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# THE SCATTERING OF UNPOLARIZED X-RAYS

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## Abstract

Unpolarized x-rays from an x-ray tube excited at 90 to 125 kv and filtered through aluminum and copper were scattered by paraffin at angles of 75°, 97°30′ and 120°. The intensity of the scattered rays was measured by the ionization produced in a chamber containing air saturated with (1) methyl iodide (2) ethyl bromide. It was found very necessary to keep the temperature of the ionization chamber and the voltage on the x-ray tube constant during an experiment. The results are expressed by the ratios of the ionization currents produced at scattering angles of 97°30′ and 120° respectively to the current produced at an angle of 75°. The wave-lengths present in the primary x-rays were determined by measuring the absorption of x-rays in a series of thicknesses of aluminum, the x-rays entering the same ionization chamber as that which was used in the scattering experiment. The absorption curve so obtained was found to consist of two exponential curves which correspond to two wave-lengths, the intensity of the shorter wave-length being much greater than that of the longer. With these wave-lengths and their relative intensities and taking account of the change of absorption in the ionization chamber due to the Compton change of wave-length the theoretical values for the above ratios were calculated by use of (1) the Dirac scattering formula and (2) the Compton formula. The agreement between the ratios calculated from the Dirac formula and the experimental ratios was excellent. The wave-lengths when methyl iodide was in the chamber were 0.205A and 0.39A and when ethyl bromide was in the chamber were 0.26A and 0.47A. At these wave-lengths the difference between the Dirac and the Klein and Nishina formulas is so small that experimental discrimination between them was not possible.

## 1. Introduction

In 1923, A. H. Compton[1] and Jauncey[2] developed theoretical formulas for the spatial distribution of the intensity of scattered x-rays. The two formulas differ only in the higher powers of $\alpha$, which is defined by

$$\alpha = h/mc\lambda \tag{1}$$

where $\lambda$ is the wave-length of the primary x-rays and $h$, $m$ and $c$ have their usual significance. It is only in the region of $\gamma$-rays that the two formulas differ, and as this paper has to do only with the scattering of x-rays and not $\gamma$-rays, we shall use the Compton formula as also representing Jauncey's formula with sufficient exactness. According to Compton,[1] the scattering coefficient per unit solid angle in a direction $\phi$ with the direction of propagation of the primary x-rays is

$$s_\phi = \frac{NZ\rho e^4}{Wm^2c^4}\cdot\frac{1+\cos^2\phi+2\alpha(1+\alpha)\operatorname{vers}^2\phi}{(1+\alpha\operatorname{vers}\phi)^5} \tag{2}$$

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[1] A. H. Compton, Phys. Rev. **21**, 491 (1923).

[2] G. E. M. Jauncey, Phys. Rev. **22**, 233 (1923).

where $N$ is Avogadro's number, $Z$ the number of electrons in a molecule of the scattering substance, $\rho$ its density, $W$ its molecular weight and $\phi$ the angle of scattering. Eq. (2) is Compton's formula for the scattering of unpolarized x-rays. The minimum value of $s_\phi$ occurs when $ds_\phi/d\phi = 0$, that is, when

$$\cos\phi = -\alpha/2 \tag{3}$$

approximately, the square and higher powers of $\alpha$ being neglected.

In the Thomson[3] theory of the scattering of x-rays, the minimum occurs at $\cos\phi = 0$, or at $\phi = 90°$. In the case of x-rays for which $\alpha = 0.1$, the minimum according to Eq. (3) occurs at $\phi = 92°53'$, which is distinctly different from the position of the minimum on the Thomson theory.

Breit[4] attacked the problem of the theoretical formula for the scattering of x-rays from the point of view of the correspondence principle and obtained a formula differing from Eq. (2). Later Dirac,[5] using the principles of quantum mechanics, obtained a formula identical with that of Breit. The Dirac formula is

$$s_\phi = \frac{NZ\rho e^4}{Wm^2c^4}\cdot\frac{1 + \cos^2\phi}{(1 + a \operatorname{vers}\phi)^3}\cdot \tag{4}$$

According to Eq. (4), $s_\phi$ has a minimum at

$$\cos\phi = -3\alpha/2 \tag{5}$$

approximately, so that for $\alpha = 0.1$, the minimum occurs at $98°36'$. The minimum according to Dirac therefore occurs at an angle of $5°43'$ greater than according to Compton and Jauncey.

Recently, Klein and Nishina[6] have derived a formula on the basis of the quantum mechanics which is somewhat different from Dirac's formula. The Klein and Nishina formula is obtained from the Dirac formula by multiplying the right side of Eq. (4) by the factor

$$k = 1 + \frac{\alpha^2 \operatorname{vers}^2\phi}{(1 + \cos^2\phi)(1 + \alpha \operatorname{vers}\phi)}\cdot \tag{6}$$

This factor has no effect on Eq. (5) as far as the first power of $\alpha$ is concerned.

In 1924 Jauncey[7] derived a formula for the scattering of polarized x-rays. For scattering in the plane of the electric vector, the position of the minimum is given by

$$\cos\phi = \alpha \tag{7}$$

approximately, so that for $\alpha = 0.1$ the minimum occurs at $84°\ 15'$. The formulas of Breit, Dirac and Klein and Nishina all agree in giving the minimum at $\phi = 90°$, which is in accord with the Thomson classical theory.

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[3] J. J. Thomson, Conduction of Electricity through Gases, 2nd. Ed., p. 325.

[4] G. Breit, Phys. Rev. **27**, 362 (1926).

[5] P. A. M. Dirac, Proc. Roy. Soc. **A111**, 405 (1926).

[6] Klein and Nishina, Zeits. f. Physik **52**, 853 (1929).

[7] G. E. M. Jauncey, Phys. Rev. **23**, 313 (1924).

In an early experiment by Jauncey and Stauss[8] on the scattering of polarized x-rays, the experimental position of the minimum appeared to agree with Eq. (7). This experiment was repeated by Barrett and Beardon,[9] who found that the minimum occurred at angles somewhat greater than 90° and within experimental error of 90°. Another repetition was made by Jauncey and Hassler,[10] who found the minimum to be at an angle slightly greater than 90°. The results of these experiments are uncertain due to (1) the difficulty of measuring the exact position of a minimum, (2) the lack of complete polarization, (3) the wide slits which are necessary in order to obtain sufficient intensity, (4) the uncertainty of the wave-length used, and (5) multiple scattering. The wide slits have the effect of flattening the minimum, and on that account making the position of the minimum still more difficult to determine. Referring to Eqs. (3) and (7), it is seen that the Compton-Jauncey formula gives the minimum for unpolarized x-rays at an angle greater than 90°, while their formula gives the minimum for polarized x-rays at an angle less than 90°. It is possible therefore with partially polarized x-rays to obtain a minimum at 90° on the Compton-Jauncey theory. Complete polarization can only be obtained by scattering at 90° or nearly 90° from a slab of some material such as paraffin, when the slab is very thin. Since, in experiments on polarized x-rays, the x-rays are scattered twice, the intensity of the doubly scattered x-rays is quite small unless both slabs of the scattering material are fairly thick. However, multiple scattering increases as the thickness of each slab increases. This multiple scattering increases the lack of complete polarization.

Due to the above objections, the present writers have made the test on unpolarized x-rays and have abandoned the attempt to determine experimentally the position of the minimum. Instead the writers have attempted to measure the relative scattering at the three angles 75°, 97.5° and 120° as accurately as possible and to compare the experimental values with the values predicted by the various theories. It should be mentioned that in 1922 Hewlett[11] published scattering curves for certain organic liquids and that these curves show minima at about 100° to 105°. As we shall show in the present paper, it is very necessary to take account of the Compton change of wave-length in regard to its effect on the ionization produced in the ionization chamber, and this Hewlett did not do, since his paper was published before the discovery of the Compton effect.

## 2. Experimental Procedure

X-rays from the tube $A$, Fig. 1, after leaving the target almost tangentially and passing through a slit system, fall upon the paraffin slab $B$. Part of the x-rays are scattered by the slab $B$ into the ionization chamber $D$ and part penetrate through the slab $B$ and then fall upon a paraffin slab $C$. Part

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[8] Jauncey and Stauss, Proc. Nat. Acad. Sci. **10**, 405 (1924).

[9] Barrett and Bearden, Phys. Rev. **29**, 352 (1927).

[10] Jauncey and Hassler, Phys. Rev. **31**, 1120 (1928).

[11] C. W. Hewlett, Phys. Rev. **20**, 688 (1922).

of the rays falling on slab $C$ are scattered into the ionization chamber $E$. The slab $B$ is mounted on the axis of an x-ray spectrometer and the ionization chamber $D$ can be rotated about this axis. The angle which the slab $C$ makes with the primary beam and the position of the chamber $E$ remains fixed throughout the experiment. The width of the beam of x-rays entering the chamber $E$ is controlled by the adjustable slit $S$. The outer electrodes of $D$ and $E$ are connected to $+100$ and $-100$ volts respectively. The inner electrodes are connected together and to the electrometer as shown in Fig. 1. Doubling and halving the voltages on the chambers $E$ and $D$ had no effect on the observed ionization currents, so that the voltages on the two chambers were always above the voltages necessary for saturation. The chamber $D$ was first filled with air saturated with methyl iodide vapor. Later $D$ was filled with air saturated with ethyl bromide vapor. A reservoir containing either liquid methyl iodide or ethyl bromide was permanently connected to $D$. The chamber $E$ was filled with air alone. In order that the primary x-rays should be in effect completely unpolarized, the axis of the x-ray tube made an angle of 45° with the plane of scattering.

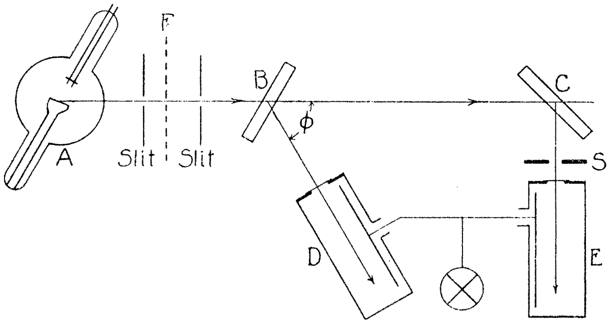

Fig. 1. Diagram of apparatus.

The angular scale of the spectrometer was graduated in quarter degrees so that settings of the ionization chamber could be made with an accuracy of 5 minutes of arc. A pin was mounted on the axis of the spectrometer and it was arranged that, when the primary beam of x-rays was observed by a fluoroscope, the shadow of the pin was in the center of the fluorescence produced by the beam. The target of the tube was turned so that the primary x-rays left the target nearly tangentially, the plane of the target being vertical. Under these conditions it was noted that the edge of the fluorescent image of the slit system as observed in the fluoroscope was sharp on one side but was somewhat indefinite on the other. Consequently we could not be certain that the axis of the spectrometer passed through the "center of gravity" of the primary beam. We therefore took readings for a given scattering angle with the chamber $D$ set first on one side of the primary beam and then on the other. Also with a given setting of the ionization chamber we took one set of readings with the paraffin slab $B$ in the Crowther[12] position and

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[12] J. A. Crowther, Proc. Roy. Soc. **A86,** 478 (1912).

a second set with the slab *B* turned through 180°, so that it was again in the Crowther position but with the opposite side presented to the primary beam. For each angle of scattering there were thus two settings of the ionization chamber and four settings of the paraffin slab. In this way, any asymmetry of the primary beam with respect to the spectrometer was corrected.

The primary voltage of the x-ray transformer was supplied by two motor generator sets. The first set consisted of an a.c. motor and d.c. generator operated by Washington University power. By means of a voltage regulator connected to the field coils of the d.c. generator a constant d.c. voltage was obtained, irrespective of variations in the a.c. voltage driving the a.c. motor. This constant d.c. voltage was applied to the d.c. motor of the second set, so that the a.c. generator of the second set gave a constant a.c. voltage. This constant a.c. voltage was applied to the primary of the x-ray transformer. Full wave rectification of the high voltage was obtained by a system of four kenotrons. Care was taken to have the filaments of the kenotrons at a temperature such that the fall of voltage across the kenotrons was too small to produce any detectable x-rays. This was necessary because irregular results were first obtained due to stray x-rays from the kenotrons. In spite of the two motor generator sets, there were occasional variations of the primary voltage across the x-ray transformer and of the milliamperes through the x-ray tube. Accordingly, we placed a choke coil in the primary circuit of the x-ray transformer. By adjustment of the length of the iron core within the coil, any variation of the voltage could be annulled. The tube was operated at maximum voltages between 90 and 125 kilovolts and a current of 7 milliamperes. The voltages were measured by the spark length between spheres of 10 cm radius. The target was of tungsten and was not water-cooled. The corona discharge from the leads to the x-ray tube was reduced by making these leads of $\frac{3}{4}$ inch flexible piping. The x-rays were made more homogeneous by passing them through a filter *F* of aluminum or copper.

The intensity of the rays scattered by the slab *B* into the chamber *D* was compared with the intensity of the rays penetrating the slab *B* and entering the chamber *E*. By means of lead shutters, the rays were first allowed to enter *D* but not *E* and then were allowed to enter *E* but not *D*. Readings of the scattered and primary rays were thus taken alternately. The time for a deflection over a given part of the scale was measured. Due to the fact that the voltages across the chambers *D* and *E* were in opposite directions, it was not necessary to ground the electrometer during a set of readings. With a given setting of the chamber, 5 readings on the scattered rays and 5 readings on the primary rays were taken. The slab *B* was then turned through 180° and the procedure repeated. This was done with a slab of given thickness and with a given voltage on the x-ray tube for the chamber angles of 120°, 97.5°, and 75° on the right side of the spectrometer and 75° 97.5° and 120° on the left side. Thus a total of 60 readings on the scattered rays and 60 on the primary rays were taken, making a total of 120 readings. In an effort to correct for multiple scattering, we then substituted a slab of different thickness and took 120 more readings. The whole of the 240 readings were taken without interruption. Since the effect of change of temperature on the ionization produced in the saturated methyl iodide or ethyl bromide vapor is quite considerable, we kept the temperature of the room constant during the whole period of the 240 readings. Also, during the same period, the voltage across the x-ray tube and the current through the tube were kept constant.

Readings of the ionization in chamber *D* were also taken with the slab *B* removed so as to obtain any effect due to stray rays. Although this effect was small, it was not negligible. The effect was different for different settings of the chamber *D*. For a given chamber angle on one side of the spectrometer and a given thickness of paraffin, the average of the readings for the scattered rays was determined. This average was corrected for the stray rays. The average for the primary rays was also determined. The ratio of this corrected average for the scattered rays to the average for the primary rays is determined for a given scattering angle on each side of the spectrometer. The average of the ratios for the two sides is then determined These are the values shown in Tables I and II.

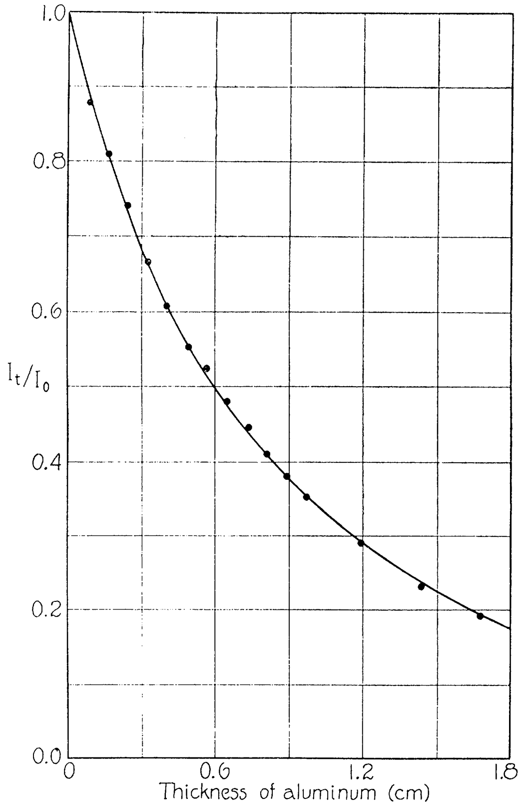

Fig. 2. Absorption in aluminum. Methyl iodide, 90 kv.

The linear width of the primary beam when crossing over the axis of the spectrometer was measured with the aid of the fluoroscope and was found to be 4 mm. Since the distance of the axis to the target of the x-ray tube is 80 cm, the angular width of the primary beam is 17′. The distance from the axis to the window of the ionization chamber $D$ is 15 cm. This window is 1 cm wide. The angle of scattering when the chamber $D$ is set at a given angle therefore has a range of 5° 32′. The angular height of the primary beam is 1° 23′, while the angular height of the scattered rays is 11° 24′. Calculation showed that the angular range of the scattering angle from a thin slab due to the height of the slits was about 30′ at each of the angles 75° 97° 30′ and 120°.

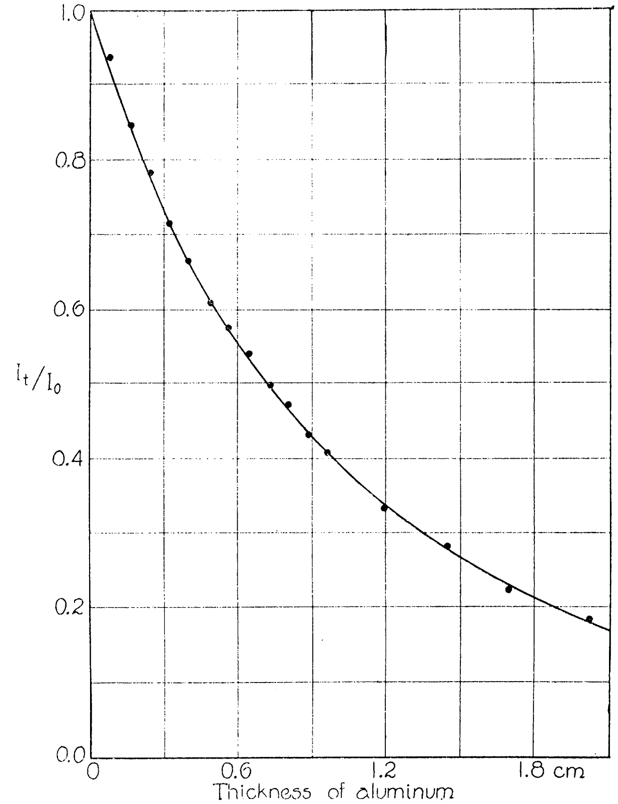

Fig. 3. Absorption in aluminum. Methyl iodide, 125 kv.

The inner electrode of the ionization chamber was placed so that no scattered rays from the slab could strike it. The diameter of the chamber was such that the rays did not strike the sides. The length of the ionization chamber was 43.5 cm.

After completing a run of 240 readings on the scattered rays and meauring the effect of the stray rays, the hardness of the primary rays was measured by placing the chamber $D$ so as to receive the primary rays. In order to cut down the intensity the rays were passed through a pinhole in a sheet of lead. Different thickness of aluminum were placed in the primary. For each thickness several readings alternately with and without aluminum were taken.

The proportion of the rays penetrating each thickness are shown by the black circles in Figs. 2, 3 and 4. The absorption curve was obtained for the same voltage across the x-ray tube and with the chamber $D$ at the same temperature as when the scattered rays were being measured.

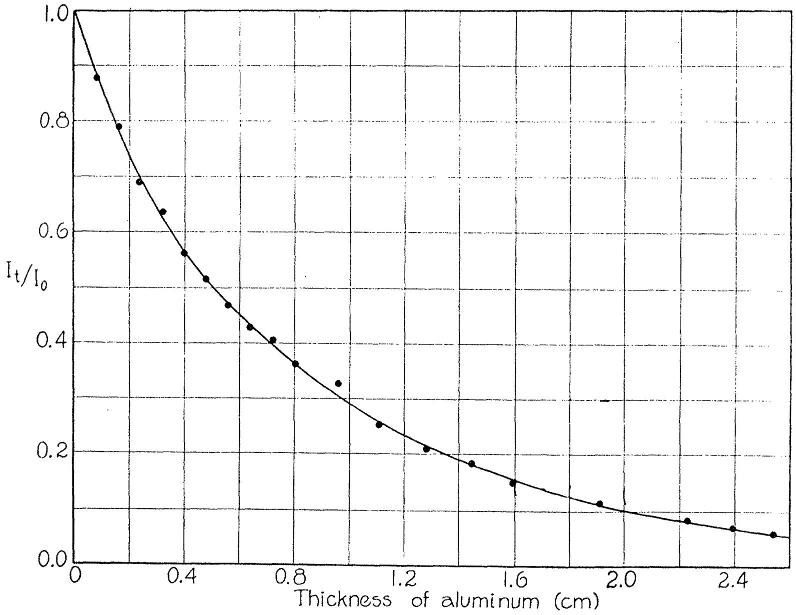

Fig. 4. Absorption in aluminum. Ethyl bromide, 95 kv.

## 3. Experimental Results

The experiment was first performed with the chamber filled with air saturated with methyl iodide vapor at a temperature of 17°C. The results are shown in Table I. Paraffin slabs of mass per unit area 0.365 gm/cm$^2$ and 0.595 gm/cm$^2$ were used. The values under 125 kv are not comparable with the values under 90 kv. The 90 kv rays were filtered through 2.92 mm and the 125 kv rays were filtered through 7.22 mm of aluminum. The results of the absorption measurements for the 90 kv rays and the 125 kv rays are represented by the black circles in Figs. 2 and 3 respectively.

Table I. *Scattering from paraffin. Methyl iodide in ionization chamber.*

| Angle | Scattering ratios<br>Potential = 90 kv<br>Mass/Area = 0.365 | 90kv<br>0.595 | 125kv<br>0.365 | 125kv<br>0.595 |
|---|---|---|---|---|
| 75° | 0.531 | 0.837 | 0.477 | 0.760 |
| 97° 30′ | 0.573 | 0.926 | 0.550 | 0.804 |
| 120° | 0.864 | 1.414 | 0.823 | 1.265 |

Later the experiment was performed with ethyl bromide in the chamber and the results shown in Table II were obtained. In this case the rays were filtered through 0.1 mm of copper and 2.0 mm of aluminum. The temperature was maintained at 24.0°C.

TABLE II. *Scattering from paraffin. Ethyl bromide in ionization chamber. Potential* 95 kv.

| Angle | Scattering ratios<br>Mass/Area = 0.365 | 0.595 |
|---|---|---|
| 75° | 0.368 | 0.645 |
| 97° 30′ | 0.397 | 0.720 |
| 120° | 0.635 | 1.150 |

The results of the absorption measurements with ethyl bromide in the chamber are represented by the black circles in Fig. 4.

## 4. COMPARISONS WITH THEORY

In order to compare the experimental results with theory it is necessary to obtain the wave-length of the primary rays. Since we used rays of the continuous spectrum filtered through aluminum or aluminum and copper, the rays were only approximately monochromatic.

Smooth curves (not the curves shown) were drawn through the experimental points shown in Figs. 2, 3, and 4. From the smooth curve in a given case values of $I_t/I_0$ were read off at each successive thickness of 1 mm of aluminum. It was found that after a thickness of about 4 mm the curve was almost exactly exponential. This exponential portion was produced back to zero thickness. Differences were then obtained between this exponential curve and the experimental curve for thicknesses less than 4 mm. It was found that these differences when plotted against thickness gave a fairly exponential curve. Thus the experimental curve appeared to be made up of two exponential portions. The solid curves shown in Figs. 2, 3 and 4 are curves calculated from the formula

$$I_t/I_0 = i_1 e^{-\mu_1 t} + i_2 e^{-\mu_2 t}. \tag{8}$$

The values of $i_1$, $i_2$, $\mu_1$ and $\mu_2$ are shown in Table III.

TABLE III.

| kv | Vapor in $D$ | $i_1$ | $\mu_1$ cm$^{-1}$ | $i_2$ | $\mu_2$ cm$^{-1}$ | $\lambda_1$ $A$ | $\lambda_2$ $A$ | $I_1$ | $I_2$ |
|---|---|---|---|---|---|---|---|---|---|
| 90 | methyl iodide | 0.67 | 0.74 | 0.33 | 2.7 | 0.205 | 0.39 | 0.68 | 0.32 |
| 125 | methyl iodide | 0.815 | 0.74 | 0.185 | 2.7 | 0.205 | 0.39 | 0.82 | 0.18 |
| 95 | ethyl iodide | 0.806 | 1.03 | .194 | 4.15 | 0.26 | 0.47 | 0.91 | 0.09 |

In Table III, $\mu_1$ and $\mu_2$ are the linear absorption coefficients in aluminum. From these the mass absorption coefficients can be found and the wavelengths $\lambda_1$ and $\lambda_2$ can be determined from Compton's table of absorption coefficients.[13] Now $i_1$ and $i_2$ are the proportions of the two wave-lengths present as measured by the ionization produced in the chamber. These are not the proportions present in the primary x-rays. At first, we thought to obtain the proportions present in the primary rays by dividing by the amount of each wave-length absorbed in the gas in the chamber. These proportions are shown in the columns headed $I_1$ and $I_2$. However, the energy of the x-rays which are absorbed in methyl iodide does not all give rise to ionization. By far the greater part of the energy goes into iodine $K$-radiation, and somewhere about 80 to 90 percent of this $K$-radiation leaves the chamber without producing ionization. According to Compton,[14] the fluorescent yield is independent of the wave-length, so long as it is short enough to excite the fluorescent radiation. Thus the same proportion of the absorbed energy goes into producing ionization in the chamber $D$ for all wave-lengths shorter than 0.375A, the $K$ critical absorption wave-length for iodine. Let $m_1$ and $m_2$ be the respective mass absorption coefficients of the wave-lengths $\lambda_1$ and $\lambda_2$ in the gas of the chamber $D$. The saturated vapor pressure for methyl iodide at 17°C is 293 mm[15], from which the density of the iodine in the methyl iodide is 0.00206 gm/cm$^3$. The values of $m_1$ and $m_2$ for iodine can be obtained from Compton's table of mass absorption coefficients by interpolation and the fraction of the radiation of each wave-length absorbed in the ionization chamber $D$ can be calculated. Let the fractions be represented by $k_1$ and $k_2$ for the wave-lengths $\lambda_1$ and $\lambda_2$ respectively. Of the fractions $k_1$ and $k_2$, the fractions $p_1$ and $p_2$ respectively appear as ionization. In the case of methyl iodide $p_1$ and $p_2$ differ because $\lambda_1$ is shorter and $\lambda_2$ is longer than the $K$ critical absorption wave-length of iodine. Hence the relative energies in the wave-lengths $\lambda_1$ and $\lambda_2$ as they enter the chamber $D$, but not as they are absorbed in the chamber, are $i_1/p_1k_1$ and $i_2/p_2k_2$. These are taken as representing the energies of $\lambda_1$ and $\lambda_2$ in the primary beam.

Now let us consider the scattering experiment. The scattered intensity $I_\phi$ when the slab $B$ is in the Crowther position is

$$\frac{I_\phi}{I} = \frac{As_\phi t \sec(\phi/2)}{R^2} \tag{8}$$

where $I$ is the intensity of the rays penetrating the slab $B$ and entering the chamber $E$. This intensity is proportional to the ionization in $E$, so long as the temperature of the room and the voltage across the x-ray tube is kept constant. $s_\phi$ in Eq. (8) is the scattering coefficient, $t$ is the thickness of the slab, $A$ the area of the window of chamber $D$, $R$ the distance of the window from the spectrometer window and $\phi$ the scattering angle.

Let primed quantities (′) represent values at $\phi'$ and doubly primed quantities (″) represent values at $\phi''$. Then, since the same slab of thickness $t$ is used at both angles, we have from (8)

$$J''/J' = (s''/s')\cdot(\sec\tfrac{1}{2}\phi'')/(\sec\tfrac{1}{2}\phi') \tag{9}$$

where $J = I_\phi/I$.

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[13] A. H. Compton, X-Rays and Electrons. p. 184.

[14] A. H. Compton, Phil. Mag. **8**, 961 (1929).

[15] Int. Crit. Tables. III p. 216.

With Eqs. (2), (4) and (6), the various theoretical values of $J''/J'$ for a given wave-length can be found. However, the ratio of the ionization currents produced in chamber $D$ is not $J''/J'$ because of the Compton change of wave-length

$$\delta\lambda = (h/mc) \text{ vers } \phi .$$

Due to this change of wave-length the proportion of the scattered rays of primary wave-length $\lambda_1$ which is absorbed in chamber $D$ is different for the scattered rays from the proportion of the primary rays of the wave-length $\lambda_1$ absorbed in $D$ in the absorption experiment. The absorbed scattered rays are proportional to $i_1k_1'/p_1k_1$ where $k_1'$ is the value of $k_1$ for the scattered rays. The ionization produced is proportional to $i_1p_1'k_1'/p_1k_1$, where $p_1'$ is the value of $p_1$ for the scattered rays. However, according to Compton,[14] $p_1' = p_1$, so long as the Compton change of wave-length does not cross a critical absorption wave-length. Hence the ionization produced is proportional to $i_1k_1'/k_1$. The values of $i_1/k_1$ and $i_2/k_2$ are shown in the columns respectively headed $I_1$ and $I_2$ in Table III.

If we have two wave-lengths, and if the $J$'s of Eq. (9) are to represent ionization currents in chamber $D$, we have

$$\frac{J''}{J'} = \frac{(i_1k_1''/k_1)s_1'' + (i_2k_2''/k_2)s_2''}{(i_1k_1'/k_1)s_1' + (i_2k_2'/k_2)s_2'} \cdot \frac{\sec(\frac{1}{2})\phi''}{\sec(\frac{1}{2})\phi'} .$$

The values of $J''/J'$ where $\phi' = 75°$ and $\phi'' = 97° \ 30'$ or $120°$ are shown for the various theoretical formula for $s_\phi$ in Table IV. The experimental values of $J''/J'$ when methyl iodide is in the chamber $D$ are also shown in Table IV.

TABLE IV. *Experimental and theoretical values of* $J''/J'$. *Methyl iodide in ionization chamber.*

| | | 90 kv | | | | | 125 kv | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Experiment | | | Dirac | Compton | Experiment | | | Dirac | Compton |
| $\phi'$ | $\phi''$ | $A$ | $B$ | Mean | | | $A$ | $B$ | Mean | | |
| 75° | 97.5° | 1.079 | 1.107 | 1.093 | 1.098 | 1.170 | 1.152 | 1.059 | 1.106 | 1.103 | 1.174 |
| 75° | 120° | 1.627 | 1.690 | 1.659 | 1.738 | 1.898 | 1.725 | 1.663 | 1.694 | 1.740 | 1.905 |

Those values shown under $A$ are for the paraffin slab whose mass per unit area is 0.365 gm/cm$^2$ and those under $B$ for the slab whose mass per unit area is 0.595 gm/cm$^2$.

During the progress of this research, a paper by DuMond[16] on multiple scattering has appeared. In this paper, DuMond discusses the problem of double scattering from a sphere, and obtains a formula for the ratio of the intensity of doubly scattered rays to that of singly scattered rays. The formula contains a function of $\phi$, the scattering angle, but this function is very nearly constant in the range $\phi = 75°$ to $120°$. We have been unable to find any certain direction in which $J''/J'$ changes with the thickness of the

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[16] J. W. M. DuMond, Phys. Rev. **36**, 1685 (1930).

slab in our experiments, and this is, we believe, in agreement with DuMond's theoretical finding that the ratio of double to single scattering does not vary with the angle in the range considered. Instead, therefore, of extrapolating the values of $J''/J'$ to zero thickness, we have merely taken the mean values as shown in Table IV.

In calculating the theoretical values, the range of scattering angles at a given setting of the ionization chamber has been neglected and the calculation been made on the assumption that the scattering angle is the angle at which the ionization chamber is set. This is allowed because the correction due to the range of the scattering angle is small since the range of 5° 32′ due to the width of the slits plus 30′ due to the height of the slits only makes a difference in the third decimal place in the theoretical values of $J''/J'$ in Tables IV and V. Further, in calculating the theoretical values it has been assumed that none of the scattered rays at the angles used are of the unmodified type. Compton[17] writes Wentzel's formula [18] for the intensity of the unmodified rays in the form

$$I_{unm} = I_e n^2 \left\{ \int u(r) \frac{\sin kr}{kr} dr \right\}^2 \tag{10}$$

where $k = (4\pi/\lambda) \sin (\phi/2)$ and $u(r)dr$ is the probability that an electron is at a distance $r$ to $r+dr$ from the center of an atom, $n$ is the number of electrons in an atom of the scatterer and $I_e$ is the Thomson value of the scattering from a single electron. In the case of helium, Pauling[19] gives the probability for each of the two electrons as

$$u(r) = 4r^2 \left( \frac{Z - s}{a} \right)^3 \exp \left\{ -(Z - s)r/a \right\} \tag{11}$$

where $Z$ is the atomic number of helium, $a$ is the radius of the normal orbit in the hydrogen atom on the Bohr theory and $s$ is the screening constant. We have applied this formula for $u(r)$ to the case of the $K$ electrons of carbon, putting $Z = 6$, $s = 0.39$ and $a = 0.53$A in Eq. (11) and $n = 2$ in Eq. (10). Performing the integration in Eq. (10), we obtain

$$I_{unm} = 4I_e/(1 + .022x^2) \tag{12}$$

where $x = (4/\lambda) \sin (\phi/2)$. For $\lambda = 0.26$A and $\phi = 75°$, $I_{unm}$ from each carbon atom in paraffin is 0.065 $I_e$. The average chemical formula for paraffin is $C_{24}H_{50}$. The $L$ electrons of C and the electrons of H do not scatter unmodified rays at $\phi = 75°$, so that the ratio of unmodified rays scattered by paraffin to the total scattering at $\phi = 75°$ is 0.7 percent. We are therefore justified in neglecting the unmodified rays in our calculated values of $J''/J'$.

In Table III it will be noted that $\lambda_2$ for the absorption curves when methyl iodide is just on the long wave-length side of 0.375A, the critical ab-

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[17] A. H. Compton, Phys. Rev. **35**, 930 (1930).

[18] G. Wentzel, Zeits. f. Physik **43**, 1 and 779 (1927).

[19] L. Pauling, Proc. Roy. Soc. **A114**, 181 (1927).

sorption wave-length for iodine. We accordingly felt rather doubtful of the reality of this wave-length. For this reason we replaced the methyl iodide with ethyl bromide. The temperature when ethyl bromide was used in the ionization chamber was 24.5°C. At this temperature the density of the bromine in saturated ethyl bromide is 0.00197 gm/cm$^3$. The primary x-rays were filtered through copper as well as aluminum in an attempt to increase the homogeneity of the rays. The absorption measurements are shown by the black circles in Fig. 4. The absorption measurements were carried to a much greater thickness of aluminum than in the case where methyl iodide was in the chamber. This was done in order to obtain the wave-lengths as accurately as possible. The solid curve of the Fig. 4 is constructed with the use of the data shown in Table III. Again as in the case of methyl iodide we have two wave-lengths, but both are on the short wave-length side of the absorption edge from bromine. The experimental and theoretical values of $J''/J'$ are shown in Table V.

TABLE V. *Experimental and theoretical values of* $J''/J'$. *Ethyl bromide in ionization chamber. Potential* 95 kv.

| | | Experiment | | | Dirac | Klein-Nishina | Compton | Thomson |
|---|---|---|---|---|---|---|---|---|
| $\phi'$ | $\phi''$ | $A$ | $B$ | Ave. | | | | |
| 75° | 97°30′ | 1.085 | 1.110 | 1.098 | 1.097 | 1.102 | 1.17 | 1.149 |
| 75° | 120° | 1.755 | 1.795 | 1.775 | 1.740 | 1.755 | 1.89 | 1.860 |

## 5. DISCUSSION

The agreement of the experimental values of $J''/J'$ with the Dirac or Klein and Nishina values as shown in Table V is excellent. At the wave-lengths used the Dirac or Klein and Nishina Formula is a much better description of the experimental facts than the Compton or Thomson Formula. The agreement of the experimental values of $J''/J'$ with the Dirac values as shown in Table IV is also good for the 90 kv x-rays. The agreement for the 125 kv x-rays, however, is not so good although the agreement of the mean values with the Dirac values is good. The corona discharge when the x-ray tube was excited with 125 kv was considerable and this seemed to introduce irregularities into the electrometer readings. This difference between the $A$ and $B$ values for $\phi''=97°\ 30'$ is due to the irregularities.

In calculating the theoretical values of $J''/J'$, two wave-lengths have been used. In the case of ethyl bromide, the longer wave-length was 0.47A. This wave-length is perhaps somewhat fictitious and we feel that the reality of the wave-length 0.26A is greater because by far the larger part of the absorption curve in Fig. 4 is due to this wave-length. If it is supposed that only this wave-length is present in the primary rays, the Dirac values in Table V become 1.098 and 1.753 for $\phi''=97°\ 30'$ and 120° respectively, while the Klein and Nishina values become 1.103 and 1.771. Also it might be objected that even the wave-length 0.26A is an average wave-length. We have therefore calculated the Dirac values in the case of ethyl bromide for $\lambda=0.22$, 0.24, 0.28, 0.30A in addition to 0.26A. The values of $J''/J'$ vary from 1.098 to 1.108 at $\phi''=97°\ 30'$ and from 1.73 to 1.76 at $\phi''=120°$. The Dirac theoretical values of $J''/J'$ are therefore practically constant for a wave-length range of 0.08A about 0.26A. The Compton values are also practically constant over the same range. This is our justification for using the average wave-length 0.26A in the calculation of the theoretical values.

If the effect of the Compton change of wave-length on the ionization produced in the chamber is neglected, the Dirac theoretical values of $J''/J'$ for 0.26A become 1.035 at $\phi''=97°\ 30'$ and 1.540 at $\phi''=120°$ in total disagreement with the experimental values. The ionization produced by the photoelectrons ejected from the brass wall at the end of the chamber and the aluminum window were neglected because with the length of chamber used this ionization is only a small fraction of the total ionization in the chamber. The aluminum window was 0.005 cm thick and the absorption in it was neglected.

We observed a rather curious effect during these experiments. The hardness of x-rays as determined by absorption in aluminum depends upon (a) the angle at which the rays leave the target of the x-ray tube (b) the nature of the gas in the ionization chamber, and (c) the temperature of the chamber if saturated vapor is used in the chamber. The length of the ionization chamber is such that the x-rays are incompletely absorbed. It is quite possible to apply a higher voltage to an x-ray tube and yet obtain less penetrating x-rays.

For the wave-lengths used in this research it is not possible to discriminate between the Dirac and the Klein and Nishina formulas. Recently, Chao[20] using $\gamma$-rays of wave-length 0.0052A has shown that the Klein and Nishina Formula fits the experimental facts much better than the Dirac Formula. This research was assisted by a grant from the Science Research Fund of Washington University.

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[20] C. Y. Chao, Phys. Rev. **36**, 1519 (1930).