because (1) we may work with $S(q')$ independent of t provided the matrix $q(q', q'')$ involves t as a parameter, (2) the equation is obviously true in the reference system of the energy where $q_{nm}(t) = q_{nm}(0) \exp [(2\pi i/h)(W_n - W_m)t]$ (3) the values of the integrals on the right and left of the above equation are indepedent of the reference system used for matrix representation. In a similar way we have

$$\frac{d\bar{p}}{dt} = \dot{\bar{p}} = -\frac{\overline{\partial V}}{\partial q}.$$

The way in which we use the matrices $q(q', q'')$ is explained very clearly in Schroedinger Sitzungsberichte der preussischen Akademie XXIV p. 417, 1930, and is treated in Weyl's book, chapter II section 15. Remembering that Schroedinger's $\psi(0)$ is our S, it is seen that at any time the averages $\overline{\partial V \partial/q}$ is taken weighing every element of the configuration space in proportion to $\psi^*(t)\psi(t)$ at that point at the particular time i.e. the average is taken in the same way as by Ehrenfest and Ruark. The same can be done for *any other equation which holds between matrix as well as classical quantities*. The only doubtful point of this shorthand proof lies in dealing with singular matrices which must be expressed by means of the δ function. This however, is usually only a formal objection since throwing away the "surface integral" contributions of partial integrations is also a necessary part of the purely wave-mechanical proof such as that of Ehrenfest quoted above.

Although the above statement is simple enough to be called trivial it seems that its physical implication is not always realized. Thus for instance the motion of a non-relativistic spinless electron in a magnetic field is governed by the same equations in quantum and in classical dynamics. The center of mass is therefore moving as the classical center of mass under the action of the mean electromagnetic field in the wave package. This has already been proved a long time ago by Kennard and elucidated by Eckart in connection with a recently suggested explanation of discrepancies between the values of e/m. Similarly with the relativistic Dirac equation the rate of change of the mechanical momentum is known to be given by a formula having the same structure as the classical formula, the classical velocities being replaced by $-c\alpha_i$. Again the mean value theorem holds. The equation which proves this is the one derived by Fock.[2]

$$\frac{d}{dt}(p_1 + eA_1/c) = e\frac{\partial A_0}{\partial x_1} + e\alpha_2\left(\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}\right) - e\alpha_3\left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right)$$

$$\frac{dx_1}{dt} = -c\alpha_1.$$

Here $p_1 + eA_1/c$ is the momentum in the direction x_1, A_0 is the scalar potential, (A_1, A_2, A_3) is the vector potential and α_1, α_2, α_3, α_4 are Dirac's four-row matrices. The equation proves that the rate of change of the average of $p_1 + eA_1/c$ of a wave package is related to the average force on the wave package by the equation of classical electordynamics

$$\boldsymbol{F} = \rho\{\boldsymbol{E} + [\mathrm{v}\boldsymbol{H}]/c\}.$$

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December 12, 1930.

[2] Fock, Zeits. f. Physik **55**, 127 (1929).

Block Structure and Ferromagnetism

In the following a model is proposed for ferromagnetic substance basedson theassumption of the existence of a "block structure" in solids as postulated by Smekal and Zwicky, and some of the properties of this model are pointed out. In a future communication it is hoped to give a more detailed and comprehensive account of the derivations and results indicated below.

A block, according to Zwicky,[1] is a region having the properties of a perfect crystal, surrounded by a surface physically different from the volume. This difference is connected with a local rearrangement of atoms such that along the surface the average interatomic distance is not that prevailing inside of a block. If such blocks exist, it is not unreasonable to assume that each one is spontaneously magnetized as predicted by the Weiss-Heisenberg theory

$$\frac{I_T}{I_0} = \tanh \frac{\mu_B N I_T}{KT}. \qquad (1)$$

[1] F. Zwicky, Helvetica Physica Acta **3**, 269 (1930).

In this expression the dependence on H is omitted as it is negligibly small for the field strengths used in most experiments, and its inclusion would add nothing to the following discussion. μ_B is a Bohr magneton. The assumption is now made, and this is the only *new* assumption, that various blocks may be magnetized in various directions, or what amounts to the same thing, any particular block may change the direction in which it is magnetized, and its magnetic moment need not be parallel to those of its neighbors. This is equivalent to saying that the electrons on opposite sides of a block surface are not ferromagnetically coupled to each other. Such a situation is not unthinkable, as such a coupling depends on Heisenberg's resonance integral which is known to be extraordinarily sensitive to variations in inter-atomic spacing, and such variations actually define the surface of a block.

Our model, then, is a group of permanent magnets, having primarily a thermomagnetic interaction. For the present, surface phenomena are neglected. In order to simplify the problem, the blocks are assumed isotropic, so that there are no "directions of easy magnetization." These last two assumptions should be dropped in a more exhaustive discussion. Under these conditions the blocks may be expected to interact in such a way that the magnetization is approximately given by

$$\frac{I}{I_T} = \coth x - \frac{1}{x} \qquad x = \frac{\mu}{KT}(H + LI). \quad (2)$$

Here μ is the block moment and is given by $I_T v$, the saturation intensity at the temperature T multiplied by the volume of a block. L is a factor varying between 0 for long thin blocks whose axis is parallel to H, and 4π for disk shaped blocks. LI represents the internal field. Eqs. (1) and (2) define the behaviour of our model.

The first point of interest is that this model is essentially that of Ewing, with the exception that the elementary magnets, instead of being of atomic dimensions, are somewhat larger. Thus the model is fundamentally capable of describing a magnetization curve and a hysteresis loop, as Ewing demonstrated experimentally. The details are determined by the only two arbitrary constants, appearing in the equations, μ and L. To get an idea of their order of magnitude it is possible to assign values to them which will give observed values for K_0, the initial susceptibility. Using experimental data obtained on iron, nickel and cobalt for K_0 and I_0, and substituting extreme values of L, we find that the block volume contains between 10^2 and 10^6 atoms. An intermediate value of L would give an intermediate block size.

The next point of interest is that this model gives the approach to saturation correctly. In a very thorough investigation Weiss[2] showed that most substances approach saturation according to the formulae

$$I = I_T(1 - a/H) \quad (3)$$

$$I_T = I_0(1 - AT^2). \quad (4)$$

The first of these formulae follows immediately from Eq. (2), and from the constant a it is possible to compute the number of atoms per block. ($a = KT/\mu$; $\mu = vI_T$). With Weiss's data for nickel and iron we get approximately 10^5 atoms per block. Weiss examined the approach to saturation as a function of the temperature between 100°K and 300°K. In this region I_T/I_0 as given by Eq. (1) is very approximately a straight line if plotted as a function of T^2. From the constant A it is possible to compute the molecular field constant N, and the values obtained are quite reasonable. It is true that an extrapolation of (4) for $T=0$ leads to somewhat too high values for I_0, but in general the error so committed will be small.

Further, this model has two Curie-points, a property of most ferromagnetic substances as pointed out by Forrer.[3] θ_p the paramagnetic Curie-point is defined by the modified Curie Equation $x = c/(T - \theta_p)$ for $T > \theta_p$. θ_f, the ferromagnetic Curie-point is in general less than θ_p, and is the temperature at which spontaneous magnetization disappears. θ_p is given in the usual way by Eq. (1). On the other hand Eq. (2) will itself have a Curie-point

$$\theta_f = \frac{\mu L I_T}{3K}.$$

That is, for $T > \theta_f$ the spontaneous magnetization of the whole material[4] will disappear, whereas for $T > \theta_p$ the blocks themselves lose their moments. From this it is evident

[2] P. Weiss, Ann. de Physique **12**, 20 (1929).

[3] R. Forrer, Journ. de Ph. et le Rad. **1**, 49 (1930).

that $\theta_f]_{T=\theta_f} < \theta_p$ as it is actually observed in most substances. A detailed examination shows that $\theta_p - \theta_f =$ approximately 15°C is consistent with blocks containing about 10^5 atoms.

Such an interpretation of θ_f brings with it the necessity of a new explanation of the thermal phenomena at the Curie-point. Such an explanation it is impossible to give until more definite assumptions can be made regarding the surface energies of the blocks, the effect of strains, etc. The fact that acceptance of the proposed model would require the abandonment of our present explanation of thermomagnetic phenomena at the Curie-point, would be a telling argument against such a model, were it not that just here the Weiss Theory is in part not quite satisfactory (in its interpretation of the change in specific heat for $T=\theta$) and in part purely thermodynamic (in the relationship it establishes between the specific heat and the magnetocaloric effect).

In conclusion I wish to point out that blocks containing approximately 10^5 atoms, as here postulated for the explanation of three ferromagnetic phenomena, are of the same order of magnitude as those postulated by Smekal and Zwicky to explain certain mechanical and electrical phenomena.

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[4] Spontaneous magnetization of the sample need not, of course, include the whole sample. It may be spontaneously magnetized in different directions in different regions. It would be sufficient that large groups of units, in this case larger groups of blocks, should possess a resultant moment in the absence of externally applied fields.

Secondary Electrons from Molybdenum

The writer has continued work begun by Soller (Phys. Rev. **36,** 664, 1930) on the distribution in energy of electrons emitted by a molybdenum target bombarded by a narrow, homogeneous beam of electrons of relatively low energy (20 to 100 volts).

The apparatus was a modification of that of Soller, employing the method of magnetic analysis. The target was heated by electron bombardment for a total of 700 hours. Observations were taken at intervals throughout this outgassing period. With the primary energy held constant, measurements were taken of the number of secondary electrons (N_ϵ) of energy ϵ. By changing the magnetic field, ϵ was made to vary in steps of 0.1 to 0.2 volts from zero up to the primary energy. Curves were plotted for $N_\epsilon/N \times$const. as a function of ϵ where N is the total number of primary electrons.

Curves plotted from the results obtained for a cold target show, in addition to the usual large group of secondary electrons with energy approximately that of the primaries, a low maximum in the region 5 to 15 volts and a second very broad and somewhat higher maximum at an energy approximately one-half that of the primaries. For low primary energies these two groups become merged. In addition there are three well defined maxima corresponding to electrons possessing energies 4.7, 11.2, and 23.2 ($\pm$.4) volts less than the energy of the true reflection maximum. These last maxima are similar to those found by Rudberg (Proc. Roy. Soc. **A127,** 111, 1930) for various metals and indicate the presence of reflected primary electrons which have lost a discrete amount of energy in the process of reflection. The height of these peaks increases as the velocity of the primary electrons decreases. These peaks appeared only after considerable outgassing and became more and more marked with further heating.

In attempting to eliminate the effects due to gas occluded on the surface of a cold target, the following procedure was adopted. With the magnetic field set to correspond to a given value of ϵ the target was heated to a bright yellow for thirty seconds and N_ϵ measured thirty seconds after the bombarding current was cut off. (The target ceased to be visibly red after about 20 seconds.) This procedure was followed through the whole range of values of ϵ. A typical curve obtained from these data was that for a primary energy of 60 volts. Observations taken in this way show a very large group of low velocity secondaries of energy from three to twenty-five volts, and N_ϵ/N now reaches a maximum value at 8 volts which is 27 times as great as the corresponding maximum at 8 volts for a cold target. For higher values of ϵ the values