will be of importance if the two curves shown in Fig. 1 of the above mentioned article reach their asymptotic values at something like the same value of r, which may often be expected to be the case. Then it is seen that, unless the asymptotic values are the same within the order of magnitude of the average relative translational energy of two at the temperature considered, the intersection of the two curves will occur so high up that it will be out of reach of an average pair of colliding atoms even though the initial state be represented by the upper curve; of course if the initial state is represented by the lower curve, the transition cannot occur with an average pair of atoms unless the curves are close enough together so that the difference in energy can be made up by conversion of the limited translational energy available to electronic energy, but this consideration shows that transitions will often be prevented even when the conversion is in the other direction, from internal to translational energy. It is of interest to note that in the cases considered by Kallmann and London (Zeits. f. physik. Chem. **2B**, 226 ff. (1929)) the energy transferred from translational to internal or *vice versa* is, in the case of those transitions which occur with exceptional probability, of the order of magnitude of the translational energy of an atom at room temperature.

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May 7, 1931.

Continuous Spectrum of Hydrogen Molecular Ion (H_2^+)

The negative glow is generally aceepted as a good source of ionized spectra, due to the probable high concentration of ions at this portion. In hydrogen, therefore, there may be some hope to obtain some spectrum at the negative glow, which had, if any, H_2^+ as an emitter. The hydrogen molecular ion has been dealt with theoretically by a number of investigators. Recently E. Teller[1] has calculated the excited states produced when the ion is separated step by step into a proton and a hydrogen atom in the ground state. According to him almost all the potential energy curves show no minima, except those corresponding to the $1s\sigma$ and the $3\ d\sigma$.

Following the interpretation of the ordinary hydrogen continuous spectrum, we can expect some continuous spectrum which can be explained by transitions to any one or all of the unstable states of the H_2^+ molecule. With such an expectation a spectroscopic examination of the negative glow in hydrogen, at a pressure of about 0.2 mm Hg was carried out. Although the ordinary continuous spectrum is absent in the negative glow, an indication of the continuous spectrum, even if not so intense, is observed between H_β and H_γ.

The probable high concentration of the H_2^+ molecule suggests that the above mentioned continuous spectrum may be due to the H_2^+ molecule. The energy involved in this region of spectrum is not inconsistent with such an explanation. As the spectrum was not observed at so extremely low pressures as in the cases of Herzberg[2] and Brasefield,[3] it is not in danger of being confused with that of the fluorescence light on the walls of the tube. So that it will not be inadequate to ascribe this continuous spectrum to the H_2^+ molecule. Details will be published with reproduction in the Science Reports of the Tôhoku Imperial University, **20** (1931).

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May 4, 1931.

[1] E. Teller, Zeits. f. Physik **61**, 458 (1930).
[2] G. Herzberg, Ann. d. Physik **84**, 560 (1927).
[3] C. J. Brasefield, Phys. Rev. **33**, 925 (1929).

New Covariant Relations Following from the Dirac Equations

1. In an article appearing in this issue of the Physical Review the authors derive by means of the spinor calculus four invariant relations between the four wave functions which are independent of the potentials. (Eq. (6), (11), (12), (13) of Ch. III.) The first of these is

$$\frac{\partial j^\lambda}{\partial x^\lambda} = 0 \qquad (1)$$

where j^l is the four-current. The second relation with which we shall concern ourselves in this letter, may be written, using ordinary tensor notation:

$$\frac{\partial k^\lambda}{\partial x^\lambda} = \frac{mc}{h} J \qquad (2)$$

where k^l is also a four vector quadratic in the wave functions and defined by the equation preceding Eq. (17) of our article. In Eq. (14) two complex quadratic invariants Δ and $\bar{\Delta}$ were introduced; from these we form the two real combinations:

$$\tfrac{1}{2}(\Delta + \bar{\Delta}) = I$$
$$\tfrac{1}{2}i(\Delta - \bar{\Delta}) = J. \qquad (3)$$

These quantities are identical with the invariants I and J of Darwin. We also prove in our paper that

$$k^\lambda j_\lambda = 0;\; k^2 = -j^2. \qquad (4)$$

Further we proved (Eq. (17)) that:

$$k^\alpha M_{\alpha l} = \frac{h}{mc} j_l \cdot J;\; j^\alpha M_{\alpha l} = \frac{h}{mc} k_l \cdot J. \qquad (5)$$

Here M_{kl} is the antisymmetric tensor of the polarization (Darwin, Gordon), whose components are again quadratic in the wave functions.

2. For the sake of a possible physical interpretation and to facilitate comparison with recent work of Fock, it seems desirable to express the above relations in term of matrices. It is easily seen, however, that the original α matrices of Dirac are unsuited to express covariance relations. We use instead the form of the Dirac equations

$$(\Gamma^\lambda p_\lambda + mc)\psi = 0 \qquad (6)$$

which is used by Weyl and Sommerfeld. (see our paper, Eq. (1), (2), (3), Introduction). The Γ^l matrices satisfy the well known relations:

$$\Gamma^k\Gamma^l + \Gamma^l\Gamma^k = 2g^{kl}. \qquad (7)$$

We adopt now the first point of view mentioned in the introduction of our paper, and regard the ψ as invariant and the Γ^l as a matrix four-vector. Using this and the relations (7), we can now form the following covariant quantities.[1]

$$\begin{array}{ll} 1 & \backsim I \\ \Gamma^i & \backsim j^i \\ \Gamma^i\Gamma^k & \backsim M^{ik} \\ \Gamma^i\Gamma^k\Gamma^l & \backsim k_m \\ \Gamma^i\Gamma^k\Gamma^l\Gamma^m & \backsim J \end{array} \qquad (8)$$

None of the indices are allowed to be equal. We assert that the quantities in the right column are obtained by multiplying the corresponding quantities in the left column with ψ' from the left and ψ from the right, (ψ' is the transposed ψ; see[2]). The relations (1) and (2) can now be derived independently from the Dirac equation (6); for (1) this is well known, whereas Eq. (2) becomes:

$$\frac{\partial}{\partial x_1}\psi'\Gamma^2\Gamma^3\Gamma^4\psi\,\pi - \frac{\partial}{\partial x_2}\psi'\Gamma^3\Gamma^4\Gamma^1\psi + \cdots = \frac{2imc}{h}\psi'\Gamma^1\Gamma^2\Gamma^3\Gamma^4\psi. \qquad (9)$$

Relations (4) and (5) can now be derived immediately with the help of (7).

3. One can also easily, though not so symmetrically, express the quantities (8) in terms of the Dirac α and the Dirac-Darwin ψ. For j and M we refer to Gordon's[2] paper. For k, I and J one obtains the following expressions:

$$k^1 = -i\alpha_2\alpha_3 \quad k^2 = -i\alpha_3\alpha_1$$
$$k^3 = -i\alpha_1\alpha_2 \quad k^4 = -i\alpha_1\alpha_2\alpha_3$$
$$I = \alpha_4 \quad J = \alpha_1\alpha_2\alpha_3\alpha_4$$

We see therefore that the *spacial* components of k are identical with the three-vector σ as introduced by Dirac[3] and further used by Fock[4] and that the time component of k is equal to ρ_1 (Dirac) or ρ_a (Fock), whereas I is equal to ρ_3 (ρ_c) and J to $\rho_2(\rho_b)$. It thus appears that from the view point of covariance neither σ nor ρ are true vectors.

4. It follows from the equality of $k_{1,2,3}$ with σ_1, σ_2, σ_2 that the spacial components of the four-vector k are equal to the components of the magnetic moment of the electron. On the other hand it is well known that the spacial components of the tensor M^{ik} represent the magnetic moment of the electron also. When neglecting relativity (i.e. considering only the large ψ_3 and ψ_4) the k_1, k_2, k_3 become indeed equal to M_{yz}, M_{zx}, M_{xy} respectively. Thus it is seen that the

[1] For this systematization (without the above identifications) compare J. Schouten, Proc. Amsterdam **32,** 105 (1929); A. Proca, Journ. de Phys. **1,** 235 (1930). F. Sauter, Zeits. f. Physik **63,** 803 (1930) used it very profitably for the solution of special problems.

[2] Gordon, Zeits. f. Physik **50,** 630 (1928).

[3] P. A. M. Dirac, Principles of Quantum Mechanics, page 242.

[4] V. Fock, Zeits. f. Physik **68,** 522 (1931).

three-dimensional vector of the magnetic moment can be completed relativistically in two ways, either to a four-vector or to an antisymmetric tensor. In the corpuscular theory already L. H. Thomas[5] called attention to these two possibilities. What the exact physical meaning of this duplicity in character of the moment and especially of the fourth component of k is remains unclear to us. Perhaps it is connected with the two-fold interpretation of the spin as angular momentum or as magnetic moment.

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May 16, 1931.

[5] L. H. Thomas, Phil. Mag. **3**, 1 (1927).

On the Gas-Temperature in the Positive Column of an Arc

For the study of thermionization in the positive column of an arc the possibility of measuring exactly the gas temperature is of great importance. We have therefore developed a new method of ascertaining this temperature avoiding any errors of previous methods, which permits measuring the temperature by gas-density.

The density of the arc-gas in the axis of a stabilized (by whirling gas according to Schoenherr) straight d.c.-arc of 2 amp. in air and also nitrogen of atmospheric pressure were measured. This was done by testing the absorption of a soft x-ray (of about 6A) passing along the axis of arc. The intensity of the x-ray was observed by the Geiger-counter. In a comparative test without the arc, the gas density was diminished by evacuating the arc-space until the x-ray intensity became the same as with the arc. The intensity found thereby is also the average density of the arc-gas traversed by the x-ray. The disturbing spaces next to the electrodes were eliminated by measuring arcs of different lengths (5-20 cm).

With the ideal gas-law this density gives a temperature in the positive column of 5270 $\pm 300°K$ in the air-arc, and of $5460 \pm 320°K$ in the nitrogen-arc. Considering the dissociation at these high temperatures, one gets a gas-temperature of $5000 \pm 400°K$ in the air-arc and of $5200 \pm 450°K$ in the nitrogen-arc. By pyrometric measurements a pure carbon (2 mm diameter) inserted in the arc axis had 3100°K in the air-arc and 2300°K in the nitrogen-arc. The temperature of the carbon is not identical with that of surrounding gas; clearly it depends on chemical reactions at its surface (oxidation!).

An extensive report on our investigations will be published in the next number of the "Wissenschaftliche Veröffenlichungen aus dem Siemens-Konzern."

Alfred von Engel
Max Steenbeck

Wissenschaftliche Abteilung,
der Siemens-Schuckertwerke A.-G.,
Berlin,
April 28, 1931.