the primary rays on the plates was always $\phi/2$.

After the determination of the angular distribution of the scattered radiation for each of the four materials, comparisons of the scattered intensities at 90° between the different materials were made. The ratio of the intensity scattered by paraffin at 90° to that by aluminum at 90° was found to be 6.56. The mass per unit area of the paraffin plate was 0.483 and that of the aluminum plate was 0.0659g/cm². Scattering from the incident face of a 0.812 mm thick aluminum plate was compared with the scattering from the copper and lead plates, each at 90°. The ratio of the intensity from the aluminum plate to that from the copper was found to be 2.64. The intensity from aluminum compared with lead was 2.84. The intensities between copper and lead were also compared directly and found to be in the ratio 1.075, in exact agreement with the ratio 2.84/2.64 from the comparisons with aluminum.

The linear absorption coefficient of the primary rays in aluminum by measurement was 2.51, and in paraffin 0.192. The absorption in paraffin was measured in the block from which the scattering plate had been cut. At the 90° position the rays scattered from paraffin and aluminum had absorption coefficients differing from the coefficient for the primary rays by a negligibly small amount. The hardening of the rays by absorption in the scattering plates was approximately annuled by the increase of wave-length of the rays on scattering. The absorption coefficients for the scattered rays in copper and lead were calculated respectively as 67.8 and 258. Measurement of the absorption in aluminum for the rays scattered from copper and lead gave the same value of absorption coefficient at 30°, 60°, and 90°. The absorption coefficients of the rays scattered from paraffin and aluminum varied slightly with direction of scattering, but were sufficiently constant to produce only a very small change in the corrections made for absorption of the scattered rays in the scattering plates. Consequently these corrections were calculated on the basis of the value of the absorption coefficient at 90°.

Calculations and Results

The scattering per gram of rays penetrating a thin plate of material oriented so that the normal to the plate makes an angle $\phi/2$ with the primary beam is given by

$$s/\rho = (I_\phi/i_\phi)(\cos\phi/2)(1/\omega m), \tag{1}$$

where I_ϕ is the intensity scattered in a direction ϕ, i_ϕ is the intensity of the primary rays penetrating the plate, ω is the solid angle subtended by the ionization chamber window, and m is the mass per unit area of the plate. i_ϕ is given by

$$Ie^{-m\mu/\rho\cos(\phi/2)}$$

where I is the primary ray intensity at the incident face of the scattering plate, and μ/ρ is the mass absorption coefficient of the primary rays in the plate. The ratio of the scattering per gram in any direction compared with that in a direction at right angles to the primary beam may be set up from Eq. (1), and then I does not appear in the ratio, since it is the same at all angles of scattering. This ratio applies to the cases of paraffin and aluminum scattering in the experiment. A similar ratio derived from Eq. (1) will give the scattering per gram at 90° from aluminum compared with paraffin.

For a plate in which the primary rays are completely absorbed and which is oriented with the incident face at an angle $\phi/2$ with the primary beam, the scattering per gram is

$$s/\rho = (\mu/\rho)I_\phi/I \tag{2}$$

Eq. (2) will thus give the ratio for copper and lead of the scattering per gram at any angle compared with that at 90°. The intensity scattered from the incident face of the 0.812 mm aluminum plate is, however, less than the intensity that would be scattered from a plate of aluminum thick enough to absorb all of the primary radiation. If the maximum path of the x-rays in the aluminum plate is s, the scattered intensity is proportional to $1-e^{-\mu s}=0.437$ for the rays scattered at 90°. The measured intensities of the Al/Cu and Al/Pb scattering at 90° must thus be divided by 0.437 to get the true ratio of the relative intensities to use in calculating the scattering per gram from copper and lead compared with that from aluminum.

When nonhomogeneous radiation is scattered, the wave-lengths entering the ionization chamber will differ for different materials because of the difference in absorption of the different wave-lengths present in the primary beam. The effective wave-length scattered from each material was determined from the spectrum of the scattered beam in each case, according to the equation

$$\lambda_{\text{eff.}} = \int_0^\infty \lambda I_\lambda d\lambda \Big/ \int_0^\infty I_\lambda d\lambda. \tag{3}$$

This meant plotting of the spectral curve for each case of scattering and the curve whose abscissas were the same as the abscissas of the spectral curve, but whose ordinates were equal to the fraction $\lambda I_\lambda/I_\lambda$ of the ordinates of the spectral curve, then measuring the areas under each curve with a planimeter and using their ratio for the effective wave-length. In this way the wave-lengths were found to be

0.32A scattered from paraffin
0.32A scattered from aluminum
0.26A scattered from copper
0.27A scattered from lead.

In the comparison of the scattering from aluminum with that from copper and lead a correction must be made to the measured intensities because of the difference in the wave-lengths scattered. The effective wave-length scattered from the 0.812 mm aluminum plate was estimated to be 0.29A. Thus the measurement of intensities gave the following scattering ratios.

(Al at 0.29A)/(Cu at 0.26A)
(Al at 0.29A)/(Pb at 0.27A)

The ratios desired are, since the angular distributions of the scattered intensities were determined at an effective wave-length for aluminum of 0.32A, (Al at 0.32A)/(Cu at 0.26A)

(Al at 0.32A)/(Pb at 0.27A).

Hence we need to multiply each of the measured ratios by the ratio

(Al at 0.32A)/(Al at 0.29A).

Coade[1] has measured the mass scattering coefficient for a number of metals at several different wave-lengths. He gives values for aluminum from which the variation of the coefficient with wave-length may be determined. By use of his data it is found that the scattering from aluminum at 0.32A is about 1.14 times the scattering at 0.29A. Hence the intensity ratios comparing the scattering from copper and from lead with that from aluminum were increased 14 percent in calculating values of the scattering per gram ratios, to correct for the actual difference in scattering.

Because of the difference in the wave-lengths entering the ionization chamber in getting the Al/Cu and Al/Pb ratios there will be a different fraction of the rays absorbed in the chamber and an additional correction must be made for this fact. Compton[2] has shown the means of comparing the intensities of two x-ray beams of different wave-lengths in terms of the ionization which they produce. His expression for the intensity ratio has the form

$$I_1/I_2 = (i_1/i_2)(f_1/f_2)(R_2/R_1)(e^{-x_2}/e^{-x_1}) \quad (4)$$

where i_1 and i_2 are the ionization currents, f_1 and f_2 are the fractions of the beams absorbed in the ethyl bromide, R_1 and R_2 are the ratios of the energy spent in producing ionization to the total absorbed energy, e^{-x_1} and e^{-x_2} are the fractions of the beams penetrating all the material in the path of the rays. The ionization current ratio is known from measurement, the fractions of the two beams absorbed in the chamber may be calculated. Compton[2] shows how to calculate the ratio of the energy spent in producing ionization to the total absorbed energy. The absorption in filters and scatterers is known, so the last term may be calculated, neglecting the absorption in air which is very small. The ratio of the intensities was thus found to be about 18 percent greater than it should be for the measured values of aluminum compared with copper, hence the experimental ratio was divided by the factor 1.18. The ratio of the scattering per gram of wave-length 0.32A by aluminum to wave-length 0.26A by copper is thus

$$\frac{2.64}{0.437} \times \frac{1.14}{1.18} \times \frac{(\mu/\rho)_{\text{Al}}}{(\mu/\rho)_{\text{Cu}}} \cdot$$

The ratios of the scattering per gram at 90° calculated from the experimental intensity ratios after making these corrections for difference of scattered wave-length and for difference of ionization are, relating each metal to paraffin as unity,

[1] E. N. Coade, Phys. Rev. **36,** 1109 (1930).

[2] A. H. Compton, Phil. Mag. **8,** 961 (1929).

Paraffin	[1.0]	at 0.32A
Aluminum	1.05	at 0.32A
Copper	1.47	at 0.26A
Lead	4.11	at 0.27A

Fig. 2 shows the curve for scattering from paraffin on the basis of the Dirac theory which gives the angular distribution of intensity proportional to $(1+\cos^2\phi)(1+\alpha \operatorname{vers}\phi)^{-3}$. The experimental points are shown along with the theoretical curve. It is seen that there is good agreement between the experimental and theoretical values in the range 60° to 120°. This is in accord with the results of Jauncey and Harvey[3] who found very good agreement between the scattering from paraffin and the Dirac theory near 90° and for

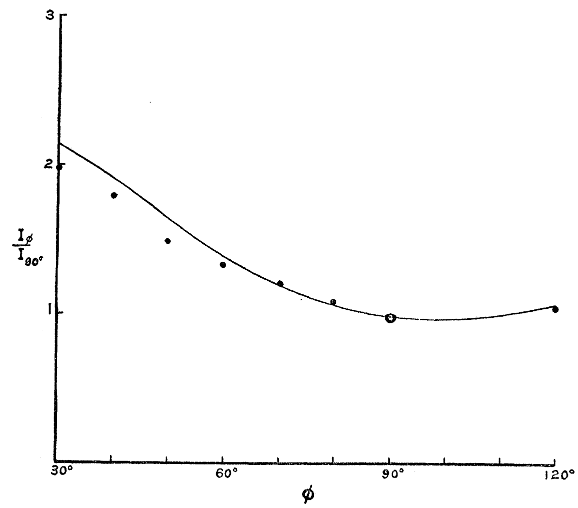

Fig. 2. Scattering from paraffin. Curve from Dirac theory, dots are values from experiment.

wave-lengths of about 0.3A. For the scattering per gram on the Dirac theory we have

$$s/\rho = 0.0238(Z/W)(1 + \cos^2 \phi)(1 + \alpha \operatorname{vers} \phi)^{-3}. \quad (5)$$

for unpolarized x-rays. Here Z is atomic number, W atomic weight, $\alpha = 0.0242/\lambda$. The average formula for paraffin is $C_{24}H_{50}$ and hence Z/W has a value of 0.5, whereupon the scattering per gram from paraffin at 90° is 0.0095, for the wave-length 0.32A. Using this combined with the experimental distributions in angle and ratios at 90°, the values of the scattering for each of the four materials have been calculated and are shown in Table I.

The scattering per electron relative to the classical scattering from a free electron is perhaps of most theoretical interest. For modified radiation this is given by

$$S = (1 + \alpha \operatorname{vers} \phi)^{-3}. \quad (6)$$

[3] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **37**, 698 (1931).

TABLE I. *Scattering per gram.*

Angle	Paraffin 0.32A	Aluminum 0.32A	Copper 0.26A	Lead 0.27A
30°	0.0189	0.0332	0.0542	0.234
40	.0170	.0235	.0391	.169
50	.0142	.0173	.0293	.104
60	.0127	.0134	.0236	.0752
70	.0115	.0116	.0173	.0534
80	.0105	.0109	.0151	.0417
90	.0095	.0100	.0140	.0390
120	.0106	.0114	.0142	.0362

At the wave-length used in the experiment the scattered rays from paraffin are almost all modified, and at 90° the value of S given by Eq. (6) would be a true value for paraffin. If we compare the value of S at 90° for one material with the value for another material at the same angle we shall have

$$S_1/S_2 = (s/\rho)_1(Z/W)_2/(s/\rho)_2(Z/W)_1. \tag{7}$$

Hence the S values at 90° are related as follows

Paraffin	[1.0]
Aluminum	1.09
Copper	1.60
Lead	5.13

In Table II are shown the values of the scattering per electron based on the value 0.803 for paraffin at 90° by the Dirac formula.

TABLE II. *Scattering per electron, S, relative to the classical scattering from a free electron.*

Angle	Paraffin 0.32A	Aluminum 0.32A	Copper 0.26A	Lead 0.27A
30°	0.91	1.66	2.85	14.10
40	.90	1.30	2.27	11.22
50	.84	1.07	1.91	7.75
60	.85	.94	1.73	6.34
70	.86	.91	1.43	5.06
80	.86	.93	1.35	4.28
90	.80	.88	1.28	4.12
120	.68	.80	1.05	3.05

DISCUSSION

Fig. 3 shows the scattering per electron plotted against values of $(\sin \phi/2)/\lambda$. Curve I is for paraffin and shows that the scattering for the wave-length and range of angles investigated is approximately in accord with a theory of the scattering from free electrons. Curve II is for aluminum, Curve III for copper, and Curve IV for lead. These indicate that the electrons in the atoms of the elements cooperate to an increasing amount as the atomic number of the scatterer is increased. They also show that the cooperation of the electrons becomes more effective as the direction of scattering approaches the forward direction of the incident primary radiation, and how particularly effective is their cooperation in heavy elements such as lead, where the electrons are concentrated near the center of the atom. Were there no interference effects between neighboring atoms the cooperation at zero angle should be such that each electron would scatter Z times as much as a free electron scattering independently, where Z is the atomic number of the atom. It is possible that where the atoms are close together constructive interference between rays from neighboring atoms will increase the value of S beyond that of the atomic number of the scattering atom.

The probable experimental error in the intensity measurements is estimated to be about 3 percent. The absolute values of the scattering from cop-

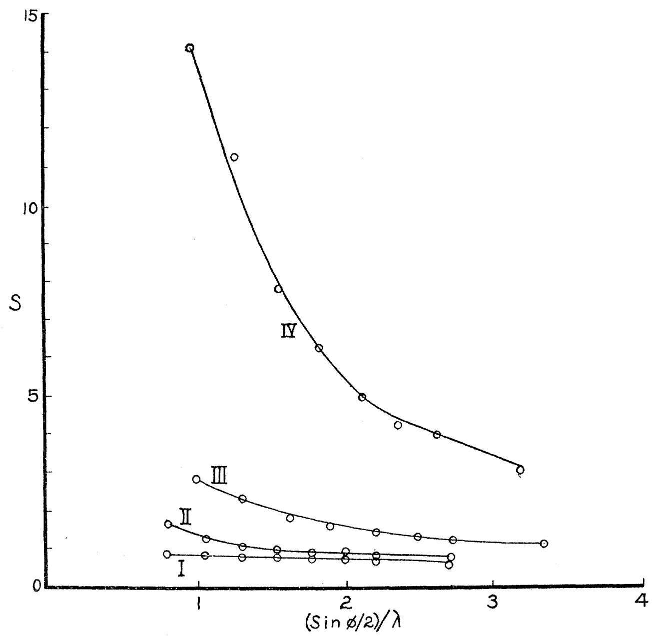

Fig. 3. Scattering per electron relative to the classical scattering from a free electron. Curve I paraffin, curve II aluminum, curve III copper, curve IV lead.

per and lead, depending as they do upon the determination of the Al/Cu ratio which was corrected by a slightly uncertain calculation allowing for the difference in the scattered wave-lengths, may be slightly more in error than the paraffin and aluminum values.

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