AN EXPERIMENTAL STUDY OF THE NATURAL WIDTHS OF THE X-RAY LINES IN THE L-SERIES SPECTRUM OF URANIUM

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Abstract

The half widths at half maximum of the rocking curves in parallel positions of the double x-ray spectrometer with calcite crystals reflecting in the first order have been investigated as a function of wave-length and this function is shown to be linear.

The rocking curves of U $L\alpha_1$ in three different antiparallel positions of the instrument give a natural width which is practically independent of the dispersion. The natural widths of U $L\alpha_1$ and U $L\beta_1$ have been observed as a function of voltage and no significant dependence was noted.

The half widths at half maximum of twelve lines in the uranium L-series spectrum were studied and the results are:

Line U L	α_1	α_2	β_1	β_2	β_3	β_4	β_5	β_6	γ_1	γ_2	γ_3	γ_6
$\Delta\lambda$ in X.U.	0.439	0.494	0.299	0.369	0.382	0.726	0.252	0.487	0.242	0.57	0.47	0.233
ΔV in volts	6.56	7.20	7.17	8.04	9.40	16.1	5.94	9.71	7.96	19.7	16.2	8.18

Possible correlations with the electron transitions are suggested and the predominance of nuclear effects are evident from the greater widths of lines involving the elliptical orbit L_I.

The observed widths are compared to the classical and quantum theory widths and the size of the nucleus necessary to explain the discrepancy is given. The possibility of observing the splitting of the energy levels due to nuclear spin as suggested by Breit is rejected. The actual shape of the line U $L\alpha_1$ has been investigated and found to approximate that predicted by the classical theory.

THE natural widths of x-ray lines in the K-series of molybdenum have been measured by Ehrenberg and Mark,[1] Ehrenberg and von Susich,[2] Davis and Purks,[3] Allison and Williams,[4] and Mark and von Susich[5] with the double x-ray spectrometer. Experiments on the relative widths of some x-ray lines in the L-series of lead and thallium have been carried out by Allison.[6] This paper is a report of experiments performed on the natural widths of the x-ray lines in the L-series spectrum of uranium. The accurate measurement of a greater number of lines in the L-series offers an opportunity to study any dependence on the transitions involved.

[1] Ehrenberg and Mark, Zeits. f. Physik **42,** 807 (1927).

[2] Ehrenberg and von Susich, Zeits. f. Physik **42,** 823 (1927).

[3] Davis and Purks, Proc. Nat. Acad. Sci. **13,** 419 (1927); **14,** 172 (1928); Phys. Rev. **34,** 181 (1929).

[4] Allison and Williams, Phys. Rev. **35,** 1476 (1930).

[5] Mark and von Susich, Zeits. f. Physik **65,** 253 (1930).

[6] Allison, Phys. Rev. **34,** 176 (1929).

Apparatus

The double x-ray spectrometer used in these experiments has been previously described.[7] The following changes have been made. The vertical slits, limiting the divergence of the beam in a plane perpendicular to the axes of rotation of crystals A and B, were 2 mm in width. The horizontal slits, limiting the divergence of the beam in a vertical plane including the axes of rotation, were of various widths which will be given later.

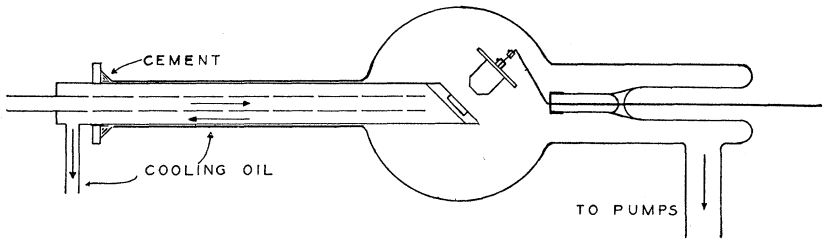

Fig. 1. Diagram of uranium target x-ray tube.

The x-ray tube used in these experiments is illustrated in Fig. 1. The tube was designed to give a large focal spot which would remain fixed on the target. These two properties of the tube are highly essential in the measurement of line widths as will be discussed later. The focal spot was at a distance of 50 cm from crystal A. The filament holder was designed from the type used by Professors Ross and Clark at Stanford University. It enabled the filament to be placed within one cm of the target and allowed the x-rays to be observed at an angle of 135° from the electron beam. The uranium was obtained from the Westinghouse Electric and Manufacturing Company. Many attempts were made to obtain good thermal contact between the uranium or some other metal but without success until finally the method used by S. K. Allison[8] was adopted. A piece of uranium was hydraulically pressed below the surface of a large copper block and the edges of the copper were peaned over while the uranium was under pressure. This served satisfactorily as a target up to a maximum power input of 0.4 kw. The tube was cooled by pumping cold kerosene through the hollow target holder.

The power for operating the tube was developed by a 5 kw. 550 cycle motor generator and the high potential supplied by a transformer was rectified by kenetrons and smoothed by condensers. The calculated voltage fluctuations were 0.6 percent at 48 kv and 6 milliamperes.

The calcite crystals used were those investigated in a previous experiment.[4] These crystals were preserved in a dessicator over dehydrated calcium chloride and sodium hydroxide, and it has been found that they have deteriorated only very slightly in the course of a year.

[7] Williams and Allison, J.O.S.A. and R.S.I. **18,** 473 (1929). For a further description see reference 4.

[8] Allison, Phys. Rev. **32,** 1 (1928).

Operation of the Instrument

The technique of measuring line widths has been previously developed. The degree of accuracy obtained with the double spectrometer can be seen by comparing the values of 0.144 and 0.163 X.U. for the half width at half maximum of Mo$K\alpha_1$ obtained by Mark and von Susich[5] and the value of 0.147 X.U. obtained by Allison and Williams.[4] The method of operation used in these experiments was that previously described.[4] The method of setting the crystal faces vertical was retained. Frequent readings of the temperature were taken while observations were being made and no significant changes were noted.

Reduction of Observed Results

The majority of the results have been obtained in the (1,1) position following a notation given in a previous paper.[9] The dispersion D of the double spectrometer is given by

$$D \equiv \frac{d\theta_B}{d\lambda} = \frac{n_A}{2d\cos\theta_A} + \frac{n_B}{2d\cos\theta_B} \tag{1}$$

n_A and n_B are the orders in which crystals A and B are reflecting respectively and θ_A and θ_B are the corresponding glancing angles. The sign of n_A is always taken positive. n_B is negative when the first incident and last reflected rays are on opposite sides of the first reflected ray,[10] otherwise it is taken positive. The dispersion D is therefore finite in all anti-parallel positions.

The first correction to be applied to the observed widths in anti-parallel positions is the geometric width due to the vertical spread of the beam. Schwarzschild's[10] equation for the "geometric breadth," $\delta\theta_B$, may be written

$$\delta\theta_B = \tfrac{1}{2}D\lambda\phi^2. \tag{2}$$

In this equation, $\delta\theta_B$ is the total angular range through which crystal B may be turned while reflecting the wave-length λ. If the widths of the slits limiting the divergence of the beam in a vertical plane are h_1 and h_2 ($h_2 \geqq h_1$), and the distance between them is L,

$$\phi = \frac{h_1 + h_2}{2L}. \tag{3}$$

The correction to be applied to the observed widths has been previously deduced[4] from Eqs. (2) and (3). Let ξ be the angular deviation of crystal B from the position of reflection for the central ray, (that ray passing through the geometrical centers of the slit apertures). Then the value of ξ at which the geometrical rocking curve drops to half maximum is given by[4]

$$\xi_{1/2} = \tfrac{1}{2}D\lambda\phi^2\left\{\frac{h_2}{h_1 + h_2}\right\}^2. \tag{4}$$

The values of $\xi_{1/2}$, in seconds of arc, calculated from Eq. (4) are arbitrarily subtracted from the observed full widths at half maximum.

[9] Allison and Williams, Phys. Rev. **35,** 149 (1930).

[10] Schwarzschild, Phys. Rev. **32,** 162 (1928).

The second correction to be applied to the resultant widths in anti-parallel positions is that due to the finite resolving power of calcite crystals. If it is assumed that the line has a Gaussian error curve distribution of intensity we have[11]

$$W = (W_A^2 + W_B^2 + D^2W_\lambda^2)^{1/2} \tag{5}$$

In the above equation, W represents the half width at half maximum in angular measure of the observed rocking curve after the geometric correction has been applied, W_A the half width at half maximum in angular measure of the curve representing the intensity of reflection from crystal A as a function of the deviations from the Bragg angle, W_B is the same quantity for crystal B, D is the dispersion given by Eq. (1), and W_λ is the half width at half maximum in linear measure of the line. The quantities W_A and W_B are observed in positions of zero dispersion (i.e. $n_B = -n_A$) and will be discussed under the next heading. These two corrections are the only ones applied to the observed widths if the deviation of the crystal faces from verticality has been reduced to zero.

Study of Parallel Positions

In the positions of zero dispersion the observed width W is from Eq. (5)

$$W = (W_A^2 + W_B^2)^{1/2}. \tag{6}$$

Assuming the crystals to be the same, $W_A = W_B = W_C$

$$W = 2^{1/2}W_C. \tag{7}$$

The values of W_C have been studied as a function of λ for the first order of reflection from calcite. There is no geometric correction to be applied to the observed widths in parallel positions if the deviation of the crystal faces from verticality is reduced to zero. The method of imposing this condition has been previously described.[4] The crystals were first adjusted as accurately as possible by the cathetometer method and a rocking curve was taken. Crystal B was then rotated through a minute of arc about an axis lying in a horizontal plane and a second rocking curve taken. This procedure was repeated until a position of crystal B was found which gave a rocking curve of minimum width. From this observed value of W the interference pattern width of a single crystal, W_C, was obtained by the use of Eq. (7).

The values of W_C have been obtained with the crystals reflecting the wave-lengths 908, 718, and 613 X.U. in the first order. These are plotted in Fig. 2. W_C is seen to be a linear function of the wave-length.

Darwin[12] and Ewald[13] have considered the problem of reflection of plane monochromatic waves from an ideal crystal. The extent of 100 percent reflection is given by

$$\Delta\theta = 4\delta \operatorname{cosec} 2\theta \tag{8}$$

[11] Schwarzschild, reference 10. Obtained by combining Eqs. (43) and (45) of his paper.
[12] Darwin, Phil. Mag. **27,** 325 and 675 (1914).

where θ is the glancing angle and δ is the deviation of the index of refraction from unity. Since [14]

$$\delta = 1 - \mu = Ne^2\lambda^2/2\pi mc^2 \tag{9}$$

where N is the number of electrons per cm^3, e is the electronic change in e.s.u., m is the mass of the electron, and c is the velocity of light, and from the Bragg law

$$\operatorname{cosec}\theta = 2d/n\lambda$$

we have

$$\operatorname{cosec} 2\theta = d \sec\theta/n\lambda. \tag{10}$$

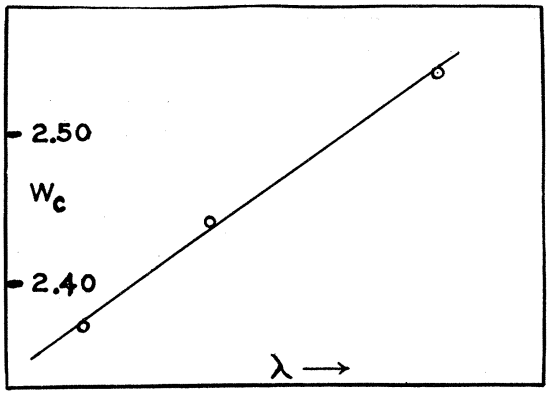

Fig. 2. The interference pattern width of a single crystal, W_C, as a function of the wave-length.

Substituting from Eqs. (9) and (10) in Eq. (8)

$$\Delta\theta = k\lambda \sec\theta \tag{11}$$

where

$$k = 2Ne^2d/\pi mnc^2.$$

Thus $\Delta\theta$ is an approximately linear function of λ as may be shown from the Bragg law and expansion in powers of λ.

$$\Delta\theta = k\lambda[1 + \tfrac{1}{2}(n\lambda/2d)^2 + 3/8(n\lambda/2d)^4 + 5/16(n\lambda/2d)^6 + \cdots]. \tag{12}$$

This series is rapidly convergent for first order reflection from calcite in the region examined.

The experimental value of W_C was found to be 1.6 times as large as that calculated for a perfect calcite crystal by a method given in a previous paper.[15] The value of W_C for the calcite crystals reflecting the wave-length 908 X.U. in the second order was 0.90 seconds of arc. This decrease of W_C for calcite with reflection in the second order is in agreement with the results of Davis and Purks[16] and Allison and Williams.[17]

[13] Ewald, Phys. Zeits. **26,** 29 (1925).
[14] A. H. Compton, X-Rays and Electrons, page 205.
[15] See reference 4. Eqs. (10) and Tables III and IV.
[16] Davis and Purks, Phys. Rev. **34,** 181 (1929).
[17] See reference 4, page 1489.

Study of U $L\alpha_1$ in Different Orders

It has been pointed out that a systematic change in the observed natural widths when measured in positions of different dispersion may be due to the finite size of the focal spot. During the observation of a curve crystal B is rotated through small angles while crystal A is left fixed. Under the previous adjustments when crystal B is reflecting the peak of the line the radiation comes from the most intense part of the focal spot. As crystal B is rotated up to and through this central position, the radiation comes from slightly different parts of the focal spot, and hence will have a different initial intensity. In positions of higher dispersion greater errors would arise from this cause, and the magnitude of the effect can be calculated. In order to overcome this difficulty the broad focus tube was used and the radiation was taken at an angle of 45° from the target. To obtain a measure of the intensity distribution of the focal spot the following experiment was performed. The double spectrometer was adjusted to reflect the peak of a given line and was left fixed in this position. The x-ray tube, which was mounted on a moveable carriage driven by a screw, was advanced by small steps in a horizontal direction perpendicular to the line of the slits. Thus the radiation reflected by the crystals at different steps came from successive parts of the focal spot, and the intensity could be examined directly. It was found that at a distance of 1.5 mm from the center the intensity had fallen to half of the maximum value. It is thus felt that the error arising from this source has been greatly reduced. Any further enlargement of the focal spot would decrease the intensity to a degree causing uncertainty in the observations.

The half widths at half maximum of the line U $L\alpha_1$ were investigated in three different positions of the double spectrometer. The results are given in Table I.

Table I. *Half widths at half maximum for U $L\alpha_1$ at 48 kv.*

Position (n_A,n_B)	D in ″/X.U.	h_1 cm	h_2 cm	$\xi_{\frac{1}{2}}$	W	$(W_A^2+W_B^2)$	W_λ in X.U.	Weighted average of W_λ in X.U.
(1,−2)	36.942	0.3	0.5	1.1″	16.9″	7.3″	0.453	
"	"	"	"	"	16.2	"	0.433	0.442
"	"	"	0.4	0.7	16.6	"	0.445	
(1,1)	68.877	0.3	0.3	0.7	27.6	12.9	0.398	
"	"	"	"	"	30.6	"	0.442	
"	"	"	"	"	30.9	"	0.445	
"	"	"	"	"	30.9	"	0.445	0.439
"	"	"	"	"	30.6	"	0.442	
"	"	0.2	0.2	0.3	29.8	"	0.430	
(1,2)	105.81	0.3	0.8	8.5	45.0	7.3	0.425	
"	"	0.5	0.5	3.1	46.0	"	0.433	
"	"	"	"	"	47.9	"	0.452	0.437
"	"	"	"	"	46.4	"	0.438	

The weighted averages have been secured by a personal estimate of the quality of the data.

For these observations D varies from 36.942 to 105.81 seconds per X.U. and the half width at half maximum, W_λ, varies from 0.442 to 0.437 X.U.

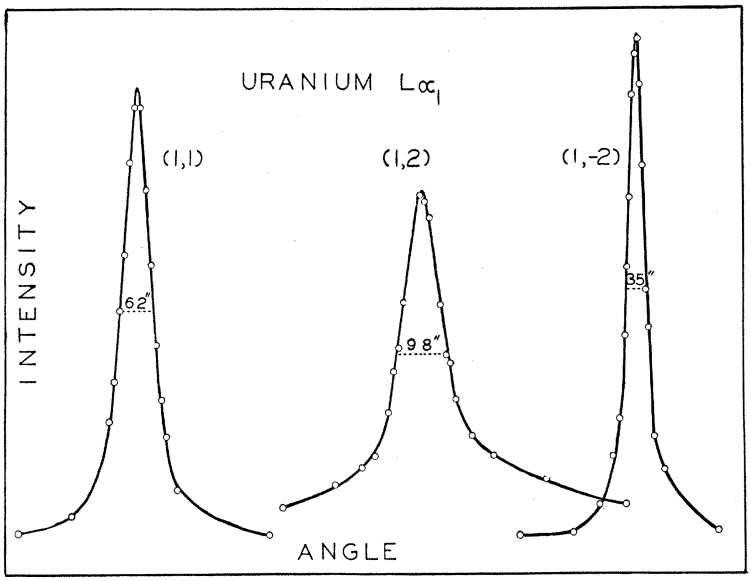

Fig. 3. Observed rocking curves of uranium $L\alpha_1$ at 48 kv. Ordinates are proportional to ionization currents, abscissae to angular settings of crystal B. The vertical scale is different for each curve.

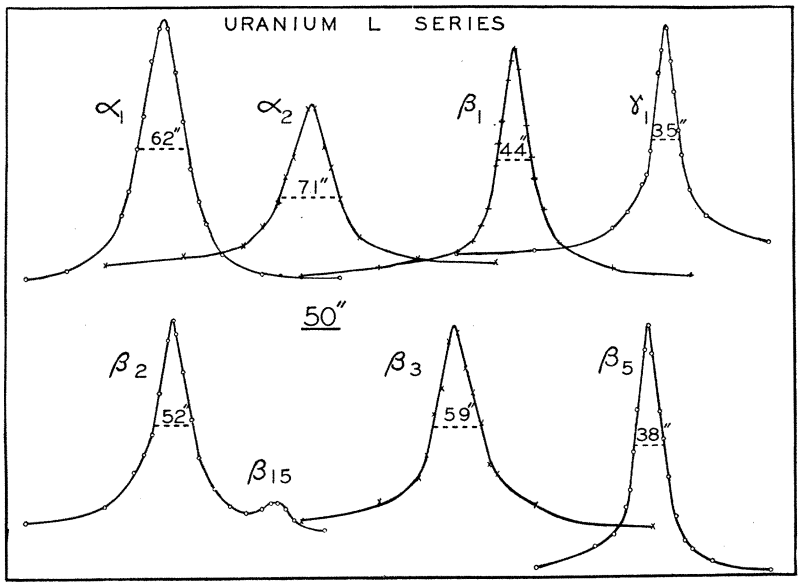

Fig. 4. Observed rocking curves in the (1, 1) position for seven lines in the uranium L spectrum at 48 kv. Ordinates are proportional to ionization currents, abscissae to angular settings of crystal B. The vertical scale is not the same for all the curves.

There is a trend towards narrower widths at positions of higher dispersion but this decrease is well within the experimental error, and the magnitude of this effect is less than formerly noted.[4] Typical curves of the line U $L\alpha_1$ in three different positions of the instrument are shown in Fig. 3.

The Natural Widths of the *L*-Series Lines of Uranium

Twelve lines of the *L*-series spectrum of uranium have been observed in the (1,1) position of the double x-ray spectrometer. Seven of these are shown in Fig. 4.

The half widths at half maximum of these lines have been calculated from the observed rocking curves in the (1,1,) position at 48 kv and are given in Table II. The details given in Table I are partially omitted but all corrections have been applied. The weighted averages of W_λ are obtained from a personal estimate of the quality of the data and the probable errors were calculated from the least squares formula.

The natural widths of U $L\alpha_1$ and U $L\beta_1$ have been examined as a function of voltage. Rocking curves in the (1,1) position were taken at different voltages under identical geometric conditions. The experimental error is greater at lower voltages due to the decrease in intensity as the excitation voltage is approached. The results are given in Table III where again the details of the calculation are omitted though all corrections were applied. An estimate of the experimental error of W_λ is also given.

Table II. *Half widths at half maximum for the L-series lines of uranium at 48 kv.*

Line	D in ″/X.U.	$(W_A^2+W_B^2)$	Number of observations	Weighted average of W_λ in X.U.
U$L\alpha_1$	68.877	12.9″	6	0.439±0.004
α_2	68.895	13.0	5	0.494± .006
β_1	68.580	11.9	8	0.299± .002
β_2	68.628	12.1	8	0.369± .003
β_3	68.568	11.8	5	0.382± .009
β_4	68.618	12.0	5	0.726± .010
β_5	68.590	11.9	6	0.252± .006
β_6	68.680	12.3	4	0.487± .009
γ_1	68.448	11.2	7	0.242± .003
γ_2	68.438	11.2	4	0.57 ± .02
γ_3	68.430	11.1	4	0.47 ± .01
γ_6	68.426	11.1	5	0.233± .005

Table III. *Half widths at half maximum of $UL\alpha_1$ and $UL\beta_1$ as a function of voltage.*

Voltage in kv.	W_λ of U$L\alpha_1$ in X.U.	W_λ of U$L\beta_1$ in X.U.
48	0.440±.005	0.295±0.005
39	0.445±.005	0.294± .005
30	0.439±.010	0.287± .010
24	0.435±.015	0.30 ± .02

Table III shows that there is no significant change of width with voltage for U $L\alpha_1$ and U $L\beta_1$ in the voltage region studied. Thus the part of the width of these lines due to satellites arising from multiply ionized states of the atom

is negligible. These lines involve the final orbits L_{III} and L_{II} and it is reasonable to assume that their behaviour is representative of lines arising from transitions to these levels. The lines involving the orbit L_I have such a relatively weak intensity that it is not practicable to measure their widths at voltages less than 48 kv.

Discussion of the Natural Widths

In order to facilitate the discussion of the natural widths and to make possible any correlations with the electron transitions involved the results are shown in Table IV.

In the third column the energy breadth ΔV in volts is calculated from

$$\Delta V = 12336\Delta\lambda/\lambda^2 \tag{13}$$

where $\Delta\lambda$ and λ are in Angstroms.

The fourth column of Table IV gives the initial and final quantum states of the atom for the transition involved. The fifth column gives the eccentricity of the initial and final orbits on the classical quantum theory measured by k/n.[18]

The $\Delta\lambda$s of the lines vary from 0.233 to 0.726 X.U. and the range of wavelengths from 592.6 to 920.14 X.U. The half width at half maximum predicted by the classical theory[19] is 0.059 X.U. and is independent of the wavelength in question.

A study of Table IV has suggested the following possible correlations. U $L\beta_3$, U $L\beta_4$, U $L\gamma_2$, and U $L\gamma_3$ involving the highly elliptical orbit L_I have the greatest energy widths observed. U $L\alpha_2$ and U $L\beta_1$ involve transitions from the same initial orbit and the eccentricities of the orbits involved are the same; the energy widths agree. U $L\beta_2$ and U $L\gamma_1$ involve transitions from similar initial orbits and the eccentricities of the orbits are the same as well as the changes in quantum numbers; the energy widths agree. For the lines U $L\beta_5$ and U $L\gamma_6$ the changes in the quantum numbers and the eccentricities of the orbits involved are the same but the energy widths are very different. There is however an agreement between their $\Delta\lambda$s. U $L\gamma_2$ and U $L\gamma_3$ involve orbits of the same eccentricity and there is a rough agreement between their energy widths; however the changes in quantum numbers are different.

These correlations are by no means complete but the wave-length region involved is so relatively great that the validity of a direct comparison between the energy widths or $\Delta\lambda$s and the transitions involved is doubtful. It is felt that no simple complete empirical correlation can be given. Coster[20] has qualitatively considered the question of the width of x-ray lines on the classical quantum theory. He concludes that the width of lines involving transitions between elliptical orbits is greater than those involving circular orbits. If ΔV is considered to be a measure of the width of the line these predictions are born out in general by the results shown in Table IV. The great

[18] Sommerfeld, Atombau und Spektrallinien, 4th edition, p. 125, Eq. 14.

[19] See reference 4, Eq. (20).

[20] Coster, Zeits. f. Physik **45**, 797 (1927).

TABLE IV. *Collected results on the half widths at half maximum of the observed rocking curves in the* (1,1) *position of the L-spectrum of uranium at 48 kv.*

Line	$\Delta\lambda$ in X.U.	ΔV in volts	Initial / Final	n / n	l / l	j / j	ϵ / ϵ	Δn	Δl	Δj
$UL\alpha_1$	0.439	6.56		3 2	2 1	5/2 3/2	1 1	1	1	1
α_2	0.494	7.20		3 2	2 1	3/2 3/2	1 1	1	1	0
β_1	0.299	7.17		3 2	2 1	3/2 1/2	1 1	1	1	1
β_2	0.369	8.04		4 2	2 1	5/2 3/2	3/4 1	2	1	1
β_3	0.382	9.40		3 2	1 0	3/2 1/2	2/3 1/2	1	1	1
β_4	0.726	16.1		3 2	1 0	1/2 1/2	2/3 1/2	1	1	0
β_5	0.252	5.94		5 2	2 1	5/2 3/2	3/5 1	3	1	1
β_6	0.487	9.71		4 2	0 1	1/2 3/2	1/4 1	2	1	−1
γ_1	0.242	7.96		4 2	2 1	3/2 1/2	3/4 1	2	1	1
γ_2	0.57	19.7		4 2	1 0	1/2 1/2	1/2 1/2	2	1	0
γ_3	0.47	16.2		4 2	1 0	3/2 1/2	1/2 1/2	2	1	1
γ_6	0.233	8.18		5 2	2 1	3/2 1/2	3/5 1	3	1	1

width of U $L\gamma_2$, U $L\gamma_3$, and U $L\beta_4$, the last as previously observed by Allison[6] in lead and thallium, are the best illustrations of the effect of an elliptical final orbit. The lines U $L\alpha_1$, U $L\alpha_2$, U $L\beta_1$ involving both initial and final circular orbits have the smallest ΔVs observed with the exception of U $L\beta_5$.

Professor J. R. Oppenheimer[21] has calculated the width of U $L\alpha_1$ to be expected on the quantum mechanics from the Einstein probability coefficients of the states involved. He gives

$$\begin{aligned} \frac{\Delta\nu}{\nu} &= \frac{\sum A_n}{\nu} \\ &\sim 3^3 2^{14} \alpha^3 Z^2 / 5^{12} \\ &\sim 6 \times 10^{-6} \end{aligned} \tag{14}$$

where ν is the frequency of the line considered. A_n is the Einstein probability coefficients of spontaneous emission from the levels n to the L level in ques-

[21] From private conversation with Professor Oppenheimer. I am greatly indebted to him for his interest in these results.

tion, and the summation if taken over all filled levels of the atom lying higher than the L levels, and α is the fine structure constant. On the basis of this reasoning $\Delta\nu/\nu < 10^{-5}$ whereas exprimentally $\Delta\nu/\nu \sim 5 \times 10^{-4}$.

Thus the observed width must be due to other phenomena decreasing the life time of the excited state and consequently increasing the line breadth. Some possible causes may be the changes of the energy levels due to Hund coupling with the valence electrons in the atom; interactions with the neighboring atoms in the metal; or the effects of nuclear spin suggested by Breit.[22] Since the L levels of uranium are so deeply buried in the atom it seems that the nuclear effects must predominate in broadening the line. Professor Oppenheimer has suggested the following possibilities due to coupling with the nucleus.

$$\begin{aligned} &(1) \quad \text{Electron interchange} \quad \Delta\nu/\nu \sim \left(\frac{a_n}{a_0}\right)^2 \frac{1}{Z} \\ &(2) \quad \text{Induced dipole} \quad \Delta\nu/\nu \sim \left(\frac{a_n}{a_0}\right)^3 \\ &(3) \quad \text{Quadrapole moment} \quad \Delta\nu/\nu \sim \left(\frac{a_n}{a_0}\right)^2 \frac{1}{Z}. \end{aligned} \tag{15}$$

In these equations a_n is the effective radius of the nucleus, a_0 is the mean radius of the orbit, and Z is the atomic number. The first of these arises from energy considerations of electron interchange between the orbital and nuclear electrons.

If a_0 is calculated from the Bohr formula

$$a_0 = \tau^2 h^2 / 4\pi^2 m e^2 Z \tag{16}$$

where τ is the total quantum number and the experimental value of $\Delta\nu/\nu$ is assumed then the following values are obtained for the radius of the nucleus.

$$\begin{aligned} &(1) \quad a_n \sim 4.5 \times 10^{-11} \text{ cm} \\ &(2) \quad a_n \sim 1.8 \times 10^{-11} \text{ cm} \\ &(3) \quad a_n \sim 4.5 \times 10^{-11} \text{ cm} \end{aligned}$$

From this reasoning alone the probability of finding a nuclear electron at these distances from the center of the nucleus is large whereas from the Rutherford scattering experiments the Coulomb law of force holds down to 10^{-13} cm. The possibility of an unexpected penetration of the orbits is thus suggested.

In Fig. 4 the line U $L\beta_{15}$, first resolved by Allison,[6] is almost completely resolved but no reliable estimate of its width could be given. It is of the order of magnitude of 0.3 X.U. Its separation from U $L\beta_2$ was found to be 1.89 X.U. as the average of three measurements.

The line $L\beta_5$ on the energy level diagram is a doublet O_V, O_{IV} to L_{III} with a relative intensity of 9 to 1 on the Burger-Dorgelo rules. Although the line is relatively narrow no evidence of the weaker component could be found.

[22] Breit, Phys. Rev. **35**, 1447 (1930).

U $L\beta_3$ and Mo $K\alpha_1$ have nearly identical wave-lengths, 708.4 and 707.768 X.U., but their widths are greatly different, 0.382 and 0.147 X.U. respectively. The width of an x-ray line is therefore not a function of wave-length alone, but depends on the atom and transition involved. The half widths at half maximum of the L-series lines of uranium are from 1.6 to 5 times as great as that of Mo $K\alpha_1$. This is in agreement with the work of Allison[6] on thallium and lead but in his work the crystals used were relatively poor and the adjustments of the instrument imperfect thus causing higher values of the widths.

The actual shape of the line U $L\alpha_1$ was investigated by taking many points on a rocking curve when the value of $\xi_{1/2}$ was reduced to negligible proportions. The classical theory gives for the shape of a line[23]

$$I_\nu = I_0/\{k^2 + [(2\pi\nu)^2 - (2\pi\nu_0)^2]\}. \tag{17}$$

In the derivation of Eq. (5) Schwarzschild[10] has assumed a Gaussian error curve distribution of intensity,

$$I'_\nu = I_0 e^{-k^2(\nu-\nu_0)^2} \tag{18}$$

The values of k and I_0 were selected so as to make the theoretical curves fit the actual curve at the maximum and half maximum ordinates. It was found that Eq. (17) gave a much better fit than Eq. (18). However the tails of the observed curve and the classical curve did not agree, the observed curve falling off more steeply than that given by Eq. (17).

Breit[22] has considered the effects of nuclear spin on x-ray terms and has predicted the splitting of the L_{II} level of uranium to be of the order of magnitude of 4.0 volts. This prediction is based upon the assumptions that the uranium nucleus has a spin 9/2 and a magnetic moment 9/2 proton units and that screening effects are neglected. If one attempts to apply this prediction to the line U $L\beta_1$ for example, it is found that the natural breadth of the line is so great that no appreciable evidence of splitting would be expected.

In conclusion, I should like to express my thanks to Professor S. K. Allison for the suggestion of this problem and his helpful correspondence during the course of the investigation. My thanks are also due Professor R.B. Brode for his kindly criticism and interest in this research.

[23] W. C. Mandersloot, Jahrbuch der Rad. und Elek. **13**, 1 (1916).