PLASMA-ELECTRON RESONANCE, PLASMA RESONANCE AND PLASMA SHAPE

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Abstract

Plasma-electron resonance is a completely internal oscillation in a plasma and has a frequency dependent only on the electron density, whereas plasma resonance depends upon boundary conditions as well. The frequency of neither is influenced by the Debye-Hückel ion cloud. Previous theory relating plasma-resonance frequency to plasma shape is amplified and demonstrated experimentally.

I. The Distinction Between Plasma Resonance and Plasma-Electron Resonance

In A recent paper[1] the plasma-resonance frequency was shown to be dependent upon the shape of the plasma boundary. In that article plasma-resonance was defined[2] as the actual resonance of the system comprising the plasma and the condenser of which it necessarily forms part of the dielectric. On that basis it was shown theoretically in II, section IV, that resonance occurred when $\eta' = 2$ where

$$\eta' = (1 + \theta)\eta = (1 + \theta)\omega_0^2/\omega^2,$$

and θ is the fraction of the condenser volume occupied by the plasma,

$$\omega_0 = (4\pi Ne^2/m)^{1/2}, \tag{1}$$

that is, 2π times the plasma-electron frequency, and ω is the impressed angular frequency. Thus neglecting the small value of θ (about 0.056 in the actual experiments)

$$\omega = \omega_0/2^{1/2}$$

at resonance. This is the same result arrived at in II, section IX, where the natural frequency of electrons in a cylindrical plasma for parallel displacements was considered. The identity of the two results simply shows that the only function of the plane parallel condenser is to furnish a uniform electric field so that the electron displacements will be parallel.

As a consequence of these considerations the definition of plasma resonance can be generalized as the resonance of the plasma in response to a definite field configuration. The dependence of the resonant frequency on this configuration has been shown theoretically through the introduction of the shape factor, ϕ, in II, section IX, and it will be shown experimentally here.

[1] L. Tonks, Phys. Rev. **37**, 1458 (1931). This paper will be referred to as II.
[2] Reference 1, footnote 7.5.

It is important to note that any difference between the plasma-electron frequency and the plasma-resonance frequency arises from the fact that the finite extent of the plasma enters as a fundamental consideration in the latter case, whereas in the former the extent of the plasma is effectively infinite compared to the size of the oscillating region, a condition which is fulfilled whenever at every point of the plasma boundary the normal component of the electric field is zero. Under this head come the two modes of oscillation already mentioned in a previous paper,[3] namely those for which throughout a cylindrical or spherical portion of a plasma of any shape or size the electron displacements are radial and a function of radius only. It is for this case, too, that the generalized treatment in I, section I, leading to Eq. (5C) proves the independence of natural frequency and configuration.

The question naturally arises as to what significance can be attached to an oscillation which by its very nature is accompanied by no external field—what can start it and how can it be detected? Just as the thermal energy of a neutral gas may be looked upon as made up of the energy of a large number of sound waves traversing the gas, so it may be that an ionized gas possesses the additional mode of heat storage represented by plasma-electron oscillations.[4] In addition, each ionization of an atom initiates an oscillation of this type because the two electrons involved leave the scene of ionization so rapidly that in effect a positive charge has been suddenly created. Detection may well occur by means of relatively small stray fields.

The reasoning leading to I, Eq. (4), has been criticized recently by Sven Benner, "Ueber die Eigenschwingung freier Elektronen in einem Konstantem Magnetfeld," Dissert., Stockholm, May 26, 1931. He points out (p. 9) that the integration constant neglected in that derivation represents a field variable in time, although uniform in space. But he fails to note that such field would give a uniform displacement of electrons throughout the whole volume of the plasma and would, therefore, inevitably introduce boundary conditions. In addition such a displacement is inconsistent with Tonks and Langmuir's assumed nonuniform displacement throughout a portion only of the plasma. Benner's expressions for plasma resonance in cylinders and spheres are consistent with the formulas of II, section IX, and are correct if it is understood (which is not absolutely clear from the content of his paper) that parallel rather than radial electron displacements are involved.

II. The Impossibility of the Plasma Resonance Frequency being Affected by the Debye-Hückel Ion Cloud

Tonks and Langmuir have been further called to task for not including the Debye-Hückel field which arises from the ion cloud surrounding each electron and which opposes the electron motion. This field ordinarily acts as a viscous force. There is the possibility of its acting like an elastic force if in each cycle the oscillating electron encounters any remnant of the cloud

[3] L. Tonks and I. Langmuir, Phys. Rev. **33,** 195 (1929). This paper will be referred to as I.

[4] I. Langmuir, Proc. Nat. Acad. Sci. **14,** 631 (1928).

established in its previous cycle. But this is far from being a possibility, for the oscillatory velocities of the electrons must be much less than their thermal velocities to avoid local ionization effects, whence it is evident that an electron free path is only slightly undulating and any effect of the Debye-Hückel cloud must be almost exclusively viscous. Even if the electron temperature was lowered to the point where the oscillatory velocity was the larger, no elastic effect could be expected because of the way that this temperature affects the "Debye distance" also. Thus the root-mean-square electron velocity is

$$\bar{v} = (3kT_e/m)^{1/2} \tag{2}$$

Using the plasma-electron frequency

$$\nu = (Ne^2/\pi m)^{1/2} \tag{3}$$

because it is an upper limit of possible plasma-resonance frequencies, we find that the minimum net distance travelled by an electron in one period is

$$\lambda_c = \bar{v}/\nu = (3\pi kT_e/Ne^2)^{1/2}. \tag{4}$$

Finally, the "Debye distance" is

$$\lambda_D = (kT_e/8\pi Ne^2)^{1/2} \tag{5}$$

whence

$$\lambda_c/\lambda_D = 15.4 \tag{6}$$

at a minimum so that even if the cloud could have been formed in the short time available and could have persisted for the interval of one cycle, the returning electron would still be far outside its field.

III. Dependence of Plasma Resonance on Field Configuration—Experimental

The change in plasma resonance frequency arising from a change in shape factor has been observed by using an elliptical plasma between the condenser plates of the apparatus described in II and shown there in Fig. 4. A cylindrical glass tube was hand-worked to the desired elliptical cross section, was flared to circles at the ends, and was then inserted loosely in the cylindrical tube which formed an integral part of the discharge tube. This structure was used to avoid exposing the elliptical tube to the crushing effect of atmospheric pressure. The internal dimensions of the elliptical cross section were 11.7 by 26.0 mm.

Fig. 1 shows the resonance characteristics obtained for the plasma positions indicated. It will be noted that the puzzling double resonance occurs in the 0° position but not in 90° position, the slight elastic response at 20 m.a. probably arising from a small oscillation component parallel to the major axis, made possible by an inexact setting of the tube. If the *a*-type resonance in the 0° position be assumed to correspond to the resonance in the 90° position, it is seen that the ratio of arc currents at resonance, $17/8 = 2.1$, is very nearly equal to the ratio of axes, $26/11.7 = 2.22$. Since II, Fig. 8, shows that ionization intensity is proportional to arc current, this constitutes a confirmation of the theory sketched roughly in II section IX and developed more completely below. The appearance of all three resonances and their unchanged positions at plasma orientations intermediate between 0° and 90° shows the independence of the two modes of oscillation, another feature brought out by the theory.

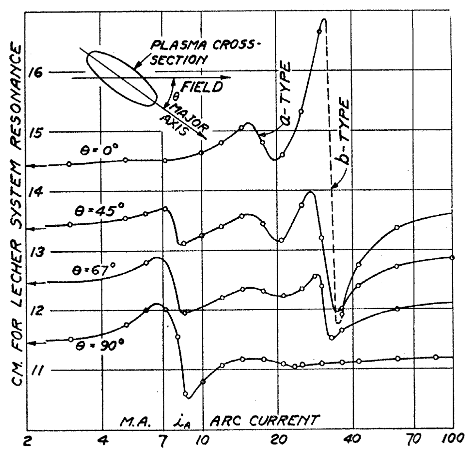

Fig. 1. Dependence of resonance in an elliptical plasma upon field orientation. Impressed frequency, 1.94×10^8 herz; condensed mercury temperature, 24 to 27°C. The zero of the ordinate scale is arbitrary, each curve being displaced 1 cm for clarity.

IV. Dependence of Plasma Resonance on Field Configuration—Theoretical

Let the x-axis of a coordinate system have the direction of the major axis, a, of an elliptic-cylindrical plasma, and the y-axis the direction of the minor axis, b, and let the impressed field $E = E_0 \sin \omega t$ make the angle θ with the x-axis as indicated in Fig. 1. Resolving the field into components, we have

$$X = E \cos \theta \quad Y = E \sin \theta. \tag{7}$$

From electrostatic theory it can be shown that if the plasma has a specific inductive capacity K_P, the electric field E_i (X_i, Y_i) inside the plasma is given by

$$X_i = X[1 + (K_p - 1)b/(a + b)]^{-1} \tag{8}$$

and likewise for Y_i. In addition, the polarization is

$$P_x = (X_i/4\pi)(K_p - 1). \tag{9}$$

Now the polarization is by definition

$$P_x = - Ne\xi \tag{10}$$

where N is the electron density and ξ the electron displacement, and the equation of motion is

$$eX_i = -m\ddot{\xi}. \tag{11}$$

Eliminating K_p, X_i, and P_x from these four equations gives

$$\ddot{\xi} + (4\pi Ne^2/m)[b/(a+b)]\xi + (e/m)X = 0 \tag{12}$$

for major axis oscillations and similarly

$$\ddot{\eta} + (4\pi Ne^2/m)[a/(a+b)]\eta + (e/m)Y = 0 \tag{13}$$

for minor axis oscillations, each independent of the other. Thus the natural frequencies corresponding to a certain electron density are less than the plasma electron frequency in the respective ratios

$$\phi_x^{1/2} = [b/(a+b)]^{1/2}, \quad \phi_y^{1/2} = [a/(a+b)]^{1/2} \tag{14}$$

where the ϕ's are the "shape factors" of II section IX.

In terms of the present experiment where N was varied to obtain resonance while ω was kept constant, we have

$$(4\pi N_x e^2/m)[b/(a+b)] = \omega^2$$
$$(4\pi N_y e^2/m)[a/(a+b)] = \omega^2$$

whence

$$N_x/N_y = a/b \tag{15}$$

as was found experimentally.