# SUPERSONIC SATELLITES AND VELOCITY

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Abstract

With increasing path length in a Pierce acoustic interferometer, minor peaks, or "satellites," were found to develop in the curves which correlate $(I_P - I_B)$, *the excess of plate current over bias current*, and $x$, *the mirror position*. Several such curves are given. Evidence that these satellites are caused by multiple reflection is presented. The acoustic velocity as determined by these satellites, differs by less than 0.06 percent from the theoretical value, $V = (\gamma p/d)^{1/2}$, whereas the major peaks yield a velocity which, in every case, *exceeds* the theoretical value by more than 0.5 percent. Apparently the velocity approaches a limiting minimum value with decreasing *intensity*, but is independent of *frequency*.

In A previous paper[1] the author refers to positive deviations from the theoretical value of the supersonic velocity in air. It has now been found that these deviations shrink so as to fall within the limits of uncertainty in the measurements, provided the "satellites," which are discussed in this paper, are used to compute the velocity. Since no other investigator has reported such satellites they will be discussed in some detail. The problem is that of finding their significance. The purpose of this paper is to present evidence that they are caused by higher order reflections, the waves being weakened in intensity and diminished in velocity as they travel back and forth. A further purpose is to determine the minimum velocity at each frequency and to demonstrate a variation in velocity with *intensity*. A test as to whether the *minimum* value of the velocity varies with *frequency* is also involved.

The general method of the investigation is the same as that used previously by the author[2] except that a careful study of the satellites is added. With this exception similar work has been reported by G. W. Pierce,[3] Charles D. Reid,[4] J. C. Hubbard,[5] E. Klein and W. D. Hershberger[6] and others.

## Apparatus

Quartz piezoelectric oscillators were used as sources. Some of these were sputtered with platinum and others with silver. The metal films serve as electrodes which are forced to oscillate with the same amplitude as that of the quartz surface. Thus a sound beam is radiated from the largest face of each crystal. The ratio of face area to the acoustic wave-length is large enough to insure approximately parallel sound beams. In the least favorable case the diagonal of the square radiating surface was 53 mm and the wave-length was

[1] W. H. Pielemeier, Phys. Rev. **36,** 1005 (1930).
[2] W. H. Pielemeier, Phys. Rev. **34,** 1184 (1929).
[3] G. W. Pierce, Proc. Am. Acad. Arts and Sciences **60,** 271 (1925).
[4] Charles D. Reid, Phys. Rev. **35,** 814 (1930) and **37,** 1147 (1931).
[5] J. C. Hubbard, Phys. Rev. **36,** 1688 (1930).
[6] E. Klein and W. D. Hershberger, Phys. Rev. **37,** 633 (1931).

1.09 mm. This results in an angle of about three degrees to which the central beam of the diffraction pattern is confined. With such a narrow beam and the rather short path lengths there are many reflections back and forth before the wave intensity becomes negligible on account of the spreading wave front. The crystals were operated in a UX-201-A vacuum tube circuit. A hair hygrometer was kept in the cast iron air chamber. The humidity was kept low with $CaCl_2$. Some tests were made with a padded mirror, the intention being to diminish the number of reflections. The mechanical backlash in the micrometer screw was measured by focussing a microscope on a sharp edge of the plate glass reflector. Some of the crystal frequencies were checked by means of standard crystals at the college broadcasting station, WPSC, and by the incoming waves from other stations.

## Results

The results are presented in the form of curves and tables. The curves, Figs. 1 to 4, show the excess of plate current over the galvanometer bias current, $(I_P - I_B)$, as ordinates and the mirror positions, $x$, as abscissas.

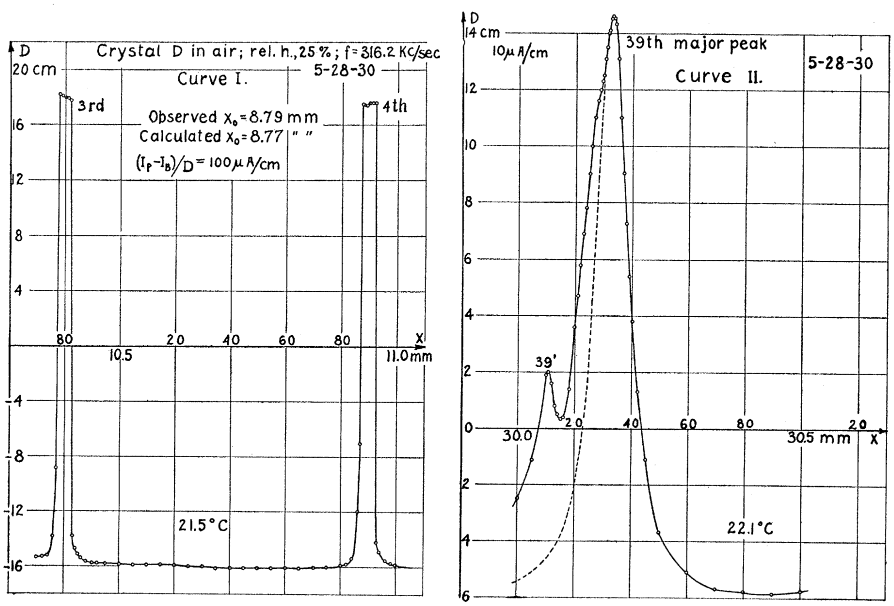



Fig. 1.

Reid[4] reports a peak displacement which might be termed an "*acoustic* backlash." Such a backlash was observed with the lower frequencies listed in Table I. The largest observed backlash was less than 0.02 mm (approximately $3\frac{1}{2}$ screw divisions). If there was any acoustic backlash with the *highest* frequency it was less than 0.005 mm. Near the tenth peak it appears to be roughly $0.01\lambda$ at each frequency. It is found by subtracting the *mechanical* backlash from the *total* backlash as indicated by the peaks when the screw is reversed.

The same type of curve was obtained with metal blocks of different thickness under a *given* crystal. For two *different* crystals the peak patterns are dif-

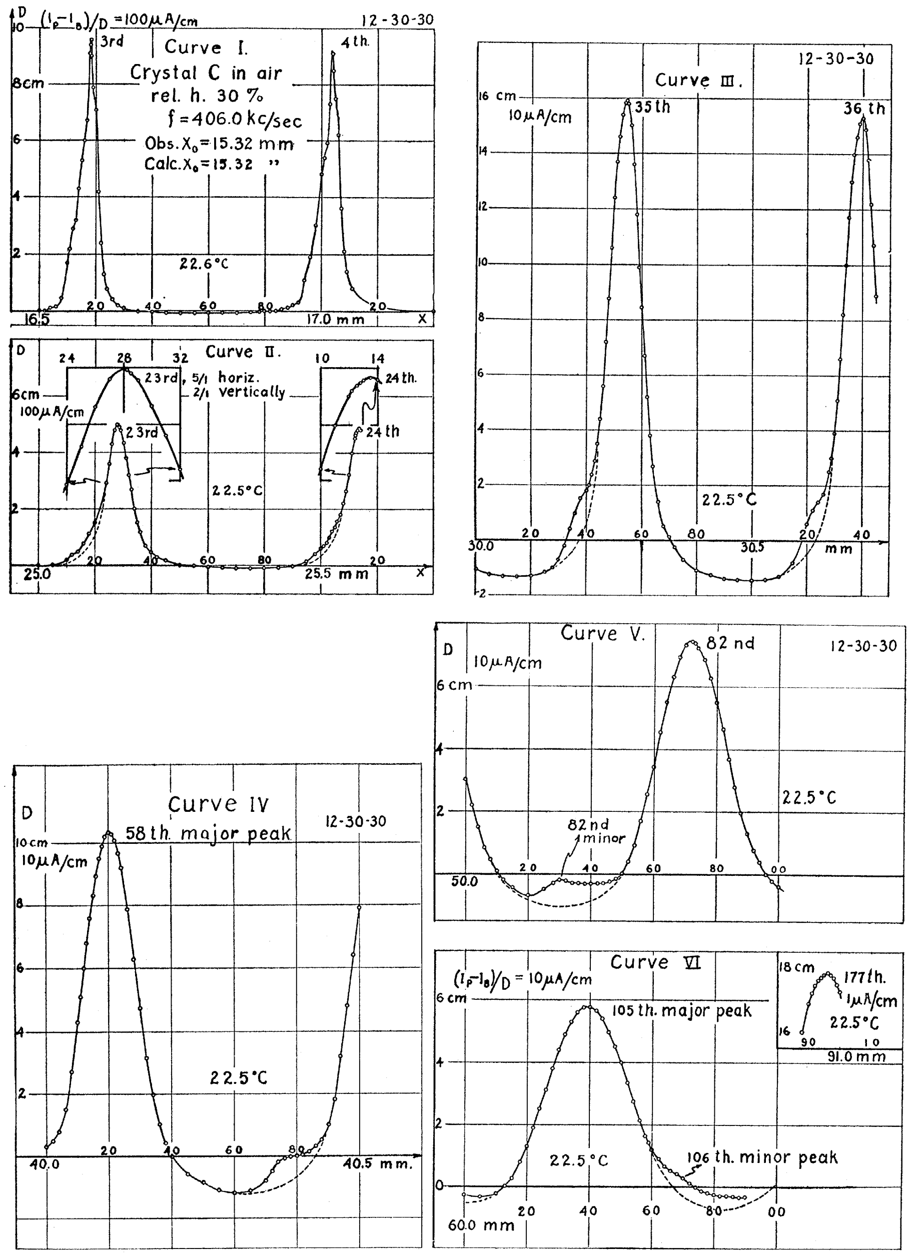



Fig. 2.

ferent for the same portion of the screw. These facts indicate that the screw is not the cause of the peculiarities of the curve.

TABLE I.

| $f$ Kilocycles | $t$ °C | $h$ % | $V_t$ m/sec. | $V_0$ m/sec. | $V_0'$ m/sec. | Date |
|---|---|---|---|---|---|---|
| 316.2 | 26.8 | 29 | 347.8 | 331.7 | 334.2 | 5–21–30 |
| 316.2 | 21.6 | 25 | 344.5 | 331.6 | 333.9 | 5–28–30 |
| 389.3 | 19.5 | *20 | 343.3 | 331.65 | 333.6 | 3–27–30 |
| 389.3 | 25.6 | 19 | 347.0 | 331.6 | 333.8 | 7–11–30 |
| 406.0 | 22.5 | 30 | 345.1 | 331.6 | 333.6 | 12–30–30 |
| 406.0 | 22.5 | 28 | 345.1 | 331.6 | 333.5 | 1– 9–31 |
| 655.5 | 25.7 | 30 | 346.9 | 331.5 | 333.9 | 2–18–31 |
| 647.9 | 26.8 | 26 | 347.7 | 331.6 | 333.4 | 4–15–31 |
| 647.9 | 24.6 | 25 | 346.3 | 331.6 | 333.3 | 4–15–31 |
| 1216. | 19.8 | 31 | 343.5 | 331.7 | 333.3 | 10–29–30 |
| 1216. | 27.4 | 34 | 348.0 | 331.6 | 333.2 | 4–17–31 |
| 1216. | 25.9 | 23 | 347.3 | 331.6 | 333.1 | 5–14–31 |

NOTE: $f$ = frequency; $t$ = temperature; $h$ = humidity; $V_t$ = velocity at $t$°C and $V_0$ = velocity at 0°C determined by the satellites. $V_0'$ = velocity at 0°C determined by main peaks.
* estimated.

## DISCUSSION OF RESULTS

The work of Reid[4] and Hubbard[5] indicates that there is no variation of velocity with frequency, in other words, that acoustic dispersion does not exist. The same may be said of the satellite data presented in Table I. From these satellite data a limiting (minimum) velocity is calculated for each frequency. This velocity differs by less than 0.06 percent from the theoretical value, $V=(\gamma p/d)^{1/2}$. This difference is just within the limits of uncertainty in the measurements. There is no noticeable change in spacing of these satellites with increasing mirror displacement such as that observed by the writer and by Reid[4] in connection with the ordinary peaks. However, with an im-

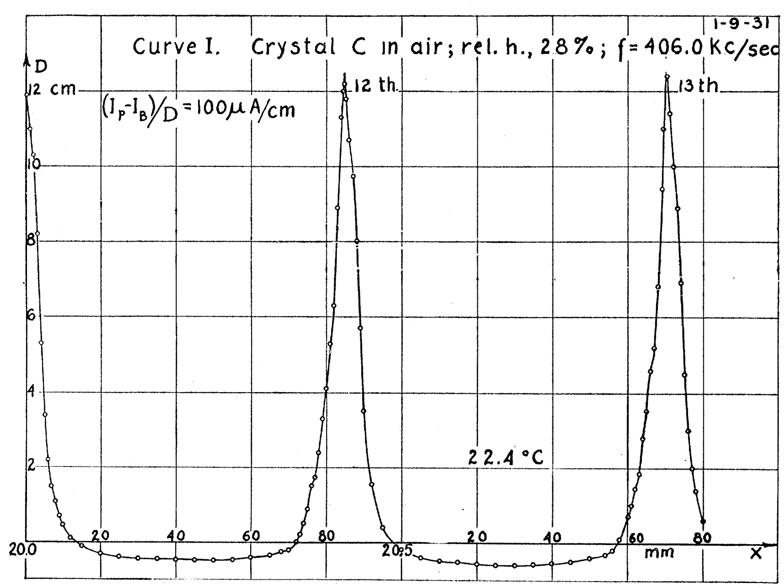



Fig. 3.

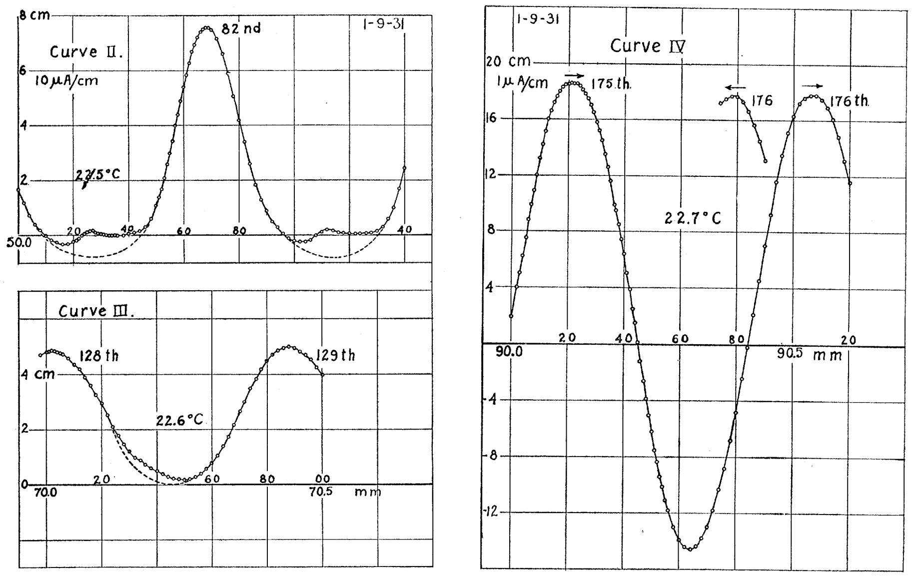



Fig. 3 (*Continued*).

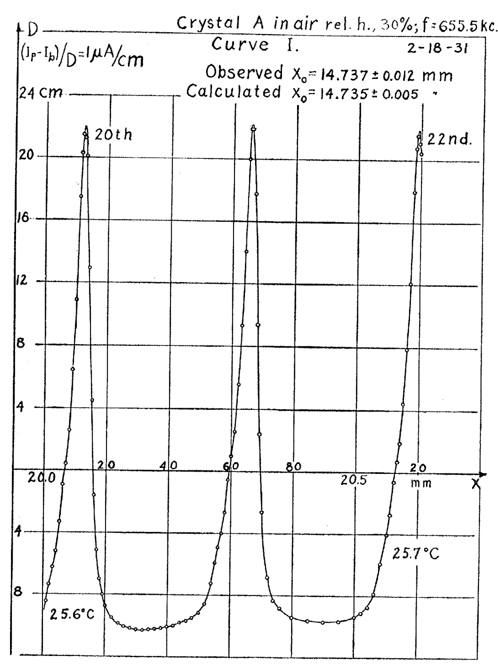
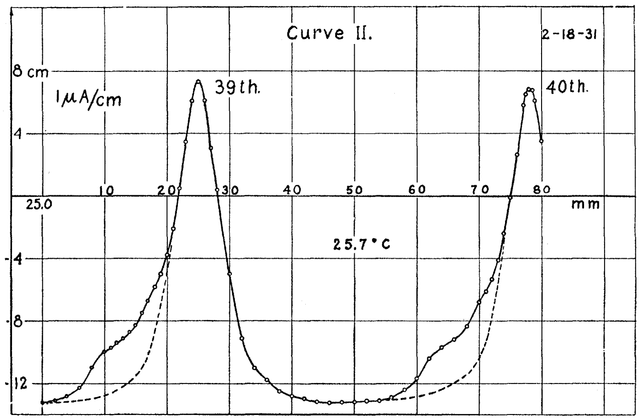
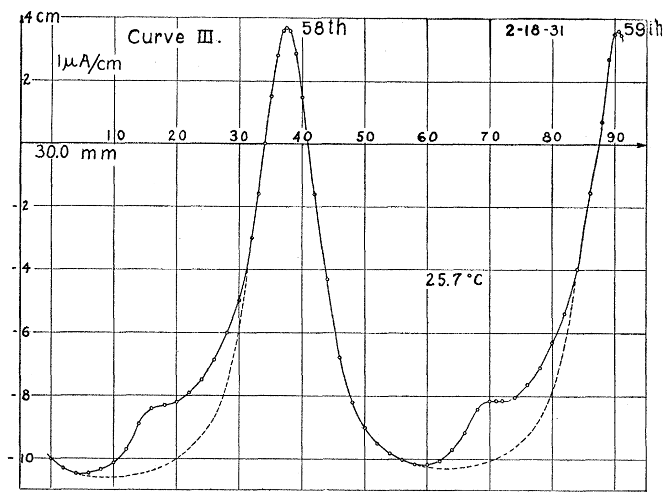



Fig. 4.



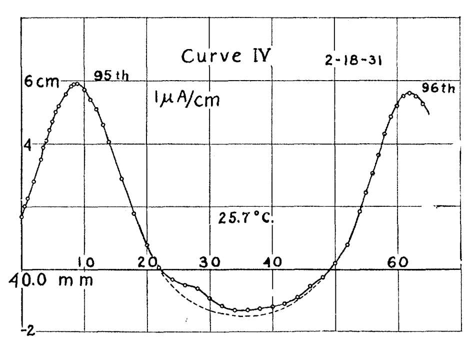




FIG. 4 (*Continued*).

proved method of holding the frequency constant, Hubbard[5] reports a *constant* spacing of the ordinary peaks. If the emitted intensity was quite low, this would be expected. Neither he nor Reid reported satellites accompanying the main peaks. In the latter case this might have been due to the rapid spread of the beam at the rather large wave-lengths used. If the author's main peak data are used, a velocity is obtained, which, in every case, *exceeds* the theoretical value by more than 0.5 percent and in some cases by more than 1.0 percent. The spacing of the *major* peaks varies with increasing mirror position as observed by Reid.[4]

The curves obtained with a padded mirror showed a slightly smaller peak spacing and there was very little tendency to form satellites. A padded mirror reflection causes a large reduction in intensity and the first return trip is probably the only effective one for all except very short path lengths.

It is not difficult to follow the satellite system through the curves of Figs. 1 to 4. The progressive lagging of the satellites behind their major peaks is one of the most striking features of these curves. At the third and fourth resonance positions shown in Fig. 1 the reaction on the crystal was sufficient to stop the oscillations as was verified by keeping the plate voltage and filament current at their ordinary value and stopping the oscillations by pressing the crystal with a rubber pad. At once $I_P$ rose to the level of the truncated peaks, although the mirror was set between two resonance positions. The twelfth peak (not shown) had a tendency toward a satellite at the left base. As shown in curve II, the 39th peak has a pronounced satellite, and a secondary maximum almost merges with the main peak. The dotted lines in these figures make the major peaks symmetrical and form an estimated base for the satellites. Readings were taken for major peak and satellite positions extending out to much greater mirror displacements. It was found that the lag persisted until it reached a half wave-length or more where the satellites were so weak that they could not be located very definitely.

In Curve I, Fig. 2, individual responses to the first few returns to the crystal are evident in each peak. The insets in curve II show parts of the 23rd and 24th peaks magnified. The smallest attempted increase in $x$ was 0.001 mm ($\frac{1}{5}$ screw head division). Definite satellites have developed in curve III. The progressive space lag is shown in curves IV, V, and VI. Curve II of Fig. 3 represents an excellent check on the data of curve V, Fig. 2.

Incomplete satellite data for oxygen indicate that the limiting velocity at 0°C is approximately 314.8 m/sec, which is also the theoretical value. The value computed from the main peaks is given in an earlier paper[1] as approximately 317.4 m/sec.

Since no other observer has reported the presence of "acoustic satellites" a few hypotheses as to their cause will now be presented.

1. The crystal may continue to vibrate at its natural frequency until the mirror reaches such a position as to produce resonance to a slight degree and therefore a sufficient added load, or reaction, might thereby be applied to the vibrating crystal to suddenly diminish its frequency and thus cause a slight drop in the plate current. As the mirror displacement increases the frequency

might approach its lower limit thus making the crystal more capable to withstand the increased load caused by better resonance; i.e. by the phase coincidence of the emitted waves and those completing the *first* trip. Thus the mirror, passing through the best resonance position, would produce a major peak. Slightly greater displacements would produce large drops in plate current. This would partially account for the shape of the curves.

The objection to this explanation is that the necessary load for such a large frequency shift would stop the oscillations entirely. Reid[4] apparently failed to observe a noticeable frequency shift as a resonance position is approached.

A *small* frequency shift of this kind might account for the acoustic backlash, however. It might also slightly modify the shape of the major peaks. Such a frequency shift is referred to in the German literature on coupled electric circuits as "Ziehen und Reissen."

2. The satellites may be caused by a phase lag of certain parts of the crystal surface behind the rest resulting in a sort of rotating *surface* wave and causing the emitted *sound* wave fronts and the crystal surface to be nonparallel.

If this were the case it should be possible to adjust the tilt of the crystal so as to eliminate the satellites. It was found that in some cases an extremely slight adjustment following a visual one caused the major peaks to become more sharp and narrow for a range of mirror positions from 1.5 to 10 mm from the crystal. Apparently this had no effect on the true satellites which could scarcely be distinguished from the first few peaks. Moreover, the curves show a progressive lag of the satellites behind their major peaks of just the right magnitude to give the theoretical velocity ($V_0 = 331.6$ m/sec). By the above hypothesis this would scarcely be possible.

3. The presence of overtones may account for the satellites.

It would be very remarkable if the crystal were to perform so as to make the satellites, instead of the *main peaks*, on this hypothesis give the accepted velocity. The frequency of one of the crystals was obtained more precisely by beats with a standard crystal at station WPSC. Then the frequency was computed from the constant satellite spacing and the accepted velocity of audible sound. These two values for the frequency differed by less than 0.02 percent.

4. The satellites and the other deviations from symmetry in the main peaks may be caused by a gradual increase of velocity above the lower limit, $V = (\gamma p/d)^{1/2}$, as the intensity approaches that of the waves when they are first emitted. This *maximum* velocity would be somewhat greater than the value obtained from the major peak spacing.

This explanation is proposed by the writer as the correct one. The strongest evidence against the others becomes the best evidence in favor of this one; namely, that the uniform spacing of the satellites gives the accepted velocity of sound. Moreover, the other deviations from symmetry in the main peaks, and their excessive spacing are explained by it. Then the satellites do not represent a separate *frequency* as in spectroscopy but, rather a limiting *velocity* at the *one* existing frequency.

In an earlier article[1] it is shown that the maxima of the $(I_P - I_B)$ curves appear at the same mirror positions as do the maxima of pressure amplitude in the air at the crystal surface. Fig. 5 shows how the square of the pressure amplitude varies with mirror displacement, assuming constant wave velocity and perfect reflections at the mirror and also at the vibrating crystal.

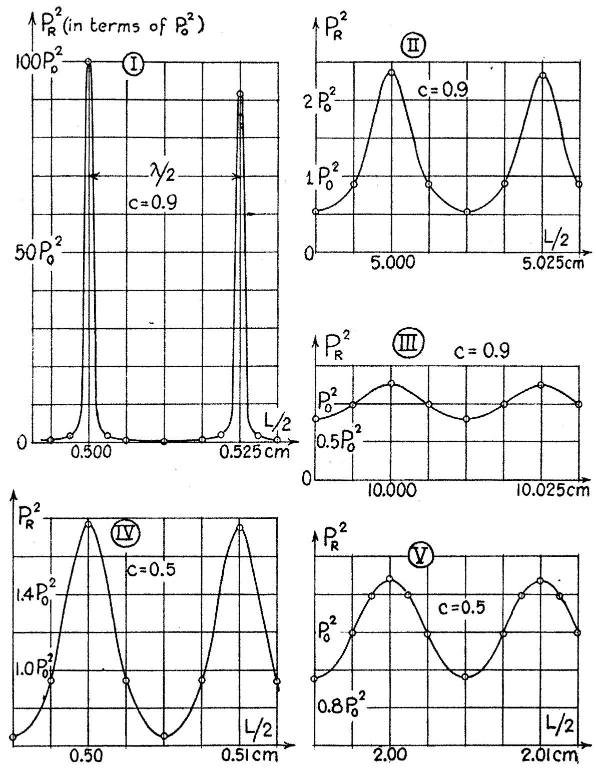



Fig. 5.

The points for this curve were obtained from the equation

$$(P_R)^2 = \left[ \sum_0^n P_0 c^{nL} \cos(-\omega nL/V) \right]^2 + \left[ \sum_0^n P_0 c^{nL} \sin(-\omega nL/V) \right]^2 \qquad (1)$$

in which $P_R$ represents the *resultant* pressure amplitude at the crystal,

$P_0$, the pressure amplitude due to the *emitted* waves only.
$n$, the number of return trips made by the wave train.
$c$, the fraction of $P_0$ left at the end of the first unit of path length.
$L$, the length of one return trip.
$V$, the wave velocity (assumed constant in this equation).
$= 2\pi f$, where $f$ is the frequency of the crystal
$-(\omega nL/V)$ = the phase angle for the returning waves as they are being reflected the $n$th time by the crystal.
$L = 2(x - x_0)$ where $x_0$ is the crystal position.

It may be seen from Eq. (1) that a variable wave velocity, $V$, will prevent perfect resonance. In other words, there will be no position of the mirror which will cause *all* the reflections at the crystal to differ in phase by a whole number of periods. The higher order reflections will require a shorter path (smaller mirror displacement) to bring them into phase coincidence with the emitted waves. If the reflections at the vibrating crystal were equally good for all phases of arrival in the returning wave trains there would probably be no satellites but rather a more gentle slope without minor peaks on the left side of each major peak. If, however, the absorption in a single return trip should reduce the intensity, and hence the velocity sufficiently, there might appear separate humps for each return trip until the limiting velocity is reached. The linear separation of such humps would depend on the path length as well as on the velocity range. Very probably, however, the reflecting power of the crystal depends decidedly on the phase at arrival. A condensation, returning to be reflected, will be largely absorbed and will feed energy into the crystal momentarily, if the crystal is contracting and therefore emitting a rarefaction at that instant. If the path length or the velocity change is great enough to produce a linear separation of the humps of more than a small fraction of one half wave-length, there will be a great reduction in the reflecting power of the vibrating crystal and the humps will fail to appear. When the mirror displacement is a little less than that required for a major peak (e.g. where the first hump might be expected) the path length is too short for resonance with respect to the first return trip. Consequently the intensity and velocity will be greatly reduced so that the mirror would need to be moved nearer the crystal to make the second return arrive in phase. But this will accentuate the reduction at the first reflection. Thus there is a strong tendency for a shift toward low intensity and the limiting velocity in all except the first return trip. Curve I, Fig. 2, shows several humps at small linear separation due to a short path length. When the mirror is at a resonance position for the waves which have reached the limiting velocity all the succeeding reflections will be in phase and a small peak is produced by the cumulative effect of all of them, there being little reduction in intensity due to the successive reflections after the limiting velocity is reached. A further decrease in intensity would merely affect the height but not the position. Sudden changes in the logarithmic decrement of peak height for the smaller mirror displacements also suggest the breaking away and disappearance of humps in the major peaks. The logarithmic decrement of the peaks in Fig. 5 (constant $V$) changes with $L$ but there are no *sudden* changes. The minor peaks caused by the waves which have reached the *limiting* velocity are considered as the true satellites. This exludes the other humps and the minor peaks caused by lack of proper mirror alignment. A careful study of Figs. 1–4, with the above explanation in mind, is rather convincing.

A decrease in wave-length and therefore in velocity with diminishing intensity for audible sound was observed by H. O. Taylor.[7] Hitchcock[8] made

[7] H. O. Taylor, Phys. Rev. **32**, 270 (1913).

[8] R. C. Hitchcock, Proc. Inst. R. E. **15**, 907 (1927).

similar observations on ultrasonic waves from crystals. Theory predicts such a variation, at least for the condensations.

Probably other investigators have failed to observe satellites because their sound beams diverged so fast that the second reflection contributed a negligible amount to the intensity. This would hardly explain Hubbard's[5] constant spacing unless the waves were *emitted* at low intensity and practically the limiting velocity.

Since there is no apparent dispersion of low intensity sound waves in air the Pierce acoustic interferometer may be used not only to measure supersonic velocities in other gases using known frequencies, but also to measure the frequency of crystals to four or five significant figures if the gas is air and if the sound intensity is known to be sufficiently low.

Gas densities are usually given for so-called standard conditions. The pressure, however, is not definite unless it is given in absolute units. Unfortunately it is not customary to state the pressure in absolute units nor to give the corresponding gravitational field intensity in connection with gas density tables. In order to compute the velocity of sound to the nearest 0.1 m/sec by the formula, $V=(\gamma p/d)^{1/2}$ the mere statement, $p=760.0$ mm of mercury, is, of course, insufficient. (Probably the best computed value and also the experimental value for low intensity sound is 331.6 m/sec at 0°C.

The data presented in this paper may have some bearing on the theory of standing waves and in some cases on the sharpness of resonance.