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SPACE CHARGE vs. IMAGE FORCE IN THERMIONIC EMISSION

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Abstract

This paper is a joint report on work by the two authors, preliminary to further and more detailed publication on different phases of the problems here discussed. Attention is called to certain deficiencies in the Schottky image force as an explanation of the thermionic work function, both on theoretical and experimental grounds. Evidence is advanced to show that space charge is more important than image force over most of the region in which the image force is thought to be valid. It is further shown that space charge with Fermi-Dirac statistics is able to account for observed phenomena of thermionic emission, including the effect of external fields. Numerical examples indicate some of the consequences of this point of view.

THE most commonly accepted explanation of the thermionic work function is that which attributes the major part of it to the "image" force of the electron (Debye, Schottky, Langmuir). Schottky[1] gave an analysis to show that space charge and structure effects, in the layer outside that in which the surface atomic structure of the emitting body plays an important part, were negligible compared with the image force. To do this it was necessary to estimate the electron density in this layer; this he calculated on the assumption that the image force law was correct, showing that under these conditions space charge and structure effects were negligible.

On the other hand, if the assumption is made that the potential and thereby the charge density in the above-mentioned layer is determined by space charge instead of by image force considerations then it may be shown that the space charge is more important than the image force. Thus we must look further for evidence by which to choose between the two explanations. As a numerical illustration of the difference involved, a calculation of the field intensities from these two contrasting standpoints at various distances from a plane tungsten cathode at 2300°K gives the following results in the absence of external field. E_i being the field intensity in v/cm computed from the image force, E_s that computed for the space charge field, we have, at a distance of 10^{-7} cm from the cathode, $E_i=3.6\times10^6$, $E_s=4.0\times10^6$; at a distance 5×10^{-7} cm, $E_i=1.4\times10^5$, $E_s=8.0\times10^5$; at 10^{-6} cm, $E_i=3.6\times10^4$, $E_s=4.0\times10^5$. Thus, for distances from the surface greater than about 10^{-7} cm, the field intensity computed from space charge considerations is greater than when computed by the image force, and this difference is accentuated as the distance from the surface increases. The concentration of electrons shows a similar behavior. The space charge treatment of course assumes a continuous distribution of electricity whereas the image force assumes the escape of only

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[1] Schottky, Phys. Zeits. **15**, 872 (1914).

one electron. As Schottky correctly points out, since in practice we find a situation intermediate between these two view-points it is necessary to examine which of these two extremes is most nearly approached in practice.

A reconsideration of this matter seems desirable, not only in view of the above fact, but also because the earlier work on the subject postulated a Maxwell distribution law in the electron atmosphere. Since the advent of the Sommerfeld electron theory it becomes desirable to find whether the degeneracy of the electron gas at and presumably also near the emitting surface affects the analysis. Among the chief reasons for the general acceptance of the image force explanation have been (1) the observed constancy of the work function for a given pure metal under moderate applied accelerating fields, i.e. the thermionic saturation current, (2) for pure metals the fairly accurate verification of the variation of thermionic current with stronger applied fields (Schottky effect) as computed by Schottky, (3) the estimation of a reasonable value for the work function, with the opportunity of characteristic values for different materials. Its chief drawback at the present time appears to be in its application to emission from coated filaments, i.e. in failing to account for lack of saturation and for the Schottky effect observed in these cases without complicated, rather *ad hoc*, assumptions.

It is clear therefore that before any superiority may be pointed out a space charge analysis should also lead to confirmation with experiment in the above directions. In deriving the thermionic current, extending through the Schottky effect, by use of space charge considerations alone, if Poisson's equation is combined with a Maxwellian distribution of electron velocities the results show definite lack of agreement for pure metals. But if a Fermi distribution is substituted for the Maxwellian, the calculated dependence of current on temperature and applied field is found to be in reasonable agreement with experiment. This may be taken to indicate the constancy of the work function calculated by this method, but it seems more reasonable to deal with directly observable quantities, and compare calculated with observed currents under varying conditions. Furthermore, the variation of the current with the nature of the emitter can be explained more readily by space charge than by image force. According to the latter the field becomes constant within a certain critical distance from the surface, this distance being so chosen that the total work function thus calculated agrees with the experimental value. As justification for this step it is pointed out that close to the surface structure effects would be important. Now, assuming the space charge method valid right down to the surface, different currents from different metals are explained by the different electron concentrations within the metals, and the work function comes out to be of the correct order of magnitude according to the Sommerfeld theory (i.e. $=W_a$) without further assumption. This leads, incidentally, to a simple explanation of the observed interdependence of A and b, the constants in the usual thermionic emission equation. Moreover it appears that the use of Fermi-Dirac statistics takes account, in part at least, of structure effects. For a simple calculation will show that an electron gas is degenerate when the electrons are so closely crowded that the mutual potential energy of neighboring pairs is large compared with their kinetic energy of thermal agitation, and obeys classical laws when the potential energy is small. For example, in an electron gas at 1600°K for which the Fermi A is 10^{-3}, i.e. the gas nearly in the classical state, the average kinetic energy is roughly twenty times the potential.

Thus it appears that the space charge calculation based on Fermi-Dirac statistics is a more proper method of dealing with pure metals, and it may be further pointed out that the same calculation based on classical statistics, assuming a low electron concentration in the emitter, seems capable of explaining results obtained with coated filaments. This latter point is being investigated further.

Following out their preliminary investigations on this subject[2] the authors have carried the work further, one of them taking exclusively the problem of deducing the thermionic current under applied accelerating fields from zero to fields operative in the Schottky effect, the other treating exclusively the case of the statistical equilibrium resulting under retarding applied fields (including zero). Both deal only with the case of an infinite plane emitting surface.

The former (R. S. Bartlett) has handled the problem after the manner of Langmuir and Fry by postulating the emission of electrons with velocities distributed according to the Fermi statistics, these electrons being subject only to the space charge field, and obeying the law of continuity of current. This calculation is rendered troublesome by the necessity of graphical computation of the Fermi integrals for the region of transition from a degenerate to a classical state, and thus are not at present in a form suitable for detailed report in this communication. The general results have been mentioned above, and appear encouraging, especially in pointing the way to a single explanation of thermionic currents from zero applied field up through fields of Schottky intensity.

The latter author (A. T. Waterman) starting with Poisson's equation and the Fermi analogue $A = A_0 e^{-v\epsilon/kT}$ of the familiar Boltzmann relation, has evaluated the potential and the electron concentration at points outside the surface, where the atmosphere is in the classical state (assuming a degenerate state within the metal). In the presence of a retarding field E the solution takes different forms on either side of a critical distance, which, expressed in terms of potential difference between this distance and the surface, is $V' = 2 \log E/\beta$ where $\beta^2 = 16\pi(2\pi m)^{3/2} h^{-3} (kT)^{5/2} e^{W/kT}$ where W is Sommerfeld's W_i, the thermodynamic potential of the electrons within the body. Thus for $V > V'$ (thus including zero field), V (at distance x) $= -2kT/\epsilon \log (1 + \epsilon\beta/2kT\, x)$ and the number of electrons per cc, $n = kT/2\pi\epsilon^2 x^2$. For $V < V'$, $V = -kT/\epsilon(\epsilon Ex/kT - \log 4E^2/\beta^2)$ and $n = E^2/2\pi kT\, e^{-\epsilon Ex/kT}$. It will be noted that on the space charge conception the concentration of electrons in equilibrium with a hot body depends only upon the distance and the temperature,—in particular not upon the nature of the hot body, for distances at which the electron gas is in the classical state (roughly $> 10^{-7}$ cm). Examination as to

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Abstracts 52, 53 Phys. Rev. **35**, 668 (1930).

the above-mentioned critical distance introduced by an applied retarding field, which is a measure of the distance inside which the applied field has no appreciable effect on the electron atmosphere, results in the observation that applied retarding fields of the order existing in cold extraction currents would control the electron atmosphere extending into the region where the electron gas becomes degenerate very near the surface. A prediction of the work is that the ratio of the anode potentials which would give equal thermionic currents under zero applied field from two different emitters under otherwise identical conditions (temperature and geometry of tube) should be $V_1/V_2 = \log \beta_1/\log \beta_2$. If Sommerfeld's electron theory is assumed, then this ratio should show appreciable difference from unity, e.g. $V_{Ca}/V_W = 0.65$; on the classical theory the ratio should always be very nearly unity, e.g. $V_{Ca}/V_W = 0.98$ in the instance quoted.