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ON THE AXIAL ROTATION AND SPECTRA OF STARS

By Ross Gunn
Naval Research Laboratory, Washington, D. C.

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Abstract

Struve's remarkable relation between high observed axial rotations of stars and their bright line spectra is considered theoretically. It is shown that the two phenomena are closely related to the same electromagnetic effects that account for the anomalous solar rotation. High electromagnetic winds, resulting from crossed electric and magnetic fields, account for the high apparent rotational velocity and transfer sufficient momentum to the star in the course of stellar time to account for the high true rotations necessary to produce fission. The large electromagnetic wind velocities require the presence of comparatively large radial electric fields and it is shown that these can add sufficient additional excitation energy to the atmospheric ions to produce bright line spectra. Thus, high apparent axial rotations, high true rotations and bright line spectra in stars are intimately related and the existence of one, usually demands the presence of the others. An approximate expression is derived for the time rate of increase of angular momentum in a star.

IN TWO recent papers,[1] Otto Struve has drawn attention to a remarkable correlation between the spectra of stars and their apparent rotation as derived from measurements of Doppler displacements. Dr. Struve points out that emission lines are found principally in the O and B type stars and that in these "Excessive rotations, estimated at . . . (. . . 250 km/sec or more), are frequent. Apparently bright lines occur preferentially in stars having rapid axial rotation." The very high peripheral velocities indicated by his observational data suggest that certain stars are rotating so fast that they are on the verge of division to form binary systems. This conclusion is in accord with statistical data on binary systems which show that most close binary systems are B type stars or later. In the following discussion we will assume that the internal constitution and atmosphere of a given star is not greatly different from that of our sun and show that Dr. Struve's strange observed relations follow directly from a consideration of electromagnetic effects which we have shown to account for certain observed solar peculiarities.

Superposed Electromagnetic Winds

In a series of papers dealing with the electricity and magnetism of the sun[2] it has been shown that the observed variations of the apparent solar rotation with latitude, altitude, and time are readily explained in terms of atmospheric motions superposed upon the body rotation of the sun proper. These systematic motions of the atmosphere result from the interaction of

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[1] O. Struve, Astrophys. J. **72**, 1 (1930); **73**, 94 (1931).

[2] R. Gunn, Phys. Rev. **35**, 635 (1930); **36**, 1251 (1930); **37**, 283 (1931); **37**, 983 (1931); **37**, 1129 (1931).

crossed electric and magnetic fields with the atmospheric ions and we may express its magnitude u by[2]

$$u = \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2(1 + (R/\lambda)^2)} \tag{1}$$

where E is the radial electric field of the star at the observed level and B is the magnetic field (both quantities being expressed in e.m.u.), R the radius of the spiral generated by the moving ion, and λ the mean free path. We will hereafter refer to this systematic superposed atmospheric motion of the ions as an "electromagnetic wind."

The electrical energy dissipated in the solar atmosphere is an appreciable fraction of the total radiated energy.[2] This seems probably to be true in all stars having magnetic fields and indicates that stars with high surface temperatures (high radiation per unit area) are surrounded by a comparatively large radial electric field. Thus in O and B type stars with surface temperatures ranging from 35,000° to 12,000° electrical effects would be expected to be very important. We have no knowledge of the magnetic field of a star and can only suggest that its magnitude and distribution is not unlike that of the sun. If this is the true situation then we would expect the ratio of the magnitudes of the electric and magnetic fields in early type stars to be large and high velocity electromagnetic winds to result. Dr. Struve's observation that excessive rotations are common in O and B type stars is consistent with the above conclusions.

EXCITATION OF RADIATION BY ELECTRIC FIELDS

Eddington[3] has concluded that bright line spectra in a non-nebulous star is evidence for the existence of "thunderstorms" in the atmosphere of the star, but he was unable to make his suggestion quantitative. We proceed to calculate the electrical energy added to an ion during its free path as a result of the presence of an electric field. An ion in the atmosphere of a star having electric and magnetic fields does not move in the direction of the electric field, on the average, but describes a cycloidal path and progresses in a direction at right angles to both the impressed electric and magnetic fields.[2] Due to the presence of the magnetic field, an ion cannot acquire more than a definite amount of energy from the electric field, even if the ion describes an infinitely long free path. The mean energy W added to an ion by the electric field is given by[2]

$$W = mvu[1 + u^2/v^2]^{1/2} \tag{2}$$

while the maximum energy W_m is considerably more than twice as much or

$$W_m = 2mvu(1 + u/v) \tag{3}$$

where m is the mass of the ion, v the component of its initial velocity in a plane perpendicular to B the magnetic field and u the magnitude of the elec-

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[3] Internal Constitution of the Stars, (1926).

tromagnetic wind given by Eq. (1). If we neglect the relatively less frequent case of successive collisions where an ion of initially large energy collides with another and starts it off with abnormally high velocity, we can express Eq. (2) in terms of the electromagnetic wind velocity u and the surface temperature of the star. Thus if we express W in terms of equivalent electron volts V, Eq. (3) becomes

$$V = \frac{u(2mkT)^{1/2}}{10^8 e}\left[1 + \frac{u^2 m}{2\,kT}\right]^{1/2} \tag{4}$$

where e is the electronic charge expressed in e.m.u., k the Boltzmann constant, and T the atmospheric or effective surface temperature. Table I gives the average increase in energy of an ion due to electrical fields in terms of equivalent electron volts, assuming that the initial energy of the ion when it starts its path is purely thermal. Many of the ions and electrons will have more than these energies and can produce additional excitation.

It will not be necessary to discuss in detail the exact mechanism whereby bright emission lines are produced, for it should be clear from Table I that ions in the atmosphere of a star can absorb sufficient energy from the star's electric field to excite bright line radiation. The table and Eq. (2) bring out clearly that the electrical excitation energy and hence the prevalence of bright lines increases rapidly with increase in the magnitude of the electromagnetic wind which is proportional to the apparent rotational velocity. Thus the theory that has been developed to account for the observed anomalies of the sun's rotation leads directly to a quantitative explanation of Struve's observed relation between high rotational velocities and bright line spectra in stars.

TABLE I. *Average increase in energy due to electric field.*

	Hydrogen ion Surface Temperature		Electron Surface Temperature	
u km/sec	10,000°	20,000°	10,000°	20,000°
10	1.74 volts	2.19 volts	0.031 volts	0.044 volts
25	7.80 "	8.46 "	0.079 "	0.111 "
50	28.8 "	29.4 "	0.16 "	0.22 "
100	112. "	113. "	0.32 "	0.45 "

ADDED ANGULAR MOMENTUM AND BODY ROTATION

With the possible exception of close binary stars in which the axial and orbital periods are identical, it seems impossible to determine the true rotational period of a star, either by observing the motion of its magnetic pole or by a determination of its departure from a true spherical form. We must have knowledge of the true body rotation in order to separate the various components of the motion and we are forced to assume that the initial rotation of a star is small and estimate the body rotation from the observed apparent motions, the physical properties of its atmosphere and its age. The directions of the electric and magnetic fields of stars are undoubtedly related to the direction of their rotation in the same manner as those of the sun and earth. The electromagnetic winds therefore blow in the direction of the body rotation, but much faster, and add momentum to the star proper. Thus, we definitely abandon the principle of the conservation of momentum in an evolving star. In connection with the sun this effect was considered[2] and it was shown that since the sun was formed, sufficient angular momentum has been transferred to the sun proper by its own electromagnetic winds to account for its present axial rotation.

The superposed systematic momentum of an ion in a star's atmosphere is derived entirely from the radial electric field, and the magnetic field serves only to change the direction of the moving ion. The mechanism is therefore one for converting radial momentum into angular momentum. There can be no reacting momentum transferred to the star's magnetic field by the mechanism described, for the magnetic force on an ion is always at right angles to its motion and the magnetic field cannot transfer energy to the ion or the ion energy to the magnetic field. Momentum, therefore, cannot be transferred by this means and we are left to consider only the momentum transferred to a star by viscous forces between the star proper and its fast moving atmosphere. This transfer of momentum is always in such a direction as to accelerate the axial rotation and calculations indicate that in the course of stellar time the added momentum is adequate to increase the angular velocity until a star becomes rotationally unstable.

The data available for a typical star are not sufficiently complete to warrant a detailed calculation and in the following we shall make a rough calculation which will serve to indicate only the approximate magnitude of the effects. The electromagnetic winds of the star's atmosphere transfer momentum to the star proper which we assume to be a semi-rigid gaseous body held together by gravitational, radiative, and electromagnetic forces. The torque T applied to the star as a result of the systematic motion of its atmosphere is

$$T = \eta A R_0 \frac{du}{dr} = \frac{d\Omega}{dt} \tag{5}$$

where η is the mean coefficient of viscosity of the transition layer between the atmosphere moving with a velocity u and the surface proper, A the effective areas in contact, R_0 the radius of the star, du/dr the radial velocity gradient and $d\Omega/dt$ the time rate of increase of angular momentum. It is clear from Eq. (1) that the superposed electromagnetic wind velocity u drops rather abruptly when the ion pressure increases sufficiently to make the mean free path λ as small as R, the radius of the spiral generated by the ion moving in the magnetic field. If the low lying regions of the star's atmosphere are in gravitational equilibrium, then the difference in altitude Δr, corresponding to a fractional change in the ion pressure of unity, is given by

$$\Delta r = \frac{kT}{zhg} \tag{6}$$

where k is the Boltzmann constant, T the temperature of the atmosphere, z its mean atomic weight, h the mass of a hydrogen atom, and g the surface acceleration due to gravity. The fractional change in the superposed velocity u in the interval given by Δr is also approximately unity so that we may write

$$\frac{\Delta u}{\Delta r} = \frac{zh\gamma M u_0}{kTR_0^2} \tag{7}$$

where u_0 is the superposed electromagnetic wind velocity in the observed regions of the star's atmosphere, γ the gravitational constant and M the mass of the star.

The coefficient of viscosity η of the transition layer is

$$\eta = \frac{(3zhkT)^{1/2}}{2^{3/2}\pi\sigma^2} \tag{8}$$

where σ is the diameter of the gaseous ions or molecules according to kinetic theory. We assume that the effective area in contact is a ring around the equator of the star of width R_0. Combining the foregoing relations and making obvious substitutions we have that the rate of increase in the angular momentum of the star proper is

$$\frac{d\Omega}{dt} = \left(\frac{3zh}{2kT}\right)^{1/2} \frac{zh\gamma u_0 R_0 M}{\sigma^2} \tag{9}$$

or if ω_t is the present value of the angular velocity

$$\omega_t = \omega_0 + \left(\frac{3zh}{2kT}\right)^{1/2} \frac{zh\gamma R_0 u_0 \tau}{\sigma^2 d^2} \tag{10}$$

where ω_0 is the initial angular velocity of the star and will be set equal to zero, d is the radius of gyration of the star about its axis of spin and τ is the age of the star. We have assumed a steady state and have made no attempt to allow for greater values of u_0 during the youth of the star or for its variation with latitude.

Now if V_0 is the apparent or observed peripheral velocity of the star and $\omega_t R_0$ the peripheral velocity of the surface proper then $V_0 = u_0 + \omega_t R_0$ and

$$\frac{V_0}{u_0} = \left[1 + \left(\frac{3zh}{2kT}\right)^{1/2} \frac{zh\gamma R_0^2 \tau}{\sigma^2 d^2}\right]. \tag{11}$$

Numerical data for calculating V_0/u_0 from Eq. (11) are lacking and we must assume that the star is of uniform density, its atmosphere is not unlike that of the sun and its age approximates 2×10^{20} sec. or the mean indicated age of all stars. Substituting therefore $z = 3.3$; $h = 1.67\times 10^{-24}$ gm; $k = 1.37\times 10^{-16}$; $\sigma = 10^{-8}$ cm; $\gamma = 6.67\times 10^{-8}$; $\tau = 2\times 10^{20}$ sec; $d^2 = 0.4R_0^2$ and for a typical B type star $T = 15{,}000°$ we find $V_0/u_0 = 4.1$ and therefore the true rotational velocity is roughly 3/4 the apparent observed value. A similar calculation for the sun yields 6.2 while the observed ratio is 4.0. It is clear from the results of our calculation that, unless B type stars are much younger in actual years of existence than their state of evolution indicates the true rotations are large fractions of the apparent rotations and therefore many of these types of stars are on the verge of division to form binary systems. It seems clear that high electromagnetic winds necessarily produce high speed axial rotations in a star of moderate age and the calculations of the present paper are not in disagreement with Dr. Struve's conclusion that many B type stars are almost rotationally unstable.

The observed high apparent axial rotations in O and B type stars, together with the well established fact that close binary systems with few exceptions are B type stars or later, seem significant, and strongly suggest that stars in this period of their evolution divide as a result of a tremendous increase in their angular momentum. The increase in angular velocity appears to be accounted for by electromagnetic effects rather than by an increase due to contraction of the star. The contraction of the star may play an important part; but there is great difficulty in accounting for the original angular momentum, and a mechanism to add momentum to the system must be provided somewhere in the plan of stellar evolution. It seems quite possible that the above mechanism for adding angular momentum to a star represents physical reality but further supporting evidence is desirable.

Conclusion

Dr. Struve's values for the apparent peripheral velocity of type B stars (approximating 250 km/sec) together with the present calculations suggest that the electromagnetic winds may have velocities of 50 km/sec or more and therefore in these stars the ratio of the magnitude of the electric and magnetic fields must be very large. The large electric fields are capable of adding considerable energy to the ions constituting the star's atmosphere and emission lines should be observed. The very high superposed atmospheric motion of the star will, as well, transfer momentum to the star proper and we should expect this stage of development of a star to be characterized by a large increase in angular momentum resulting ultimately in the formation of binary systems. Statistical evidence supports this conclusion. We see therefore that high apparent axial rotations, high true rotations and bright line spectra in stars are intimately related and the existence of one usually demands the presence of the others.