ELECTRONIC VELOCITIES IN THE POSITIVE COLUMN OF HIGH FREQUENCY DISCHARGES

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Abstract

Theoretical discussion of the mechanics of individual electrons in the positive column of high frequency discharges with special consideration of the effect of elastic impacts proves that small mean values of the electric force do not prohibit the production of electrons of sufficient velocity to excite or even ionize gas molecules.

In a recent paper Charles J. Brasefield[1] has published measurements of the potential drop at the electrodes and the electric force in the positive column of high frequency discharges in mercury, helium, and neon for different gas pressures and frequencies of oscillation. The procedure of measuring the electric force in the positive column was the same as that used by the author.[2] and others in earlier work on high frequency discharges. It consisted in measuring the total voltage between the electrodes for various distances of the electrodes and a given current. The mean electric force of the positive column can then be calculated in the known manner, if one assumes that the drop in potential at the electrodes is, for a given current, independent of the separation of the electrodes.

Discussing his interesting results Brasefield concluded that "these results showed that in general, the magnitude of the electric force was too small to produce electrons whose velocity would be sufficient to ionize or excite the gas." This statement would be of the greatest importance for the theory of the positive column in high frequency discharges if it were the only reasonable inference to be drawn from the experimental results. But an explanation, and a very plausible one, may be obtained by calculations similar to those given by the author for a similar problem in an earlier paper.[3]

In order to look clearly into this matter it is necessary to give the experimental results, the calculations and the proposed solution of the problem in essential detail.

Brasefield, as others, observed that for a given current and a given frequency there exists an optimum pressure at which the conductivity is a maximum. The electric force at this gas pressure and at gas pressures near to it is very small. He then calculates the electric force necessary to give an electron the ionizing velocity in an electric field alternating with high fre-

[1] Charles J. Brasefield, Phys. Rev. **37**, 82 (1931).
[2] E. Hiedemann, Verh. D. Phys. Ges. (3) **7**, 47 (1926); Ann. d. Physik **85**, 649 (1928).
[3] L. Ebeler and E. Hiedemann, Ann. d. Physik (5) **5**, 625 (1930).

quency. His calculation gives values for the necessary minimum electric force, which is much higher than the electric forces measured at or near the maximum of conductivity.

It may be pointed out at first that this calculation is based on the assumption that the electric force in the positive column of the high frequency discharge is the same at any point of the column, which supposition is by no means proved or even probable. But supposing that the above-mentioned procedure of measurement really gives the electric force in the positive column, not merely the mean value of it, a more comprehensive calculation gives a result, which differs slightly but not unimportantly from the result of Brasefield.

If E_0 is the amplitude of the electric force and f the frequency of oscillation, then the following equation is valid:

$$m\ddot{x} = eE_0 \sin 2\pi ft. \tag{1}$$

Integration of Equation (1) gives

$$\dot{x} = -\frac{e}{m}\frac{E_0}{2\pi f}\cos 2\pi ft + C. \tag{2}$$

If at the time zero, the electron has the velocity v_0 in the initial direction of the electric field[4] then

$$C = v_0 + \frac{e}{m}\frac{E_0}{2\pi f}\cdot$$

The velocity of the electron in the direction of the field is then given by Equation (3):[5]

$$\dot{x} = \frac{e}{m}\frac{E_0}{2\pi f}(1 - \cos 2\pi ft) + v_0. \tag{3}$$

The maximum velocity in the direction of the field will be obtained when $t = 1/2f$ and is given by

$$v_{\max} = \frac{e}{m}\frac{E_0}{\pi f} + v_0. \tag{4}$$

[4] By the direction of the electric field that direction is meant, in which the e.m.f. accelerates an electron.

[5] Brasefield gives the equation:

$$\dot{x} = \frac{e}{m}\cdot\frac{E_0}{2\pi f}\cdot[\cos\delta - \cos(2\pi ft+\delta)]$$

where δ is a phase constant depending on the value of E at the instant the velocity of the electrons is zero. Brasefield's equation includes therefore only those electrons which, at the time zero, have no velocity or a velocity contrary to the initial direction of the acceleration and smaller than the maximum velocity obtainable in half a cycle by the acceleration produced by the electric field. Equation (3) on the contrary includes electrons of all directions and velocities.

The total maximum velocity of the electron is the velocity in the resulting direction of the electron. If this direction forms an angle θ with the direction of the electric force, then the total maximum velocity $V_{\max}$ will be given by

$$V_{\max} = \frac{1}{\cos\theta}\left(\frac{e}{m}\frac{E_0}{\pi f} + v_0\right). \tag{4a}$$

The real velocity V of the electron at any moment is the essential element in all processess of exciting or ionizing gas molecules. For the following discussion, however, it will be sufficient to regard only the velocities in, or contrary to, the direction of the electric field. To simplify matters, Eq. (4) instead of (4a) will therefore be used from now on.

The essential difference between the result of this calculation and that given by Brasefield is that, to the velocity an electron may obtain in half a cycle the velocity of the electron at the beginning of the half cycle must be added, with the right sign of course. To calculate the electric force necessary to give to an electron the ionizing velocity v_i, we must use

$$E_0 = \frac{v_i - v_0}{e} m\pi f. \tag{5}$$

Now the electric forces measured by Brasefield demand —near the maximum of conductivity at least—velocities v_0 which are not small compared with v_i. That means that electrons of nearly sufficient velocity must be present at the beginning of half a cycle. How can such electrons be produced? This can easily be seen, if one remembers that an electron will make elastic impacts with gas molecules before it has obtained the excitation energy.[6] By these elastic impacts it will lose insignificant amounts of energy only, but the direction of its motion will be changed. To simplify the discussion only those impacts will be regarded, for which an electron will after the impact have a component of velocity v' in the direction contrary to that before the impact.

Let us consider now an electron which, at the time zero, has the velocity zero. After half a cycle it will have obtained the velocity $v_{\max}$. Now this electron may have an elastic impact and may immediately after the impact have the velocity v' in the direction contrary to its direction before the impact. As v' is a part of its former velocity, we may define v' by

$$v' = v_{\max}/n. \tag{6}$$

The next half cycle will give to this electron the velocity v''

$$v'' = V_{\max} + v' = V_{\max}(1 + 1/n). \tag{7}$$

[6] Here purposely no correct discrimination is made between the possibility for an electron to get excitation and that to get ionization energy because the difference of these energies is small compared with the difference between one of these energies and the maximum energy obtainable by the e.m.f. in half a cycle. The excitation energy is of primary importance too; especially in those cases in which metastable states are produced and ionization effected by successive impacts.

As this may happen several times and the various n must not be large compared with unity, one easily sees that, as a result of elastic impacts, an electron may obtain a multiple of the maximum velocity obtainable in half a cycle.

This effect of the elastic impacts on the electronic velocities will be efficient especially at gas pressures for which the path of an electron during half a cycle is comparable with the mean free path of the electron. The path of an electron with the velocity v_0 at the time zero between $t=0$ and $t=1/2f$ has been calculated[3] by integration of Equation (3) to be

$$x = \frac{v_0}{2f} + \frac{e}{m}\frac{E_0}{4\pi f^2}\,. \tag{8}$$

The gas pressures at which x will be comparable with the mean free path of an electron can thus easily be calculated. The result gives pressures of the same order of magnitude as the optimum pressures measured.[7]

From Eq. (8) one sees that Equation (7) is not quite correct, because if an electron of the velocity zero at the time zero has made an impact with a gas molecule after $t=1/2f$ then the electron will after the impact meet a gas molecule before the second half cycle is finished. The velocity of the electron immediately before the second impact will therefore be smaller than $v'+v_{\max}$. A correct equation can be obtained without difficulty, but even without doing so it can easily be seen that matters are much too complicated to allow more than a very rough calculation of the critical order of magnitudes.

In determining the conductivity not only the electrons with excitation or ionization energy are of importance, but the positive ions and the slower electrons too. In an earlier work A. v. Hippel[8] has shown that the energy which positive ions may periodically acquire in the alternating electric field of a high frequency discharge, is to be neglected compared with the ionizing energy. He has also directed attention to the fact that positive ions oscillate with only very small amplitudes, which at the optimum pressures measured by Brasefield may be neglected in comparison with the mean free path. This means that, if positive ions have once been produced, the chance of their getting lost from the discharge is relatively small when no static electric field is present. A small loss of ions on the other hand means a high degree of ionization and a high conductivity. The degree of ionization is limited by recombination and molecular diffusion to the glass walls and to a large extent probably by the static field due to the potential of the glass walls. By elastic impacts electrons will not only get a component of velocity parallel to the direction of the alternating electric force but also normal to it. As this velocity will not be altered by the electric force and as the mobility of the electrons is large a steady current of electrons should be directed to the glass walls and produce there a negative charge. A short time after the beginning of the discharge a stationary state will be reached and a static electric force will be produced.

[7] See also Charles J. Brasefield, Phys. Rev. **35**, 1073 (1930).

[8] A. v. Hippel, Ann. d. Phys. (4) 87, 1035 (1928).

This static field will enlarge the loss of positive ions and produce a deformation of the electric field of the alternating e.m.f. Hence the supposition that the electric force in the positive column is the same at any point of the column cannot be right and any calculation based on this assumption will not give correct, but at the best only approximate results. This is another reason that the small measured mean values of the electric force in the positive column do not call for the conclusion that electrons of excitation energy cannot be produced in the high frequency discharge at the maximum of conductivity.

Another important factor in the conductivity is introduced by the oscillation of the slower electrons in the alternating electric field. Their importance for the mechanism of the high frequency discharge at low pressures was pointed out first by F. Kirchner.[9] The motion of these oscillating electrons is given by[5]

$$m\ddot{x} = eE_0 \cos 2\pi f t \tag{9}$$

and the path in a quarter of a cycle; i.e., the amplitude of oscillation; is given by

$$x' = \frac{e}{m} \frac{E_0}{4\pi^2 f^2} \cdot \tag{10}$$

By comparison of Eq. (10) with Eq. (8) it can be seen that the optimum pressure for the oscillating electrons has not quite the same value as for the non-oscillating electrons, but that it is of the same order of magnitude. The importance of the oscillating electrons for the conductivity of the discharge lies, as is known, in the fact, pointed out by Kirchner, that by oscillating in the field these electrons will remain in the discharge.

Those electrons which at the time zero have no velocity or a velocity in the direction of their acceleration in the initial half cycle, will not oscillate in the discharge, but go in one direction only (of course without taking account of the effect of impacts). During half a cycle, they will be accelerated, during the next half they will be retarded by the alternating e.m.f. but they will always go forward. At very low pressures, where the mean free path of the electrons cannot be neglected compared with the separation of the electrodes, such non-oscillating electrons produced near the electrodes may thus reach the positive column and their part in the excitation or ionizing processes in the positive column may perhaps not be neglected.

In conclusion it may be said, that the surprisingly small mean values of the electric force measured in the positive column of high frequency discharges near the maximum of conductivity do not at all force one to assume that electrons of excitation energy cannot be produced in the positive column of these discharges.

[9] F. Kirchner, Ann. d. Physik (4) **77**, 287 (1925).