THE EFFECT OF RESOLVING POWER ON MEASUREMENTS OF THE ABSORPTION COEFFICIENT OF ELECTRONS IN GASES

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Abstract

The absorption coefficient of electrons has been studied as a function of the resolving power of the experimental apparatus. An electron beam of 0.4 mm radius was used in a Mayer type of apparatus in which the opening at the end of the scattering chamber was of variable aperture. Values of the absorption coefficient were obtained in helium and in mercury vapor for 20, 40, 80 and 135 volt electrons over an angular aperture range of from 2° to 11° as measured from the end of the electron gun. All curves showed a definite decrease in the absorption coefficient with increase in angular aperture. These curves were corrected for the positive ion current, from the efficiency of ionization data of Bleakney and Smith, to give curves for the total number of electrons collected as a function of the size of the opening. These showed also a definite decrease with increase in aperture, corresponding to a preference for scattered electrons to be deflected through small angles. In helium the curves were steeper the higher the electron velocity, indicating that this preference is more decided for fast electrons than for slower electrons. In mercury vapor the curves were more nearly alike. The results do not indicate as decided a preference for small angle scattering as is indicated by the angular distribution curves of Arnot in mercury vapor. This discrepancy is not explained.

Introduction

EXPERIMENTS [1,2 3,4] designed to determine the angular distribution of electrons after collision with gas molecules indicate a decided preference for small angle scattering. In fact, most of the published curves show a a continued increase in the probability of scattering with decrease in angle, down to as small angles as have been obtained. Such curves for slow incident electrons are not so steep, in general, as are those for faster electrons, but in all cases the scattered electrons show this definite tendency to be concentrated about the forward direction.

The term "absorption coefficient" as applied to electron scattering by gas molecules has concerned itself with a determination of α from the relation $I = I_0 e^{-\alpha x p}$. In such experiments electrons of a definite velocity are caused to move in a beam (whose size and shape depend on the defining electrode system) through a scattering chamber of length x which contains a gas at a known pressure, p. On passing through this scattering chamber certain

[1] E. G. Dymond and E. E. Watson, Proc. Roy. Soc. **A122,** 571 (1929).
[2] G. P. Harnwell, Phys. Rev. **33,** 559; **34,** 661 (1929).
[3] F. L. Arnot, Proc. Roy. Soc. **A125,** 660 (1929).
[4] J. H. McMillen, Phys. Rev. **36,** 1034 (1930).

encounters with gas molecules cause some of the electrons to leave the beam, and so the beam of initial current strength, I_0, is reduced to the value I as given above. Experimentally two general methods have been used to determine α. The path of the electron beam may be a straight one as in a Mayer[5] type of apparatus, or the beam may be focused over a semicircular path by means of a magnetic field as was first done by Ramsauer.[6]

If $I'=I_0-I$ is the current to the scattering chamber, then, for low pressures, from the exponential relation given above, α is proportional to I'/I_0. Thus a determination of α for a particular apparatus is equivalent to measuring the fractional current to the scattering chamber. For low velocity electrons this current is composed merely of those electrons which have been deflected through an angle greater than some average minimum angle, $\bar{\theta}$. This angle is a function of the size of the opening at the end of the scattering chamber and of the length of the path. For electrons with a velocity greater than a certain value there may be ionization at an encounter which will result in at least three charged particles. A certain fraction of these will be collected, depending on the apparatus. If $\Phi_s(\theta)$, $\Phi_e(\theta)$ and $\Phi_i(\theta)$ are the probabilities that the initial electron, the ejected electron and the ion, respectively, will move in the direction θ after such an encounter, and if F^+ is the probability of ionization taking place, then for a Mayer type of apparatus the fractional current to the scattering chamber is

$$I'/I_0 = \int_{\bar{\theta}}^{\pi} (\Phi_s + F^+\Phi_e + F^+\Phi_i)d\theta \tag{1}$$

where the integral is taken over the possible angles of collection. If it is assumed that the ion has an equal probability of moving in any direction, Φ_i is a constant and the integral of the third term does not change much with a small change in the lower limit of integration, $\bar{\theta}$. The second term, representing the angular distribution function for the ejected electrons has not, to the author's knowledge, been investigated. This distribution may be uniform but it seems more probable that it would be somewhat similar to the distribution for the scattered primary electrons. The first term in the integrand is the probability function, $F(\theta)$, which is measured in angular scattering experiments. As has already been mentioned this function is one which in general increases with decreasing θ, being of particular weight for values of θ near θ. Hence one would expect that a small change in the lower limit of integration would produce a large change in the value of Eq. (1). Any distribution of the ejected electrons other than a uniform distribution would enhance this effect. Thus the current collected by the scattering chamber is very decidedly a function of the resolving power of the apparatus. In the Ramsauer type of apparatus the integrals of the second and third terms of Eq. (1) will probably just balance each other. Both the ejected electron and ion will have such energies that their paths will have a smaller radius of curvature than that of

[5] H. F. Mayer, Ann. d. Phys. **64**, 451 (1921).
[6] C. Ramsauer, Ann. d. Phys. **66**, 547 (1921).

the main electron beam, and hence even if initially they were directed along the electron beam they would soon leave it and balance each other at the walls of the scattering chamber. At such an encounter, or any encounter in which the initial electron loses an appreciable amount of energy it too will be removed from the beam regardless of its direction of motion immediately following the encounter. Thus a Ramsauer type of apparatus measures not only the integral of the first term of Eq. (1) but also an additional negative current due to those electrons which lose energy even though they are deflected through angles less than $\bar{\theta}$. On the other hand, in a Mayer type of apparatus the integral of the first term is partially balanced by a part of the positive ion current.

From these considerations one would expect that the magnitude of the absorption coefficient as obtained by various observers using apparatus with different resolutions should exhibit quite a wide range of values. But this does not seem to be the case. Recent measurements by Maxwell,[7] Jones[8] and Brode[9] on the absorption coefficient for electrons in mercury vapor agree much better than would be expected. The author* has also carried out a determination with a Mayer type of apparatus (separate from that used for the work reported in this paper) in which electrons scattered through angles down to 3°, on the average, were counted as having collided. The absorption coefficient was determined for electrons of from 5 to 100 volts velocity and the values agreed within a few percent with those obtained by Brode with a Ramsauer apparatus. These values are but 25 percent greater than those reported by Maxwell. His apparatus defined a collision only if it resulted in a very much larger deflection of the electron. Arnot[3] has obtained an angular scattering curve for electrons in mercury vapor, and a numerical integration of this curve as applied to each individual apparatus indicates that there should be a much greater divergence in the results. Similarly for other gases various observers obtain results more in accord with each other than would be expected.

It was then decided to build a Mayer type of apparatus in which the opening at the end of the scattering chamber could be changed in size, so that the absorption coefficient could be studied in one particular apparatus as a function of the resolving power. Such a study should throw light on the probability of angular scattering, and should enable a check of existing curves Recently Metta Clare Green[10] carried out a similar investigation, but found no consistent variation of the absorption coefficient with opening. Inasmuch as such a result is so definitely in opposition to the results of angular distribution experiments and as the magnitudes of the values obtained for the absorption coefficient differed, in most cases, so radically from accepted values,

[7] L. R. Maxwell, Proc. Nat. Acad. Sci. **12,** 509 (1926).
[8] T. J. Jones, Phys. Rev. **32,** 459 (1928).
[9] R. B. Brode, Proc. Roy. Soc. **A125,** 134 (1929).
* Work not published.
[10] Metta Clare Green, Phys. Rev. **36,** 239 (1930).

such a study should still be fruitful. Measurements were accordingly made in helium and in mercury vapor.

Apparatus

The apparatus used is shown diagrammatically in Fig. 1. It was constructed of copper. Electrons from the tip of the filament, F, were accelerated to a definite velocity between the plates D_1 and D_2, and made to travel in a beam of small cross-section through the scattering chamber, *sc*, and into the Farady cage, *FC*. The diameters of the circular openings in the gun, G, were 0.75, 0.50, 0.50 and 0.75 mm for D_1, D_2, D_3 and D_4 respectively. D_5 was about 1.5 mm in diameter, and less than 1 mm from D_4. The distance from D_1 to D_2 was 5.0 mm and from D_1 to the end of the gun, D_4, was 45 mm. The scattering chamber, from D_4 to D_6, was 4.0 cm in length, and the Faraday cage was 16 cm in length. The hairpin filament was a small tungsten wire bent sharply at the opening D_1. It required about 1.5 amperes for satisfactory electron emission. All diaphragms were slightly bevelled, and important surfaces were given a light coat of soot.

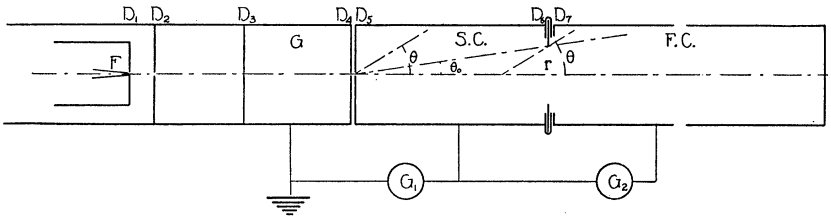

Fig. 1. Diagram of apparatus.

The variable aperture at D_6 was obtained by the use of a 30 cm strip in which were cut a series of holes of different size. The strip could be slid freely back and forth by means of a special magnetic control, which was constructed such that all magnetic material could readily be removed for each reading.

The apparatus was sealed in a Pyrex tube free from wax joints, and vacuum conditions were such that a pressure of less than 10^{-6} mm mercury would build up in 24 hours when the tube was cut off from the pumps. For the measurements in mercury vapor, a trap containing mercury was maintained at a definite temperature, and the calculations were based on the vapor pressure corresponding to that temperature as recorded in the International Critical Tables. Pressures corresponding to various temperatures up to about 20°C were used. The gas pressures for helium were determined from McLeod gauge readings. A side tube containing carbon which had been baked out for several hours at 500°C was used in conjunction with the helium measurements. This tube and the mercury trap were kept at the temperature of liquid air.

A pair of large Helmholtz coils was used to neutralize the earth's field, and the tube was mounted such that the electron beam traveled along the axis of the coils.

METHOD

In a determination of α from the exponential decrease in the intensity of an electron beam a constant, K, must be introduced to account for those electrons which would be collected by the scattering chamber regardless of gas pressure. This constant should depend only on the geometry of the apparatus. The beam of initial strength I_0 is then reduced to $I=KI_0e^{-\alpha xp}$ after traveling a distance x in a gas at pressure p. The current ratio I/I_0 was measured by the galvanometers, G_2 and G_1 (Fig. 1) for different gas pressures and for different openings at the end of the scattering chamber. From the above equation

$$\log I/I_0 = \log K - \alpha xp/2.3 \tag{2}$$

in which the logarithms are taken to the base 10. If K is a constant, independent of the pressure and electron current for a particular resolution, the log I/I_0 should exhibit linearity when plotted against the pressure. The

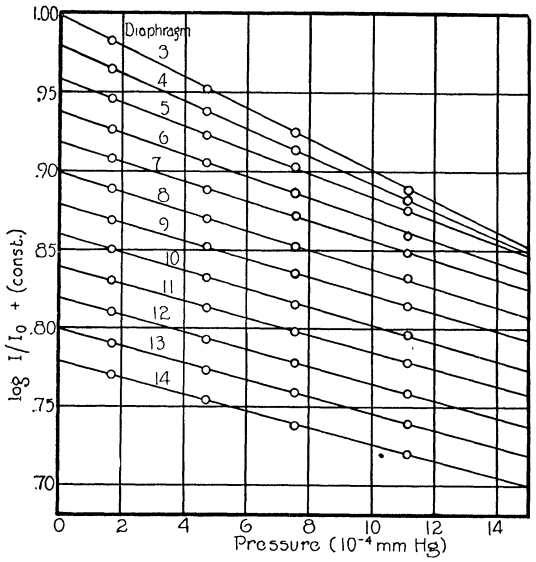

Fig. 2. To show linearity between log I/I_0 and pressure. (80 volt electrons in mercury vapor.)

slope of such a straight line relationship being $\alpha x/2.3$ makes it possible to calculate α. If the pressure used is the observed pressure corrected for thermal diffusion and reduced to the corresponding pressure at 0°C, α is obtained in the usual units, cm^2/cm^3 per mm pressure at 0°C.

Fig. 2 shows the relation between log I/I_0 (plus an arbitrary constant) and the pressure for 80 volt electrons in mercury vapor; each curve was obtained with a different opening at the end of the scattering chamber. These curves indicate that K was independent of the pressure in each case. The value of log $K=\log I/I_0$ when $p=0$ shows that K was greater than 0.99 for all openings. Thus at least 99 percent of the original beam was collected by

the Faraday cage when no gas was present. This means that the number of secondary electrons reflected into the scattering chamber from the end of the gun or the end of the Faraday cage was at a satisfactory minimum.

By applying a retarding field between the Faraday cage and the scattering chamber (Fig. 1) the velocity distribution of the electron beam was found to be quite satisfactory. For a 40 volt electron beam, over 95 percent of the electrons had velocities within a few tenths of a volt of the mean. Though this differed from 40 volts by a small amount, the beam will nevertheless be designated as a 40 volt electron beam. The correction will also be disregarded for the other velocities used.

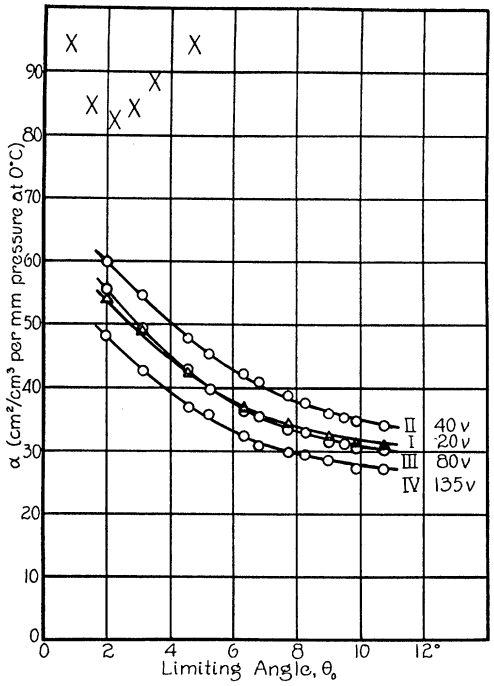

Fig. 3. "Absorption coefficient," α, as a function of the resolving power of the apparatus, in mercury vapor. (X-values given by Metta Clare Green for 37.5 volt electrons.)

The values of α in mercury vapor were for the most part obtained from such curves as those in Fig. 2, which in turn had been determined by four different pressures. Those for helium and some of the mercury values were obtained with but two pressures, one of which was usually zero. The path length value of $x=4.0$ cm was used for all the calculations.

Results and Discussion

The results of the measurements in mercury vapor are given graphically in Fig. 3, and those for helium in Fig. 4. α is plotted for 20, 40, 80 and 135 volt electrons against the limiting angle, θ_0, giving curves I, II, III and IV respectively. Referring to Fig. 1, this limiting angle is defined as $\theta_0=\tan^{-1} r/l$

where r is the radius of the opening and l is the length of the scattering chamber. If any electron is deflected through an angle less than this it will not be counted as having collided. The range of values for θ_0 corresponds to a range of diaphragm radii of from 1.4 to 7.6 mm. The experimental points for mercury are averages of two complete sets of readings, differing from each other in most cases by less than 2 percent. The values for helium are averages of

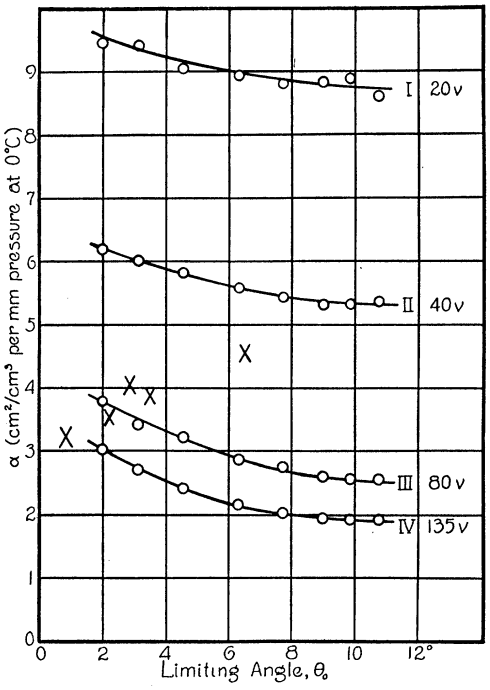

Fig. 4. "Absorption coefficient," α, as a function of the resolving power of the apparatus, in helium. (X-values given by Metta Clare Green for 97 volt electrons.)

four sets of readings with a somewhat larger experimental error. All of the curves show a definite decrease in the absorption coefficient as the size of the opening increases. This, of course, corresponds qualitatively to the tendency for scattering to take place in the forward direction.

If $K=1$, which has been seen to be practically the case, the current to the scattering chamber is $I'=I_0-I=I_0(1-e^{-\alpha xp})$. For low pressures this becomes

$$I'/I_0 = \alpha xp \tag{3}$$

or, α is numerically equal to the negative current to the scattering chamber in terms of a certain number of electrons per primary electron per cm path per mm pressure at 0°C. Thus the curves of Figs. 3 and 4 give the relative negative current collected by the scattering chamber when the size of the opening is changed. As has already been seen, this negative current is the resultant of a positive ion current combined with the scattered and ejected

electron current, the fraction which is due to positive ions depending on the probability of ionization. Hence if these curves are to be discussed with regard to the characteristics of electron scattering they must be corrected for the positive ion current. This correction is made possible by the recent work of Bleakney[11] and of Smith[12] who have determined the efficiency of ionization of electrons in various gases. If F^+ is this efficiency of ionization

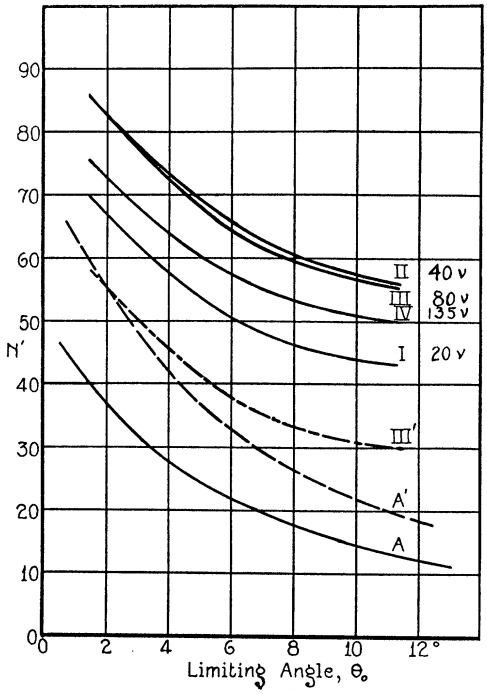

Fig. 5. Number of electrons collected by the scattering chamber, per cm path per primary electron per mm pressure at 0°C, as a function of the resolving power, in mercury vapor. (Curve *A* predicted, as the number of primary electrons scattered, by Arnot's results for 80 volt electrons.)

(given in terms of the number of units of positive charge formed per cm path per primary electron per mm pressure at 0°C) and we assume that the ion has an equal probability of going in any direction,* the positive current to the scattering chamber can be obtained by a simple integration. It is of course a function of θ_0 and is found to vary from $1.10F^+$ at $\theta_0=2°$ to $1.03F^+$ at

[11] Walker Bleakney, Phys. Rev. **35**, 139 (1930).

[12] P. T. Smith, Phys. Rev. **36**, 1293 (1930).

* This assumption seems valid providing the interior of the scattering chamber is a field-free space. A recent investigation by Arnot (Proc. Roy. Soc. **A129**, 361 (1930)) indicated that the positive ions tend to move out perpendicularly to the electron beam. It is doubtful whether his results may be applied here, for they were obtained under different conditions. Intense electron beam currents up to 25 micro-amperes were used, whereas in the work reported here this current did not exceed a tenth of a micro-ampere. However, the results to follow would be substantially the same, even though we accepted Arnot's distribution for the positive ions.

$\theta_0 = 10°$. The values of F^+ given by Bleakney for 20, 40, 80 and 135 volt electrons in mercury vapor are 12.0, 21.2, 25.0 and 22.5 respectively; those given by Smith for the respective voltages in helium are 0.00, 0.63, 1.18 and 1.24. With these values the corrections can be calculated and applied to the curves of Figs. 3 and 4. The corrected curves, representing the total number of electrons collected by the scattering chamber (per primary electron per cm path per mm pressure at 0°C), are given in Figs. 5 and 6, for mercury vapor and helium, respectively. Since these curves deal only with the electrons which are collected they may be used in a comparison of electron scattering.

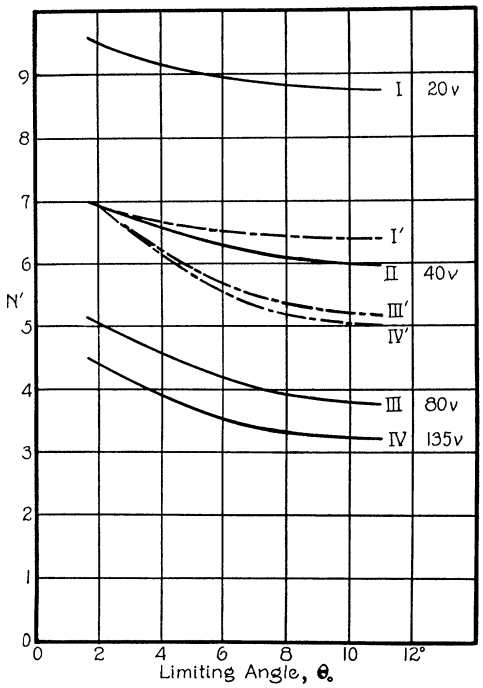

Fig. 6. Number of electrons collected by the scattering chamber, per cm path per primary electron per mm pressure at 0°C, as a function of the resolving power, in helium.

To get a better idea of the relative dependence of this scattering on angular aperture for the different primary velocities, the 20, 80 and 135 volt curves in helium (Fig. 6) have been multiplied by the proper factors to make them coincide with the 40 volt curve at $\theta_0 = 2°$, giving the new curves I′, III′ and IV′ respectively. From this set of curves we notice that as the velocity of the primary electrons becomes greater, the curves become relatively steeper; i.e., a small change in the size of the opening cuts out a relatively greater number of the scattered electrons if the primary electrons have high velocity than if they have a lower velocity. This indicates that scattering of fast electrons is more definitely concentrated in the forward direction than that for slow electrons, and corresponds to the decrease in steepness

of the angular scattering curves with decrease in primary electron velocity, as observed by McMillen.[4] Mercury vapor however, does not exhibit such a marked relative preference. The curves in Fig. 5 show about the same relative change over the range of angles used, being much more alike than are those in helium. This indicates that the angular distribution of scattered electrons in mercury vapor is quite similar over the velocity range of from 20 to 135 volts. This does not imply, however, that the angular distribution for the scattering of primary electrons would be the same over this velocity range. To say anything about this, the distribution of the ejected electrons must be taken into consideration, and it is not likely that this distribution would be the same for primary electrons of different velocities. These relations cannot be checked with existing data for, aside from our lack of knowledge of the behavior of the ejected electron, no investigation has as yet be published in which the angular distribution has been studied as function of electron velocity in mercury vapor.

From the angular scattering curves of Arnot for 80 volt electrons in mercury vapor it is possible to predict the number of primary electrons which should be collected by the scattering chamber for any size opening. He determined the angular distribution for electrons which had not lost energy and also for those which had lost energy. He assumed that these electrons were all primary electrons, even though there is the possibility that an ejected electron may be given sufficient energy such that it will be confused with the group of primary electrons which had lost energy. The calculation for the number of primary electrons collected according to his results can be done in the following way.

Let $F(\theta)d\theta$ be the number of electrons scattered between θ and $\theta+d\theta$ per primary electron per cm path at a definite temperature and pressure. The effective length of path over which electrons deflected through angle θ are collected is $l_e=l-r/\tan\theta$, as can be seen from Fig. 1. Then for N_0 primary electrons the number of electrons deflected between θ and $\theta+d\theta$ which are collected by the scattering chamber is $N_0 p F(\theta) l_e d\theta$ for a gas pressure p which is sufficiently small. The total number collected will then be

$$N' = N_0 p \int_{\theta_0}^{\pi} F(\theta)(l - r/\tan\theta)d\theta. \tag{5}$$

It may be noted that this includes all electrons scattered into the chamber, even those which are deflected through angles greater than 90° giving for them the proper effective path length which is greater than 1. If $F(\theta)$ be known, the function $F(\theta)$ $(l-r/\tan\theta)$ can be plotted against θ for different values of r, and the areas under the resulting curves will represent the relative numbers of primary electrons collected by the scattering chamber for openings of different size. The only assumption which has been made is that the scattering takes place along the axis of the apparatus.

Compton[13] has tabulated Arnot's values for the scattering of 80 volt

[13] K. T. Compton, Rev. Mod. Phys. **2**, 123 (1930).

electrons in mercury vapor in terms of the number of electrons scattered per unit solid angle at angle θ per primary electron per cm path at 20°C and 0.001 mm pressure. $F(\theta)$ is then this function multiplied by $2\pi \sin \theta$. By using these values for $F(\theta)$ and performing the numerical integrations outlined above, N'/N_0p was obtained for different values of the diaphragm radius. On correcting this result to 0°C and 1 mm pressure (since $F(\theta)$ was given for a temperature of 20°C and a pressure of 0.001 mm) and dividing by the path length, $x=4.0$ cm, one obtains the predicted values for the number of primary electrons which should be collected by the scattering chamber for various apertures. Curve A of Fig. 5 is the result of this work, plotted in terms of $\theta_0=\tan^{-1} r/l$. Curve III of the same figure represents the actual number of electrons which were collected experimentally. Assuming the validity of both curves, the difference between their ordinates for a particular value of θ_0 would represent the number of ejected electrons collected by the scattering chamber for that value of θ_0. This difference increases with increasing θ_0 which means that a larger ejected electron current would be collected the larger the diaphragm opening. Such a result can be interpreted only if the ejected electrons obey a distribution which favors scattering in the backward direction, which does not seem reasonable. It is of interest to compare the curves if we assume a uniform distribution for the ejected electrons. The correction to be made to curve III will be practically the same as (but opposite to) the correction which was made for the positive ions, and the resulting curve will be similar to the curve for α. This is given in Fig. 5 as curve III'. To compare this curve with Arnot's the latter has been plotted to a different scale giving curve A'. Both curves, of course, show the preference for small angle scattering, but Arnot's curve makes this preference much more decided. If we had assumed an angular scattering for the ejected electrons favoring scattering in the forward direction, curve III' would have differed much more decidedly from Arnot's predicted curve. It is to be mentioned that Arnot's results were obtained with primary electron currents of several micro-amperes, whereas in the present work no currents greater than a tenth of a micro-ampere were used. As was noted above, the only assumption made in the calculation was that the scattering should take place along the axis. This was approximated quite well, for tests at the end of the scattering chamber indicated that the beam was but 0.8 mm in diameter and hence all of the scattering took place within 0.4 mm of the axis. Thus there remains a definite discrepancy between the two curves which is difficult to explain.

It is of interest to note that the procedure we have just outlined may be reversed, and $F(\theta)$ may be obtained from the experimental curve. Thus, if the experimental curve gives the result

$$f(\theta_0) = \int_{\theta_0=\tan^{-1} r/l}^{\pi} F(\theta)\cdot(l - r/\tan\theta)d\theta \tag{6}$$

then $F(\theta)$ can be determined by differentiating this equation twice. This gives

$$F(\theta_0) = \sin\theta_0/l\left(\cos\theta_0\frac{d^2f(\theta_0)}{d\theta_0^2} - 2\sin\theta_0\frac{df(\theta_0)}{d\theta_0}\right). \quad (7)$$

This equation, however, is rather impractical of application for the second derivative bears too much weight. A small error in the experimental determination of $f(\theta_0)$ is amplified in the calculation, which makes $F(\theta)$ very uncertain.

The results of Metta Clare Green[10] which were previously mentioned are given by the crosses in Figs. 3 and 4, for certain electron velocities in mercury vapor and helium. These values are also plotted in terms of $\theta_0 = \tan^{-1}r/l$ where $l = 7.5$ cm in her apparatus. For a given electron velocity she found no definite variation in the absorption coefficient with change in aperture, all the values for a given curve lying within the experimental error of the mean of that group. However such a result is in direct opposition to all angular scattering experiments. In addition, the magnitudes of the values obtained for the absorption coefficient differed in most cases so radically from those obtained by any other observer that it is difficult to place much weight on the results. This may, in part, have been caused by the penetration of electric fields into the scattering chamber at both ends. Such experiments necessitate that the scattering be studied in a field-free space. Otherwise it is difficult to determine the cause of this discrepancy.

This investigation has given further evidence that when electrons are scattered by gas molecules they exhibit a preference for deflection through small angles. It has also been shown that, over the velocity range studied, fast electrons show a greater relative tendency to be deflected through small angles than do slower electrons. This is quite definitely the case for scattering in helium, whereas it is not so marked in mercury vapor. Further, the results in mercury vapor do not indicate as decided a preference for forward scattering as do those of Arnot.

The author wishes to express his gratitude to Professor Tate for his many helpful suggestions and continued interest throughout this work.