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THE REFLECTION OF THE $K\alpha$ LINE OF CARBON FROM QUARTZ AND ITS RELATION TO INDEX OF REFRACTION AND ABSORPTION COEFFICIENT

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Abstract

Measurements were made of the ratio of the intensity of the reflected ray to that of the incident ray in the case of the $K\alpha$ line of carbon reflected from a mirror of fused quartz for glancing angles of incidence between 1° 30′ and 8°. The atomic absorption coefficients of carbon, nitrogen, oxygen, and neon were experimentally determined by the writers as described in the preceding article. This gave the value of the oxygen coefficients directly and data from which the atomic absorption coefficient of silicon could be calculated. The linear absorption coefficint, μ, of quartz was then computed from its density, percentage composition, and the atomic absorption coefficients of its constituents. Then from the relation $\kappa=\mu\lambda/4\pi$, the absorption index κ, was found to be 1.68×10^{-3}. The value of $\delta=1-n$ was computed from the Drude-Lorentz formula and found to be 4.8×10^{-3}. Convenient methods for the calculation of the intensity of either the parallel or perpendicular components of the reflected ray are derived from the Fresnel equations relating to absorbing media. A curve computed with the above values of δ and κ predicts somewhat higher reflectivity than found experimentally. Use of the value $\kappa=2.5\times10^{-3}$ yields a computed curve agreeing well with the experimental one. On the other hand, assuming values of δ other than that given above leads to less satisfactory agreement.

Introduction

THIS investigation was undertaken with the view of measuring the intensity of soft x-rays reflected at various angles from a mirror and at the same time determining, either theoretically or experimentally, the magnitude of the index of refraction and the absorption coefficient of the mirror. Knowing the latter two quantities and the curve of reflected intensity plotted against glancing angle of incidence it should be possible to learn something regarding the precise validity of the extrapolations of classical dispersion formulas from the optical to the soft x-ray region.

Quartz appeared to be one of the most promising materials for investigation since it could be highly polished and particularly because the $K\alpha$ line of carbon ($\lambda=44.6$A), which is one of the most intense and easily obtained lines in this region, was so far removed from a critical frequency of either silicon or oxygen that the index of refraction could be computed from classical dispersion theories with a great deal of confidence. Hence measurements of the intensity of reflection of this line from quartz were made and the results briefly noted in a paper read before the American Physical Society.[1] However

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[1] Dershem and Schein, Phys. Rev. 35, 292 (1930).

it was not found possible to secure a film of quartz sufficiently thin to transmit a measurable amount of this radiation and permit a direct measure of its absorption coefficient. Hence attention was turned to the possibility of measuring the absorption of this line in gases and determining the law of absorption for elements of low atomic number. A description of this work is given in the preceding paper. In this way a value of the absorption coefficient of oxygen was obtained directly and that of silicon was determined from the formula $\mu_a = 1.65\, Z^{4.4} \times 10^{-23}$ which had been derived from the experimental data pertaining to other elements.

THEORY

The critical glancing angle of total reflection for hard x-rays is quite sharply defined and furnishes a convenient method of measuring indices of refraction in this case. However, with increase of wave-length the discontinuity in reflected intensity at the critical angle becomes less abrupt and finally for very soft x-rays entirely disappears. This is due partly to a more gradual change in reflected intensity at the critical glancing angle as this angle becomes larger, even in the case of substances entirely transparent to the radiation. However it is mainly due to an increase of absorption. Hence if the index of refraction is to be deduced from an experimental curve of reflected intensity plotted against glancing angle it is necessary to know the absorption coefficient and to find, if possible, the value of the index of refraction which must be assumed in order to secure agreement between calculated and experimental curves.

A formula for computing the ratio of reflected to incident intensity at various angles may be derived in the following manner. In using this formula the index of refraction and the index of absorption may be either known or assumed.

Considering first the component with the electric vector perpendicular to the plane of polarization, the ratio of the reflected to the incident amplitude is given in the following formula of Fresnel,[2] in which ϕ and r are the angles of incidence and refraction measured from the normal.

$$\frac{R_s}{E_s} = \frac{\cos\phi - n\cos r}{\cos\phi + n\cos r}\,. \tag{1}$$

By Snell's law $\cos r = (1-\sin^2\phi/n^2)^{1/2}$. Hence

$$\frac{R_s}{E_s} = \frac{\cos\phi - (n^2 - \sin^2\phi)^{1/2}}{\cos\phi + (n^2 - \sin^2\phi)^{1/2}}\,.$$

Changing to the glancing angle of incidence θ, since $\cos\phi = \sin\theta$

$$\frac{R_s}{E_s} = \frac{\sin\theta - [n^2 - (1 - \sin^2\theta)]^{1/2}}{\sin\theta + [n^2 - (1 - \sin^2\theta)]^{1/2}}\,.$$

In order to take account of absorption it is necessary to substitute a complex expression[3] for n, namely $(1-\delta-i\kappa)$ in which $1-\delta$ is the index of refrac-

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[2] Cf. Drude, "Theory of Optics" English translation, 282 (1902).

[3] Drude, loc. cit. p. 360 et seq.

tion and κ the index of absorption. Since δ and κ are small compared to unity the squares and products of these quantities may be neglected. Also θ may be substituted for $\sin\theta$. Making these substitutions and simplifications[4]

$$\frac{R_s}{E_s}=\frac{\theta-(\theta^2-2\delta-2i\kappa)^{1/2}}{\theta+(\theta^2-2\delta-2i\kappa)^{1/2}} \tag{2}$$

Setting

$$a-ib=(\theta^2-2\delta-2i\kappa)^{1/2} \tag{3}$$

$$\frac{R_s}{E_s}=\frac{\theta-a+ib}{\theta+a-ib}\,.$$

Multiplying by the complex conjugate to secure the square of the absolute value which is the desired ratio of intensities

$$\left(\frac{R_s}{E_s}\right)^2=\frac{(\theta-a)^2+b^2}{(\theta+a)^2+b^2}\,. \tag{4}$$

Squaring Eq. (3) and separately equating the real and imaginary terms

$$a^2-b^2=\theta^2-2\delta \tag{5}$$

$$b=\kappa/a \tag{6}$$

Whence

$$a^2=\tfrac{1}{2}\left\{\theta^2-2\delta+\left[(2\delta-\theta)^2+4\kappa^2\right]^{1/2}\right\}. \tag{7}$$

The numerical value of a may now be readily determined by substituting the known or assumed values of θ, δ and κ in (7). The positive value should be chosen for all square roots in the preceding discussion. Equation (6) then yields the value of b. The ratio of reflected to incident intensity is then given by substituting these values in (4).

For the case of the parallel component we have

$$\frac{R_p}{E_p}=\frac{n\cos\phi-\cos r}{n\cos\phi+\cos r}\,.$$

Handling this in a similar manner we obtain in place of Eq. (4) the following

$$\left(\frac{R_p}{E_p}\right)^2=\frac{(\theta-2\delta\theta-a)^2+(b-2\kappa\theta)^2}{(\theta-2\delta\theta+a)^2+(b+2\kappa\theta)^2}$$

a and b have the same values as in the case of the perpendicular component.

The reflected intensity of the parallel component is slightly less than that of the perpendicular component. In the case of the $K\alpha$ line of carbon reflected from quartz this is negligible for small angles, becoming about 5 percent at 8°.

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[4] An equation of this form has been given by Prinz, Zeits. f. Physik **47,** 479 (1928) but this is not a final form for numerical calculations.

Schön[5] has developed a more complicated formula for the parallel component yielding results equivalent to that above. Thibaud[6] has also published a formula of essentially the same form as that of Schön.

APPARATUS

The principal parts of the apparatus were the same as previously used and described by one of the writers[7] in an investigation of the reflection of the carbon line from glass. However certain mechanical features were added to permit direct comparisons of the incident and reflected beams.

Referring to Fig. 1, X is a water-cooled x-ray tube, the rays from which pass through the slits S_1 and S_2 and falling upon the grating G, become spread out into a spectrum of which only the inside order of the $K\alpha$ line may pass through the shield S_3 and strike the quartz mirror M. This mirror could be turned through small known angles by means of the lever L and the screw T. This screw could be turned in small steps by an electromagnetically operated ratchet controlled from outside. Another ratchet, operated by an electromagnet, was used to turn the eccentric cone bearing B on which the mirror was mounted. The mirror could thus be moved alternately into or out of the beam by a one-half turn of the bearing B. In this way a series of exposures of the direct and reflected beams could be taken alternately upon the plate P. The latter was carried on the plate holder H which could be given a step by

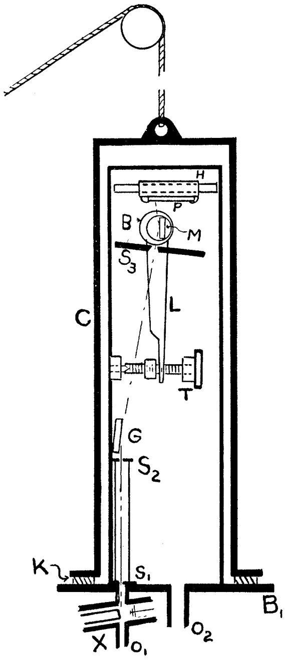

Fig. 1. Diagram of apparatus.

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[5] Schön, Zeits. f. Physik **58**, 165 (1929).

[6] Thibaud, Jour. de Physique **7**, 37 (1930).

[7] Dershem, Phys. Rev. **34**, 1015 (1929).

step transverse motion by means of another electromagnetically operated ratchet not shown in the sketch. In this way from 12 to 20 separate exposures could be made on one plate.

The apparatus could be opened or closed by raising or lowering the cylinder C by means of a rope and pulley. A vacuum seal between the cylinder and the base plate B_1 was secured by means of the sulphur free rubber gasket K. During operation a vacuum higher than 10^{-4} mm Hg was maintained by means of mercury diffusion pumps connected to the outlets O_1 and O_2.

RESULTS

The densities of the lines produced by the reflected and the direct rays were compared by photometric measurements and the ratio of their intensities determined in the same manner as described in the preceding article on absorption coefficients. The experimental points thus obtained are marked with circles in Fig. 2. The curves shown in this figure are, however, plotted from values of the ratio of reflected to incident intensity computed from Eq. (4). The values of δ and κ to be used were determined in the following way.

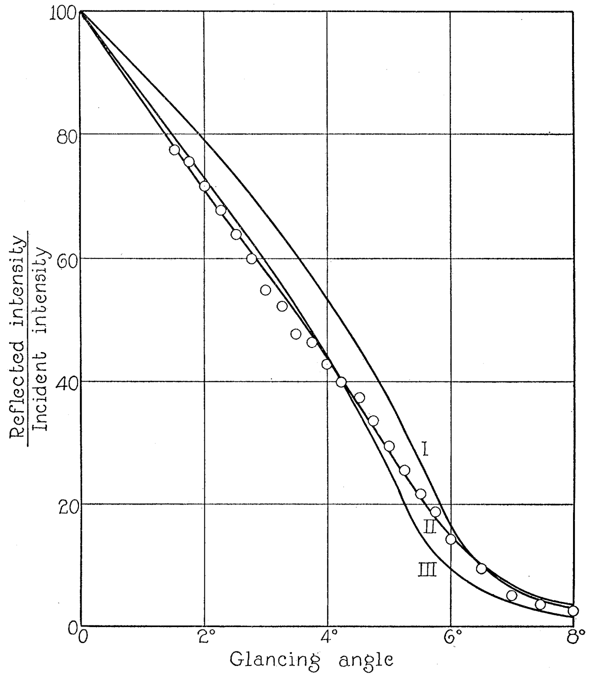

Fig. 2.

Since the K absorption frequencies of oxygen and silicon are much higher and their L absorption frequencies much lower than that of the $K\alpha$ line of carbon, δ may be computed from the simplified Drude-Lorentz formula, $\delta = e^2n/2\pi m\nu^2$ in which n is the number of electrons per cc having lower natural frequencies than that of the incident radiation. In this case the K electrons are not considered. One may also use the more extended formula, $\delta = e^2N/2\pi m \Sigma n_c/\nu^2 - \nu_c^2$ in which N is the number of molecules per cc, n_c the number of electrons per molecule belonging to the characteristic energy level c and ν_c the frequency pertaining to this level. The values of δ obtained from these formulas do not differ by so much as one percent owing to the fact that the incident frequency lies approximately midway between the characteristic K and L frequencies of oxygen and silicon. For quartz and the carbon line the value thus obtained for δ is 4.8×10^{-3}.

The value of κ was found from the experimental equation deduced in the preceding article on absorption coefficients namely, $\mu_a=1.65\ Z^{4.4}\times10^{-23}$ whence μ_a for silicon is found to be 18.2×10^{-19}. This in conjunction with the experimental value of μ_a for oxygen, 1.52×10^{-19}, and the density of fused quartz, 2.206 gm.cc, yields the value, $\mu=47200$ for the linear absorption coefficient of quartz. Then the absorption index κ, can be found from the equation $\kappa=\mu\lambda/4\pi=1.68\times10^{-3}$.

Curve I of Fig. 2 was computed with the use of the above values of δ and κ. This curve lies somewhat above the experimental points. This might be taken to indicate that the real value of the absorption coefficient was greater than that determined from the work on the absorption of gases. Edwards[8] finds a similar result when using ordinary x-rays. He suggests that the absorption of the surface layer may be greater than that of the mirror as a whole.

Other values of δ and κ were tried in an endeavor to find the best fit with experiment. Curve II in which $\kappa=2.5\times10^{-3}$ and δ has the same value as in curve I, gives a very good approximation to the experimental results. In computing for curve III the values, $\kappa=1.68\times10^{-3}$ and $\delta=4.0\times10^{-3}$ were used. This curve is distinctly less in accord with the experimental points. These curves and many others that were computed but not shown in the figure indicate that the value of δ computed from the dispersion formula is very nearly the correct one but that the effective value of the absorption index is about fifty percent higher than that determined from absorption measurements in gases. There is also the possibility that surface films or a lack of perfect smoothness have reduced the reflected intensity in much the same manner as would be the case if the absorption were really greater than the value deduced from experiment. The question might also be raised as to whether, or not, the true value of the absorption index may be thus obtained from the absorption coefficient. It is perhaps rather, to be considered remarkable that theories and formulas developed for the optical region before anything was known of x-rays or quantum absorption phenomena should yield results so nearly in accord with experiment when extended to this region where the wave-lengths are one hundred times smaller and the requirement of surface smoothness presumably so much more exacting. This work is to be continued with some changes in technique which it is hoped will somewhat increase the accuracy of measurement. Also other wave-lengths and other mirrors will be employed.

In conclusion we wish to express our appreciation to Professor A. H. Compton for his cordial interest and support throughout this investigation and the preceding one on the absorption of gases, and to thank the International Education Board for the award of a fellowship which made it possible for one of us (M.S.) to engage in the study of these problems.

[8] Edwards, Phys. Rev. **37**, 339 (1931).