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INTENSITIES OF BALMER EMISSION LINES IN STELLAR SPECTRA

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Abstract

Contours of the Balmer emission lines in the spectra of fourteen stars of spectral class *Be* have been measured on twenty-seven spectrograms. In many stars the lines are broad and flat-topped, with steep outer edges, while in others they are double. The contours are explained by Doppler effect caused by the rotation of a nebula surrounding the star. The mean relative total intensities are: $\beta:\gamma:\delta:\epsilon=10:6.2:5.2:(5.4)$. The slow decrement in intensity agrees with the hypothesis that the bright lines are caused mostly by recombination. Lines produced in this way must be strongest (other things being equal) in a medium containing a maximum number of atoms of the next stage of ionization. This agrees with the observed strength of the emission lines of Fe II and of H in the earlier subdivisions of spectral class *B*.

Introduction

APPROXIMATELY one out of every fifteen stars of spectral class *B*0 to *B*5 (effective temperature 23,000°C to 15,000°C absolute) shows, in addition to the usual absorption-line spectrum, a number of emission lines. Bright Balmer lines are observed most frequently, but occasionally emission lines of Fe II, He I and He II, Mg II, etc., are also present. The emission lines of hydrogen are usually superposed over broad and shallow absorption lines similar to those observed in normal stellar spectra.[1]

Until recently little was known concerning the origin of these bright lines. The normal reversing layer of a star's atmosphere produces absorption lines only.[2] In many stars the emission lines are conspicuously broadened. Attempts have been made by various investigators to attribute this peculiar broadening to pressure or Stark effect. But we now know that in the reversing layer the ionic Stark effect is very powerful and that it causes extremely broad absorption lines of hydrogen having wings sometimes one hundred or more angstroms in extent. The contours of the emission lines, on the other hand, are quite different. Thus, in Fig. 1 the star 25 Orionis has a typical broad emission line, with flat top and steep outer gradients. It seems evident that this type of broadening cannot be due to Stark effect.

A much better representation of the contours may be obtained by assuming that the bright lines originate in a gaseous shell outside the reversing layer of the star. This assumption agrees well with the results of theoretical investigations.[2] The broadening of the bright lines may then be explained as a Doppler effect caused by the rotation of the nebulous shell around the star.

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[1] For further details see: O. Struve, Astrophys. J. **73**, 94 (1931) and the original papers by P. W. Merrill, R. H. Curtiss and W. J. S. Lockyer.

[2] S. Rosseland, Astrophysik auf atomtheoretischer Grundlage, p. 198, 1931.

This not only accounts for the peculiar flat-topped contours of the emission lines in such stars as 25 Orionis, but it also explains the doubling of the emission lines in ψ Persei, π Aquarii and many other stars.[3] Furthermore, there appears to be a distinct tendency for broad and widely-separated double emission lines to occur in stars having "dish-shaped" absorption lines. On the other hand, narrow emission lines occur predominantly in stars having normal absorption lines of He, Mg II, etc. Since it is believed that dish-shaped absorption lines are produced by rapid axial rotation of the stars, it is impossible to escape from the conclusion that the broadening of the emission lines is also caused by rotation.

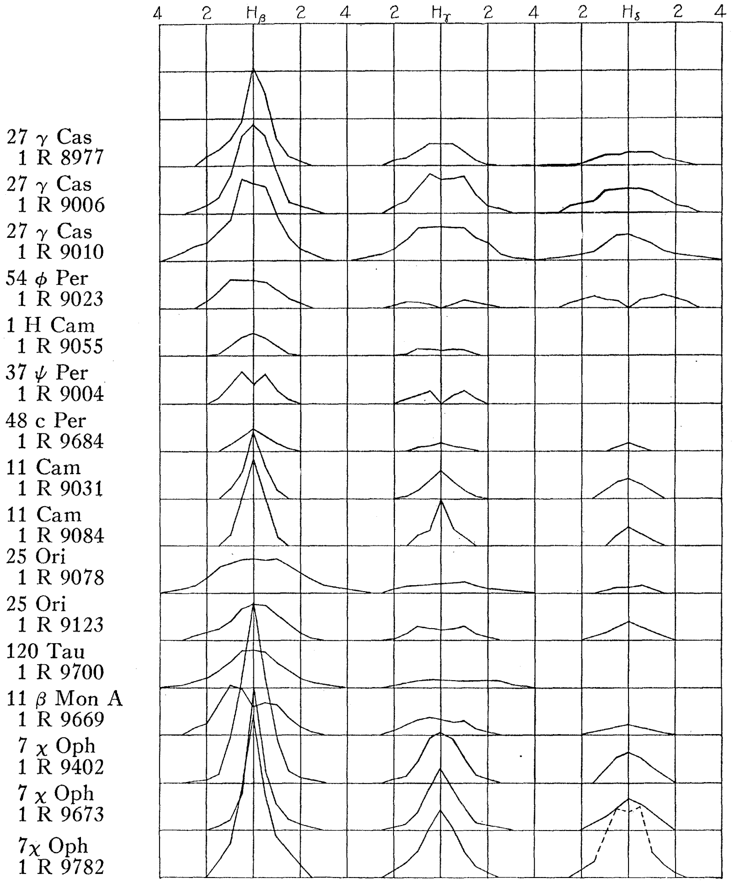

Fig. 1. Contours of hydrogen emission lines in *Be* stars. One division in the abscissa corresponds to two millimeters on the microphotometer tracing. One division in the ordinate corresponds to 0.75 of the intensity of the continuous spectrum adjoining each line. The names of the stars and the numbers of our plates are given at the left.

Additional evidence in favor of Doppler effect as the primary cause of

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[3] O. Struve, Astrophys. J. **73,** 94 (1931).

widening is presented in the work of C. D. Higgs,[4] who found that the contours of the bright Fe II lines in γ Cassiopeiae agree in width with those of the Balmer lines. It would be difficult to explain this similarity in the contours by processes other than Doppler effect.

The rotational hypothesis is sufficiently flexible to account for many of the observed contours of emission lines. Thus, a nebulous ring rotating around a central star would give rise to a double bright line, with steep outer edges.[5] On the other hand, consider a spherical nebula of uniform density, which greatly exceeds the central star in size. Suppose for simplicity that the nebula rotates as a solid. Then, all atoms which lie in a plane that is parallel to the axis of rotation and passes through the line of sight, have the same velocity component in the line of sight:

$$\rho = v \sin i \sin \theta = (x/r)v \sin i$$

where x is the distance of the plane from the central meridian, and r is the radius of the nebula. Since the nebula is transparent to its own light,[6] all atoms lying in the same cross section of the sphere contribute equally to the formation of the line. Consequently, each value of ρ appears in the contour with a weight of

$$(1 - x^2/r^2).$$

If the original contour (not affected by rotation) is very narrow, then the contour of the broadened line is directly given by

$$I = I_0(1 - x^2/r^2). \tag{1}$$

If, however, the original contour is not very narrow, the broadened contour may be evaluated by a process of graphical integration similar to the one used previously for stellar absorption lines.[7] The contour expressed by (1) has a flat top, suggestive of the observed contours of stars like 25 Orionis. In reality, the nebula would not rotate as a solid, since each atom is virtually free to move like a satellite under the attraction of the central force. However, this does not affect our conclusion that flat-topped single lines as well as double bright lines may be explained by the mechanism of rotation.

It should be mentioned that for certain stars the central depression is very narrow and deep. This is probably due to real central absorption in outlying layers of hydrogen (e.g.11β Monocerotis).[8] The doubling caused by rotation is probably responsible for widely-spaced components separated by a diffuse absorption line (e.g.31o Aquarii).

If our conclusion that the bright lines originate in a nebula around the star is correct, it becomes of interest to investigate the mechanism by which

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[4] C. D. Higgs, Astrophys. J. **70,** 251 (1929).

[5] O. Struve, Astrophys. J. **73,** 101 (1931).

[6] A. S. Eddington: The Internal Constitution of the Stars, p. 388, 1926.

[7] O. Struve, Astrophys. J. **72,** 13 (1930).

[8] It is, however, difficult to reconcile this view with Eddington's statement that nebulae are transparent to the light of the Balmer lines. For the same reason it would be difficult to attribute the sharp central depressions to absorption in interstellar space.

the lines are excited to emission. It is generally believed that the hydrogen lines emitted by gaseous nebulae are the result of recombination processes.[9] The discovery of W. H. Wright[10] that certain planetary nebulae show a continuous emission spectrum beyond the limit of the Balmer series lends considerable weight to this belief. A similar continuous emission spectrum was found by Ch'ing-Sung Yü[11] in several stars having bright Balmer lines (γ Cassiopeiae, ϕ Persei, χ Ophiuchi, etc.); it is therefore probable that the emission lines in such stars are also caused by recombination.

The observational results of the following section have been derived to provide material for a comparison of the relative Balmer-line intensities in *Be* stars and in gaseous nebulae. For the latter we use the determinations of H. H. Plaskett;[9] the former have not, to our knowledge, been previously determined.

OBSERVATIONAL RESULTS

For this investigation we have selected twenty-seven plates of fourteen stars showing fairly strong bright lines. Stars having widely-separated double bright lines were not included, but there are several in which the separation is small or moderate. The quantities to be derived are the relative total energies emitted in the Balmer lines. The spectrograms, calibrated in the usual way with a tube sensitometer, were analysed by means of a thermoelectric microphotometer, and the galvanometer deflections on the tracings were transformed into relative intensities with the use of the calibration marks. An ordinary electric lamp served as a light-source in the sensitometer, and no attempt was made to use separate calibrations for each wave-length. However, from the work of G. R. Harrison[12] and from unpublished tests made at the Yerkes Observatory, it appears that for the particular photographic emulsion (Eastman 40) used by us the characteristic curve does not vary much with wave-length, within the limits here considered.

The intensity measured at any point of the emission line consists of the intensity of the emission line proper plus the residual intensity of the absorption line forming the background on the plate. To allow for this background intensity we have extrapolated the observed wings of the absorption lines towards the center, on the original tracings, and have in each case taken

$$I(\text{emission}) = I(\text{observed}) - I(\text{absorption})$$

There is some uncertainty in the graphic extrapolation of the absorption lines. We have attempted to make them similar to the lines observed in normal *B* stars.[13] Any errors which may have come in would be greatest in the case of comparatively faint emission lines. Stars with strong lines probably

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[9] H. Zanstra, Astrophys. J. **65**, 50 (1927); H Zanstra, Zeits. f. Astrophysik **2**, 1 (1931). H. H. Plaskett, Harvard College Observatory, Circular No. 335, J. A. Carroll, Monthly Notices of the Royal Astronomical Society **90**, 588 (1930). A. S. Eddington; The Internal Constitution of the Stars, p. 388, 1926.

[10] W. H. Wright, Nature **109**, 810 (1922).

[11] Ch'ing-Sung Yü, Publications of the Astronomical Society of the Pacific **39**, 112 (1927)

[12] G. R. Harrison, J.O.S.A. **19**, 267 (1929).

[13] C. T. Elvey, Astrophys. J. **71**, 191 (1930).

give more reliable results. It is surprising that in several stars the central absorption, which is well visible on the original negatives, is nearly lost by allowing for the intensity of the background. In other stars, certain fine details visible on the plates were lost in the unavoidable process of smoothing during the measurements, but since it was our intention to get total energies we did not attempt to preserve these details. Nevertheless the differences in the shapes of the lines are well visible in the contours: some are narrow and have peaks in the centers, while others are broadened and flat-topped or even depressed in the centers.

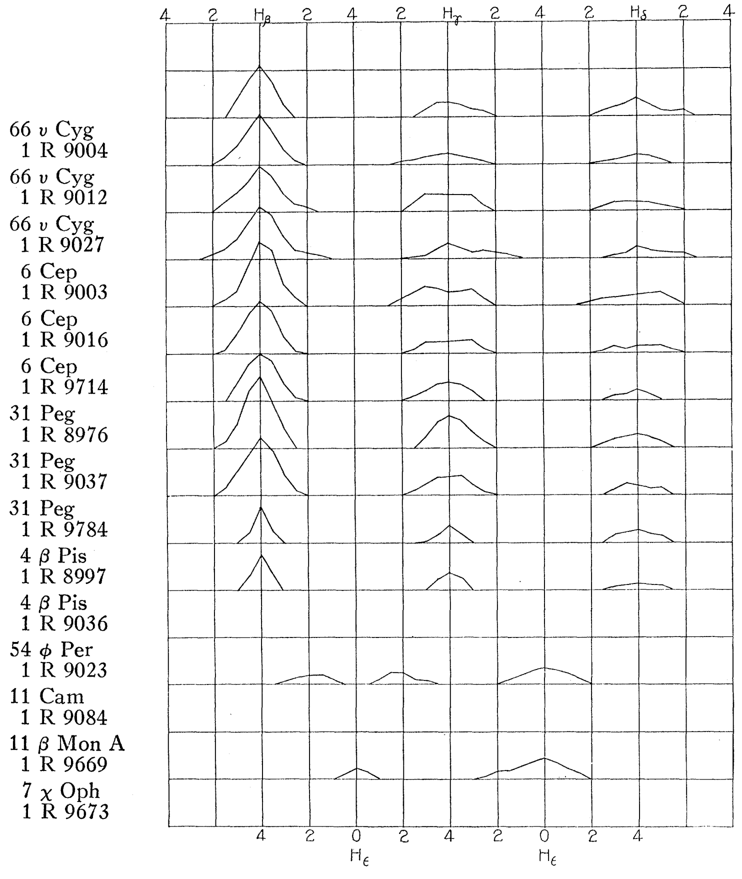

Fig. 2. Continuation of Fig. 1. The four lines at the bottom represent *Hε* in four stars.

The resulting intensities, as plotted against millimeters on the tracings, are shown in Figs. 1 and 2. One millimeter corresponds to 2.59A at *Hβ*, 1.64A at *Hγ*, 1.27A at *Hδ* and 1.09A at *Hε*. For each line the intensity of the adjoining portions of the continuous spectrum is taken as a unit. The transmission factors of the atmosphere and of the optical system are considered the same for the emission line as for the adjoining portions of the continuous spectrum. The true distribution of energy in the continuous spectrum is known since the stars may be regarded as black-body radiators. Following Russell, Dugan, and Stewart,[14] we have adopted the following temperature scale:

Spectrum:	*B0*	*B1*	*B2*	*B3*	*B5*
Temperature (°K):	23,000	21,000	19,000	17,500	15,000

A small error in the temperature would not appreciably affect our results. Planck's law leads to the following relative intensities in the continuous spectrum:

T(°K)	P_γ/P_β	P_δ/P_β	P_ϵ/P_β
23,000	1.42	1.70	1.87
21,000	1.41	1.68	1.84
19,000	1.40	1.64	1.80
17,500	1.38	1.61	1.76
15,000	1.35	1.55	1.67

Let the observed relative intensities of the continuous spectrum near $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$ be designated by C_β, C_γ, C_δ, C_ϵ, and let the intensities of the emission lines be given by β, γ, δ, ϵ. Then the observed intensities on the plate, referred to the intensity of the continuous spectrum at $H\beta$ as a unit, are:

$$\gamma C_\gamma/C_\beta;\ \delta C_\delta/C_\beta;\ \epsilon C_\epsilon/C_\beta$$

These values multiplied by the transmission factors

$$\frac{P_\gamma C_\beta}{P_\beta C_\gamma};\ \frac{P_\delta C_\beta}{P_\beta C_\delta};\ \frac{P_\epsilon C_\beta}{P_\beta C_\epsilon}$$

give the final intensities of the bright lines. The factors C_γ/C_β etc. cancel out.

TABLE I.

Star	Sp	Plate	β	γ	δ	ϵ	$\frac{C\beta}{C\gamma}$	$\frac{C\beta}{C\delta}$	$\frac{C\beta}{C\epsilon}$	$\gamma\frac{P\gamma}{P\beta}$	$\delta\frac{P\delta}{P\beta}$	$\epsilon\frac{P\delta}{P\beta}$
27γ Cas	B0p	IR 8977	10	3.7	2.4	—	0.92	1.52	—	5.25	4.08	—
		9006	10	6.0	4.3	—	0.84	1.30	—	8.52	7.31	—
		9010	10	6.0	3.4	—	0.81	1.32	—	8.52	5.78	—
54ϕ Per	B0p	9023	10	2.2	4.8	4.0	1.24	1.39	2.17	3.12	8.16	7.48
1 H Cam	B3p	9055	10	3.9	—	—	1.14	1.59	—	5.38	—	—
37ψ Per	B5p	9044	10	4.4	—	—	1.19	1.82	—	5.94	—	—
48c Per	B3p	9684	10	3.0	2.1	—	0.91	1.02	—	4.14	3.38	—
11 Cam	B3p	9031	10	5.9	4.7	—	1.06	1.19	—	8.14	7.57	—
		9084	10	4.6	2.5	3.6	1.00	1.35	3.33	6.35	4.02	6.34
25 Ori	B3p	9078	10	1.9	0.7	—	0.77	0.95	—	2.62	1.13	—
		9123	10	3.8	3.4	—	0.71	0.96	—	5.24	5.47	—
120 Tau	B3p	9700	10	3.2	—	—	0.82	1.23	—	4.42	—	—
11β MonA	B3p	9669	10	3.6	1.4	0.8	1.01	1.43	3.44	4.97	2.25	1.41
7χ Oph	B3p	9402	10	4.0	2.3	—	1.11	1.75	—	5.52	3.70	—
		9673	10	7.1	4.4	3.5	0.77	1.05	2.44	9.80	7.08	6.16
		9782	10	5.6	6.6	—	1.26	3.12	—	7.73	10.63	—
66υ Cyg	B3p	9004	10	4.0	4.2	—	1.14	1.59	—	5.52	6.76	—
		9012	10	3.4	2.1	—	0.95	1.49	—	4.69	3.38	—
		9027	10	5.7	2.4	—	0.78	1.20	—	7.87	3.86	—
6 Cep	B3p	9003	10	3.2	2.1	—	1.07	1.92	—	4.42	3.38	—
		9016	10	4.7	3.2	—	1.10	2.00	—	6.49	5.15	—
		9714	10	4.0	1.8	—	1.10	1.78	—	5.52	2.90	—
31 Peg	B3p	8976	10	4.5	1.7	—	1.10	1.70	—	6.21	2.74	—
		9037	10	5.2	2.3	—	1.15	1.56	—	7.18	3.70	—
		9784	10	4.3	1.8	—	0.55	1.00	—	5.93	2.90	—
4β Pis	B5p	8997	10	6.3	7.6	—	0.83	1.33	—	8.50	11.78	—
		9036	10	6.4	4.4	—	1.15	1.47	—	8.64	6.82	—
									Mean	6.17	5.16	5.35

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[14] Russell, Dugan and Stewart, Astronomy **2,** 753 (1927).

The relative intensities of the lines, as they appear on the plate, may be obtained by dividing the values in columns 5, 6, 7 of Table I by the values in columns 8, 9, 10 respectively. The final relative intensities of $H\gamma$, $H\delta$, $H\epsilon$ are shown in columns 11, 12, 13. The intensity of $H\beta$ was taken equal to 10.

An inspection of the contours in Figs. 1 and 2 and of the final intensities in Table I shows that within the rather large accidental errors the relative intensities are about the same in all stars. The similarity is even increased if we consider only stars with strong lines. The mean for all 27 plates is

$$H_\beta : H_\gamma : H_\delta : H_\epsilon = 10:6.2:5.2:(5.4).$$

The last value is quite uncertain and has therefore been placed in parenthesis.[15] These relative intensities are probably affected by considerable errors due to the difficulty of separating the emission from the continuous background; but they show that the decrease in intensity towards higher members of the Balmer series is at least as slow as, and perhaps even slower than, that observed in the gaseous nebulae. For the latter, H. H. Plaskett[9] finds, as the mean of eight nebulae:

$$H_\beta : H_\gamma : H_\delta = 10:4.25:1.83.$$

The comparatively slow decrease in intensity in *Be* stars agrees qualitatively with the result of R. H. Curtiss,[16] who observed the bright lines in 11 Camelopardalis as far as $H\tau$ (the 19th member). In gaseous nebulae, where the observations are easier because of freedom from the continuous spectrum and from broadening, the Balmer series was traced in some cases to $H\xi$ (the 14th member).[17]

DISCUSSION

The relatively slow decrease in the intensities of the Balmer lines in *Be* stars, together with Yü's discovery of continuous emission beyond the series limit, suggests that the Balmer lines are produced by recombination: the hydrogen atoms in the nebulous shell are photoelectrically ionized by absorption of continuous stellar radiation on the high-frequency side of the limit of the Lyman series. The recombination of these free electrons with hydrogen ions causes a number of continuous emissions at the Lyman, Balmer, Paschen, Brackett, etc., limits, and provides each quantum level with a certain population Nn which depends upon the probability of capture into a state with total quantum number n. The excited atoms thus formed lose energy in emitting the various series lines, of which the Balmer lines alone are accessible to observation.

Comparing the *Be* stars with the stimulating stars in gaseous nebulae, we notice that the latter are usually of a spectral class earlier than $B1$ while the former are frequently of class $B3$, $B5$ or even later. The question

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[15] The relative intensity of $H\epsilon$ (and to a small extent that of $H\delta$) has been somewhat exaggerated, because in forming the mean we have disregarded all plates on which it could not be measured.

[16] Publications of the Observatory, University of Michigan **3**, 256 (1923).

[17] F. Becker and W. Grotrian: Ergeb. d. exakt. Naturwiss. **7**, 24 (1928).

arises: are these comparatively cool stars sufficiently powerful in their radiation of frequencies greater than $\nu_0 = 32.84 \times 10^{14}$ to stimulate an observable amount of luminosity in the nebulae? The total number of quanta emitted by a star in such frequencies is, according to H. Zanstra:[18]

$$\frac{8\pi^2 R^2 k^3}{c^2 h^3} T^3 \int_{x_0}^{\infty} \frac{x^2}{e^x - 1} dx; \quad x = \frac{h\nu}{kT}$$

Consider two stars: $T_1 = 30{,}000$°C, and $T_2 = 15{,}000$°C. The first may be regarded as typical for the gaseous nebulae, the second for the *Be* stars. The ratio of the total energies is

$$\frac{T_1^3}{T_2^3} \frac{\int_{x_0}^{\infty} \frac{x^2}{e^x - 1} dx}{\int_{x_0}^{\infty} \frac{x^2}{e^x - 1} dx} \approx 400.$$

If all radiation beyond ν_0 were absorbed and reemitted in the two nebulae, the gaseous nebula should have a total luminosity 400 times greater than the nebula surrounding the *Be* star. But since in the former the light is spread over a considerable surface, while in the latter it is concentrated in a point-image, it would appear to be possible for a star of class *B*5 to excite a nebula so that it would be perceptible on the spectrogram.

Aside from this difference in total luminosity, there is also a difference in the average kinetic energies of the liberated electrons. According to Eddington[19] the initial electron-temperature is approximately

$$T_0 = (\tfrac{2}{3}) T (1 - 1/x_0)$$

where $x_0 = h\nu_0/kT$, and T is the effective temperature of the star. The second term in parenthesis is usually so small that it can be neglected here. The average kinetic energy of the electrons is

$$E = (\tfrac{3}{2}) k T_0 \propto T.$$

Consequently in our gaseous nebula the average kinetic energy of the electrons should be twice as great as in the nebula associated with the *Be* star.

It would be tempting to attribute the relatively slower Balmer decrement in *Be* stars, as compared to that of gaseous nebulae, to this difference in electron temperature. But we believe that our observations are not sufficiently accurate to serve as a basis for this deduction. There appears to be no experimental proof of any change in the relative intensities of the lines depending upon electron-velocities. Carl Kenty[20] and Miss Hayner,[21] have found that the line-intensities decrease rapidly with increasing electron-

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[18] H. Zanstra, Astrophys. J. **65**, 53 (1927).

[19] A. S. Eddington: The Internal Constitution of the Stars, p. 377, 1926.

[20] Carl Kenty, Phys. Rev. **32**, 624 (1928).

[21] Lucy J. Hayner, Zeits. f. Physik **35**, 365 (1926).

velocity, but their observations do not settle the question whether the relative intensities remain unaltered or not. In the central blue ring of the electrodeless discharge G. Herzberg[22] found the following intensities:

$$\alpha:\beta:\gamma:\delta:\epsilon = 1.00:0.95:0.40:0.14:0.05$$

In the outer red ring he found:

$$\alpha:\beta:\gamma:\delta:\epsilon = 1.00:0.25:0.08:0.020:0\cdot 010.$$

This difference has been interpreted[23] as being due to differences in the relative velocities of the free electrons, but there is no certainty that recombination alone is responsible for the lines throughout the entire cross section of the discharge tube.

A recent theoretical investigation by E. C. G. Stueckelberg and P. M. Morse[24] defines the effective cross-section of recombination

$$q = \sum_{n,l} A(nl)\cdot 10^{-20}\cdot Z^2/V$$

where V is the energy of the electrons. The values of $A(nl)$ were computed by Stueckelberg and Morse, and are practically independent of V. The sums $\Sigma_l A(nl)$ should be proportional to the populations in the various states of total quantum number n:

n	1	2	3	4
$\sum_l A(nl)$:	0.227	0.143	0.104	0.081 .

J. A. Carroll[25] has shown how the expected intensities of the bright lines may be computed by means of the values of $\Sigma_l A(nl)$ and of the transition probabilities computed by Francis G. Slack.[26] Unfortunately the theoretical $\Sigma_l A(nl)$ have been given only for small values of n, and the computation has therefore not been carried out. However, it appears that the expected Balmer decrement for recombination is even slower than that observed for the *Be* stars. This applies of course equally well to the observational data concerning the gaseous nebulae where there can be no reasonable doubt as to the existence of recombination.

It has been pointed out by B. P. Gerasimovič[27] that the bright lines in *Be* stars do not correspond to the state of ionization of the reversing layer. Thus bright Fe II lines are intense in classes *B5e* and *B3e*, while in ordinary absorption spectra ionized iron vanishes near *B8* and reaches maximum near *F0*. A similar situation occurs for the hydrogen absorption lines which have their maxima at *A0*. If the gases responsible for the bright lines were in thermodynamic equilibrium (similar to that believed to hold for the reversing

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[22] G. Herzberg, Ann. d. Physik **84**, 565 (1927).

[23] H. H. Plaskett, Harvard College Observatory, Circular No. 335, 14, 1928.

[24] E. C. G. Stueckelberg and Philip M. Morse, Phys. Rev. **36**, 16 (1930).

[25] J. A. Carroll, Monthly Notices of the Royal Astronomical Society **90**, 588 (1930).

[26] Francis G. Slack, Phys. Rev. **31**, 527 (1928).

[27] Private communication.

layer) we should conclude that their state of ionization is lower than that of the reversing layer which is responsible for the absorption lines. Eddington[28] has shown that the state of ionization in a nebula of temperature T and of density ρ will be the same as that of a gas in thermodynamic equilibrium of the same temperature and of density $\rho\delta$, where δ is the "dilution factor." Since, for constant temperature, lower ionization means greater density, we must have

$$\rho\delta > \rho_0$$

where ρ_0 is the density of the reversing layer. Professor Gerasimovič has pointed out to us that the relative rotational velocities from bright and dark lines show that δ should be of the order of 3. Consequently

$$\rho > \rho_0/\delta = \rho_0/3.$$

This is, of course, altogether too large a density for a nebula.

But the difficulty is only an apparent one. The ionization in the nebula is probably not much lower than in the reversing layer. In the case of recombination the maximum intensity of a line of a given stage of ionization n is reached when the medium contains a maximum number of atoms ionized $(n+1)$ times. Thus the emission lines of Fe II will appear strongest when the medium contains a large number of Fe^{++} atoms, and we should therefore expect the maximum of the bright recombination lines of Fe II to coincide with the maximum of the Fe III absorption lines, provided that $\rho\delta \approx \rho_0$. Unfortunately, no absorption lines of Fe III are known in stellar spectra, but from the two first ionization potentials of iron, 7.8 and 16.5 volts, we infer that our expectations are approximately confirmed. Similar arguments hold in the case of hydrogen.

We are indebted to Dr. Carl Eckart for various helpful suggestions in connection with this work and to Dr. C. T. Elvey for some of the observational material.

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[28] A. S. Eddington: The Internal Constitution of the Stars, p. 382, 1926.