

ABSOLUTE WAVE-LENGTHS OF THE COPPER AND
CHROMIUM *K*-SERIES

BY J. A. BEARDEN

DEPARTMENT OF PHYSICS, THE JOHNS HOPKINS UNIVERSITY

(Received April 6, 1931)

ABSTRACT

In the measurement of x-ray wave-lengths by ruled gratings two principal difficulties have been discussed which may account for the difference observed between the wave-lengths determined by this method and those determined by using crystal gratings. They are, the periodic error in the grating, and the geometrical divergence of the x-ray beam. It is now shown that the effect of the periodic error can be determined by a study of the intensities of the optical ghost lines. For a good quality optical grating it has been found that the error in x-ray wave-lengths due to the periodic error in the grating is thus of no importance. By using a suitable disposition of apparatus the effect of the geometrical divergence of the x-ray beam can be made as small as desired. Thus it is concluded that ruled gratings can be used for precise wave-length measurements of x-ray spectra.

In the present experiment the two parallel plate method has been used for determining the angles of incidence and diffraction. Five glass gratings of different grating spaces and ruled on two ruling engines have been used. The results from the various gratings on the same wave-length have agreed satisfactorily. No consistent variations of any type were observed. The final results from 172 sets of plates are given in the following tables.

Spectral line	Crystal λ	Grating λ	Limiting error	Grating λ — Crystal λ
Cu <i>K</i> β	1.38914A	1.39225A	$\pm 0.00014A$	+0.224%
Cu <i>K</i> α	1.53838	1.54172	± 0.00015	+0.217%
Cr <i>K</i> β	2.08017	2.08478	± 0.00021	+0.222%
Cr <i>K</i> α	2.28590	2.29097	± 0.00023	+0.222%

From these results the true grating space of a calcite crystal is $d = 3.0359 \pm 0.0003A$. Using this value of the grating constant, Planck's constant as determined by Duane, Palmer and Yeh is, $h = 6.573 \pm 0.007 \times 10^{-27}$ erg · sec. e/m can be determined from the dispersion of x-rays by using the absolute wave-length of an x-ray spectrum line. The mean of the values of the dispersion as given by Stauss and Larsson gives $e/m = 1.769 \times 10^7$ e.m.u.g⁻¹. The values of these constants are independent of any imperfection in the crystal. If the crystal lattice is assumed to be perfect we then have Avogadro's number, $N = 6.019 \times 10^{23}$ mol. per mole, and the charge on the electron $e = 4.806 \times 10^{-10}$ e.s.u. Using this value of e , and d as above, we find Planck's constant $h = 6.623 \times 10^{-27}$ erg · sec.

M EASUREMENTS of the wave-length of x-ray lines using ruled gratings have been made by a number of investigators.¹⁻⁷ The results obtained

¹ A. H. Compton and R. L. Doan, Proc. Acad. Sci. **11**, 598 (1925).

² J. Thibaud, Comptes Rendus **182**, 55 (1926).

³ F. L. Hunt, Phys. Rev. **30**, 227 (1927).

by this method have been in general higher than the corresponding wave-lengths measured by means of a crystal grating. Unfortunately the differences observed between the grating values and the crystal values by different experimenters have not been the same. In the best of the experiments, differences exist which appear to be greater than the probable errors in the experiments. It has been pointed out by the writer⁸ that these differences are most likely due to the quality of the gratings used and consistent errors of observation. The present experiment was undertaken in an effort to eliminate as far as possible the uncertainty which exists concerning the absolute wave-length of x-rays.

THEORY OF RULED GRATINGS FOR X-RAY MEASUREMENT

Since ruled gratings must be used at tangential incidence for x-rays, the usual grating formula $n\lambda = d(\sin i - \sin r)$ may be written in terms of the small angles θ and α of Fig. 1 as

$$\eta\lambda = 2d \left(\sin \frac{2\theta + \alpha}{2} \sin \frac{\alpha}{2} \right) \quad (1)$$

where θ is the angle between the surface of the grating and the direct beam, and α the angle between the reflected beam and the diffracted beam.

The angles θ and α for x-rays are very small so that the problem of accurately measuring x-ray wave-lengths is principally a problem of measuring very small angles with a high degree of precision. Assuming that these angles can be precisely measured, it will be of interest to investigate the possible error in the wave-length due to other causes.

The geometrical divergence of the x-ray beam perpendicular to the plane of the grating will shift the position of the spectrum line as was first pointed out by Porter.⁹ Porter's result was for the special case where the distance from the source to the center of the grating, and the distance from center of the grating to the position of the image were equal. Stauss¹⁰ has considered in a similar manner the more general case where the distances are not equal. The complete diffraction equation then becomes:

$$n\lambda = d(\cos \phi_1 - \cos \phi_2) \left[1 + \frac{3x^2}{20(\phi_2^2 - \phi_1^2)} \left(\frac{\phi_2^2}{l_2^2} - \frac{\phi_1^2}{l_1^2} \right) \right] \quad (2)$$

where $\phi_1 = \theta$, $\phi_2 = (\alpha + \theta)$, x is the length of grating used, l_1 the distance from the source to the grating, and l_2 the distance from the grating to the image. It will be seen that the correction term will be either + or - depending on the

⁴ E. Bäcklin, Inaugural Dissertation, Uppsala Universität (1928).

⁵ C. P. R. Wadlund, Phys. Rev. **32**, 841 (1928).

⁶ J. A. Bearden, Proc. Nat. Acad. Sci. **15**, 528 (1929).

⁷ J. M. Cork, Phys. Rev. **35**, 1456 (1930).

⁸ J. A. Bearden, paper presented at Optical Society of America, Charlottesville Meeting, October 1930.

⁹ A. W. Porter, Phil. Mag. **5**, 1067 (1928).

¹⁰ H. E. Stauss, Phys. Rev. **34**, 1601 (1929).

values of l_1 and l_2 . It will also be noticed that the position of the 0 order will be displaced if $l_1 \neq l_2$ thus

$$\phi_2 = \phi_1 \left[1 + \frac{3x^2}{20} \left(\frac{1}{l_1^2} - \frac{1}{l_2^2} \right) \right]. \quad (3)$$

The magnitude of this correction depends on the method used in measuring ϕ_1 and ϕ_2 .

The geometrical center of the slits is not the center of the reflecting surface of the grating. If β represents the angle between the central slit ray and a line from the source to the effective center of the grating, it has been shown¹¹ that

$$\beta = \frac{s^2}{4\phi_1 l_1^2} \quad (4)$$

where s is the slit width and ϕ_1 and l_1 are the same as in Eq. (2).

The divergence of the x-ray beam in the plane of the grating leads to a correction in the wave-length of the form

$$n\delta\lambda = \frac{d}{2} \left(\frac{\phi_1^2 h_1}{2l_1^2} - \frac{\phi_2^2 h_2^2}{2l_2^2} \right) \quad (5)$$

where $2(h_1 + h_2)$ is the length of spectrum line, and $2h_1$ the effective height of grating.

Prins¹¹ has criticized the writer's published results⁶ believing that a correction of the type shown in Eq. (2) would account for the difference in wave-length observed between the grating and crystal methods. Correspondence with Prins has shown that he had misinterpreted the writer's disposition of apparatus. This correction, as will be shown later, is of negligible importance.

There are three types of errors which are usually present in the ruling of a grating, either of which would lead to erroneous absolute wave-lengths. These errors, the erratic, the error of run, and the periodic error have been discussed by Michelson and Rowland in connection with the absolute wave-lengths of optical spectra. The erratic error is not susceptible to analytical correction but its importance can best be determined by using gratings ruled on different ruling engines and comparing the results.

The error of run can be determined in a number of ways. Probably one of the most precise methods of determining this error is the focusing property exhibited by such gratings. Assuming parallel incident light, Fagerberg¹² has derived an equation of the form

$$\frac{d_1}{d_2} = 1 + \frac{x \cos^2 \psi_2}{r(\sin \psi_1 - \sin \psi_2)} \quad (6)$$

where d_1 and d_2 are the grating spaces at the extremes of the grating, ψ_1 is the angle of incidence, ψ_2 the angle of diffraction, x the length of the grating, and r

¹¹ J. A. Prins, *Nature* **124**, 370 (1929).

¹² Sven Fagerberg, *Zeits. f. Physik* **62**, 457 (1930).

the distance from the grating to the focus. With this method it is possible to detect an error in the grating space of 10^{-8} cm. Another method which has been used by the writer in connection with the above is to rule every 4th or 7th line on the grating longer than the other lines. The distance between these lines can then be measured directly by a comparator. Also the average grating space can be accurately measured by determining the number of spaces on a grating and the distance between the extreme lines.

The periodic error of the grating will produce an asymmetrical spectrum line which always makes the angle of diffraction appear larger than it should be. This effect is of no importance in ordinary optical gratings for the error is inversely proportional to the length of the grating used. In the x-ray application, however, the length of the grating used is only a few mm so the error may be important. Fagerberg has calculated the error for x-rays and obtained

$$\delta\lambda = \frac{2m}{x} \lambda \quad (7)$$

where m is the maximum displacement of any line on the grating from an ideal grating with the average grating space m_0 , x is the length of the grating used, and λ the wave-length. He assumed m to be from 0.0005 to 0.001 mm, $x = 1$ mm and found $\delta\lambda = 0.001$ to 0.002λ which is the order of magnitude of the observed difference between grating measurements and crystal measurements. It should be pointed out, however, that the values of m assumed were, for good quality gratings, too high. m can be determined from the intensity of the Rowland ghost lines in a grating. The relation between the intensity of the ghost lines and the intensity of the main line may be written approximately

$$\frac{I}{I_0} = \left(\pi N \frac{m}{m_0} \right)^2 \quad (8)$$

where I is the intensity of the first order ghost, I_0 the intensity of the main line, N the spectral order, m the maximum displacement of a line from the ideal grating, m_0 the average grating space.

It is true that for gratings of large grating space (e.g. 50 lines/mm) one might be able to have $m = 0.001$ mm and the intensity of the ghost lines be very weak in the low orders. However if one examines the very high spectral orders the ghost lines would be very intense. One can usually observe 30 or more orders with a grating of 50 lines per mm. If we observe the 30th order of such a grating and have $m = 0.001$, the intensity of the ghost lines would be more than 20 times the intensity of the main line. In any case one can determine the importance of this error by carefully examining the intensity of the ghost lines of the grating. If one wished to get an accurate estimate of m for a coarse ruled grating, I believe it would be permissible to rule a small spaced grating (e.g. 600 lines/mm) and determine m from such a grating; thence rule the coarse grating using the same part of the ruling engine screw. There seems little reason to believe that the ruling engine would misplace the lines on a coarse grating any more than on a finely spaced grating.

Compton¹³ has considered the question of the refraction of the x-rays at the surface of the grating, and concludes that refraction could not cause any displacement of the spectral lines. It has been shown by Siegbahn,¹⁴ and has also been observed by the writer, that the type of surface influences greatly the intensity of the x-ray spectra. It is difficult to believe, however, that this could in any way displace the spectral lines. The writer has also been able to improve the intensity of the x-ray spectra many times on a lightly ruled grating by etching it. In order to test the effect of etching on the position of the spectral lines, one half of a grating was etched and the other half left unetched. The grating was mounted in such a manner that either etched or unetched part could be used. The results were the same within experimental error from both parts.

Thus, it seems from the above considerations that the measurement of x-ray wave-lengths by the use of ruled gratings can be made reliable. By a suitable disposition of the apparatus the error in the wave-length should be just the error of measuring the grating constant d and the two angles θ and α .

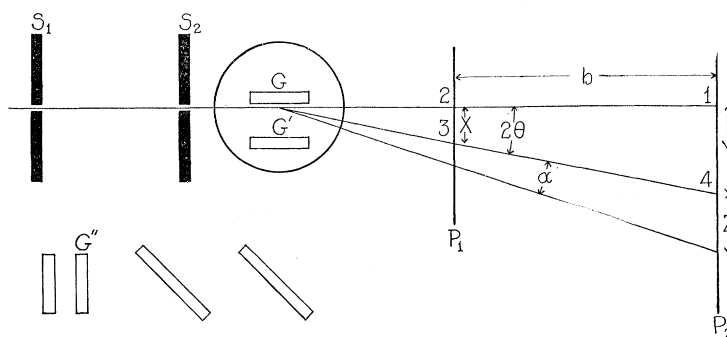


Fig. 1. Diagram of apparatus.

METHOD OF EXPERIMENT

In the present experiment the K -series of copper and chromium were chosen as a source of K -rays. The intensities of these lines can be made very great and they are the longest wave-lengths that can be worked with successfully without using a vacuum spectrograph. These wave-lengths can also be used to determine directly the grating space of such crystals as calcite and rock salt. Even though the angles of diffraction and reflection for these lines are small, it is believed that greater precision can be secured by using these wave-lengths than could be obtained by use of longer wave-lengths with a vacuum spectrograph.

There are several methods by which the angles θ and α could be measured. However, from a consideration of the error involved in each method it appears that the modification of the Uhler and Cooksey¹⁵ parallel plate method,

¹³ A. H. Compton, Jour. of the Franklin Inst. Oct. (1929).

¹⁴ M. Siegbahn and T. Magnusson, Zeits. f. Physik **62**, 435 (1930).

¹⁵ H. S. Uhler and C. D. Cooksey, Phys. Rev. **10**, 645 (1917).

used by the writer,⁶ is preferable. Fig. 1 shows the arrangement of the apparatus. This method avoids error due to any variation in the width of the slits, or non-uniformity of illumination over the slit. It will also be seen that it is not necessary that the grating be accurately on the axis of rotation as is essential in other methods. The measurements are all linear measurements and no angular motion of the grating is needed for a determination of the angles.

In order to determine the errors to be expected from this arrangement Eq. (1) may be written in terms of the measured quantities as

$$n\lambda = \frac{d}{2b^2} \left[\frac{(Y - Z)^2}{y^2} \right] (y \pm z)z \tag{9}$$

where *b*, *x*, *y*, and *z*, are as indicated in Fig. 1. In Eq. (9) the angles were used instead of the sine of the angle, and the angle was put equal to the tangent of the angle. This equation is in error by approximately 1 part in 3000 for the angles involved. It can be shown that for a given distance from the grating to the second plate that the minimum error in wave-length for a given error in measuring *x* occurs when *b* is made a maximum. This neglects the geometrical divergence of the x-ray beam and from Eq. (3) it will be seen that this must also be considered in choosing *b*. The distances *y* and *z* must be made large enough to be conveniently measured. A good comparator cannot be depended upon to better than about 0.001 mm. Thus, in order to obtain the desired precision *y* and *z* must be greater than 10 mm. This makes the distance from the grating to the second plate about 2 meters. An error in measuring the distance *x* produces an error in the wave-length of about 3 times as much as the same error in *y* or *z*. Fortunately the lines determining *x* are very narrow and are never more than 1 mm apart so *x* can be measured to a much higher precision than *y* or *z*. By assuming an error of 0.001 mm in *x*, *y* or *z* one is lead to expect an error in λ of approximately 1 part in 10,000.

In the present experiment the distance *l*₁ from the first slit to the grating was 45 cm, from the grating to the second plate 191 cm, and from the grating to the first plate 7 cm. A typical plate gives $\phi_1 = 3 \times 10^{-3}$, $\phi_2 = 7 \times 10^{-3}$ for the first order, and $\phi_2 = 16 \times 10^{-3}$ for the sixth order. Thus Eq. (2) becomes

$$n\lambda = d(\cos \phi_1 - \cos \phi_2)(1 - 1.93 \times 10^{-6}) \quad \text{1st order}$$

$$n\lambda = d(\cos \phi_1 - \cos \phi_2)(1 + 0.054 \times 10^{-6}) \quad \text{6th order}$$

Eq. (3) becomes

$$\phi_2 = \phi_1(1.0000087)$$

for the 2nd plate, and

$$\phi_2 = \phi_1(0.99964)$$

for the first plate.

On the first plate this corresponds to 0.0002 mm. From Eq. (4) $\beta = +4.15 \times 10^{-8}$ or 0.00008 mm on the 2nd plate, and Eq. (5) gives for the 6th order

$\delta\lambda = -1.64 \times 10^{-13}$ cm. Thus there are no corrections of any importance. In any case some are + and some are -, so the total correction is probably much less than any individual correction.

APPARATUS

The apparatus was mounted on a reinforced concrete block 6.5" \times 30" \times 96". The top of the block was a smooth stone. Iron blocks 1" \times 4" \times 4" were inserted about 3" from the top and at various positions in the block when it was cast. Threaded holes in the center of the iron blocks permitted iron rods to be rigidly attached to the base. The apparatus was held in position by these rods. This base was supported by 3 concrete blocks under each end. Between the blocks, rubber, felt and wood strips were inserted to damp out the vibrations from the building.

The plate holders were made from a solid brass plate 1/2" \times 4" \times 6". The surface against which the photographic plates rested was scraped to a plane surface. The photographic plates were held against this surface by a plane brass plate against which pressed two strong springs. A piece of blotting paper was inserted between this brass plate and the back of the photographic plate to equalize the pressure on the photographic plate. The plate holders were attached to a 2" square iron rod which was held rigidly to the base by two 1.25" iron rods. The photographic plates were inserted and removed without disturbing the adjustment of the plate holders.

The grating was mounted on a three point support which was attached to a slide capable of being moved through several mm. The position of the slide was determined by a scale and dial, reading to 0.001 mm. This slide rested on a table which rotated on accurately made centers. The table could only be rotated through 200°, but this was sufficient for making adjustments. It was found that such a table was far more satisfactory in rotating about an axis than the ordinary spectrometer.

The slits were made of steel with gold faces 1 mm thick. One jaw was clamped rigidly and the other jaw held against it by small springs. Thin spacers were used to obtain the desired slit widths. The two slits were attached to a brass tube which was capable of being rotated by a long arm and a tangent screw. Thus after the slits had once been made accurately parallel they could be clamped in position, and any further adjustment of tilt could easily be made by the tangent screw.

The x-ray tube was a water-cooled Coolidge type, with a side arm and a thin window of aluminum (0.01 mm thick) to reduce the absorption of the copper and chromium x-rays. The focal spot was usually about 1 to 2 mm in diameter and the tube was operated about 10 m.a. and 40 K.V. The x-rays were taken off at about 10° from the face of the target. The chromium targets were prepared by electroplating a layer of chromium on the regular copper targets. The x-ray tube was mounted on a slide in order that the source of greatest x-ray intensity could be placed in line with the slits. The tube could also be raised and lowered to bring the focal spot in line with the center of the slits, grating and plates. Evacuated tubes with thin celluloid windows were

placed between the slits and between the plate holders to reduce the absorption of the x-rays.

The experiment was carried out in a small room in which the temperature could be accurately controlled. A recording mechanism kept a continuous record of the temperature and the temperature never varied by more than 0.1°C . During a single exposure the temperature usually remained constant to within 0.02°C .

ALIGNMENT AND CALIBRATION OF APPARATUS

The alignment of the plate holders, center of grating, and center of slits in a horizontal line was accomplished by adjusting each to a given height above the top of the stone base. The height of the focal spot of the x-ray tube was adjusted by placing a horizontal slit on this line midway between the tube and second plate, thence adjusting the height of the tube until the image of the focal spot fell on the center of the second plate P_2 . This adjustment was made to within 0.5 mm.

The adjustment of the grating surface parallel to, and on the axis of rotation was very conveniently and accurately made by using the modified Michelson interferometer shown in Fig. 1. The grating which was ruled on a plane parallel plate was spluttered with gold or silver to obtain a good optical reflection. The grating was mounted with the reflecting surface against a rigid three point support on the slide of the rotating table.

If one places plane parallel plates in positions G' and G'' (Fig.1) of the same thickness as the plate on which the grating G is ruled, it will be seen that the optical path is the same whether the rotating table is in the position shown, or after it has been rotated through 180° . The grating slide and the slide which supported the second interferometer mirror were adjusted until the central white light fringe remained in the same position when the rotating table was turned through 180° . Thus the grating surface was accurately set on the axis of rotation to within 0.1 of a fringe. The grating surface was made parallel to the axis of rotation by observing the number of fringes in the field of the two 180° positions. Care was taken to make sure that the same number of fringes represented zero angle between the grating surface and the axis, and not an angle of twice as many fringes. This angle was made 0° to within 1 fringe/20 mm which is about $2''$ of arc. It will be noticed that after the adjustments are once made the grating can be taken out and replaced very easily by the use of the interferometer. It was not necessary that the grating surface be spluttered, as the fringes were always visible even with the cleanest glass surface. The interferometer was also used to check the position of the grating during an exposure.

The axis of rotation was then made approximately perpendicular to the horizontal line, which passed through the center of the slits and the center of the plate holders. This was accomplished by allowing a beam of light to pass through a slit in the center of the second plate holder P_2 and thence reflected by the grating surface back on to the slit.

The slits were then moved horizontally until the x-ray beam passed over

the axis of rotation. In order to do this, the slit S_2 nearest the grating (about 30 mm away) was made very narrow (approximately 0.003 mm). The grating was adjusted parallel to the x-ray beam by x-ray reflection, and the slits moved until the narrow beam was partially cut off.

The grating was then rotated through 90° . The grating surface was then perpendicular to the direction of the x-ray beam. A telescope fitted with a Gauss eye piece was placed about 5 meters away and adjusted until the image of the cross-hairs was reflected back by the grating on to the cross hairs. A plane parallel mirror was inserted in place of the photographic plate, and the plate holder P_1 adjusted until the image of the cross hairs was returned as above. The mirror was then placed in the second plate holder P_2 which was likewise adjusted. Thus the two plate holders were made parallel and also perpendicular to the x-ray beam to within $5''$ of arc.

The grating was rotated to a position parallel to the x-ray beam and moved completely out of the path of the x-rays. The slit S_2 was made 0.01 mm wide and the slit S_1 near the x-ray tube was made 0.015 mm. A photographic plate was then placed on a slide between the grating and the first plate holder P_1 . A short exposure of about 10 seconds was sufficient to record a sharp line on the plate. The plate was then moved about 1 mm and the grating was moved 0.001 mm into the x-ray beam. Another exposure of 10 seconds was made. This was repeated until the grating cut off the entire x-ray beam. If the slit S_2 was not parallel to the grating, the lines on the developed plate were not cut off uniformly by the grating at the top and bottom. In this way the slit S_2 could be made parallel to the grating to within $10''$ of arc. The parallelism of the slit S_1 was tested in the following manner. A photographic plate was placed in the plate holder P_1 . The direct x-ray beam was recorded. The grating was rotated through an angle of about $5'$ of arc and then put in position to reflect the x-ray beam. The reflected beam was thus recorded. If the slits were parallel to each other and parallel to the axis of rotation of the grating, the two lines on the developed plate would be parallel. Thus by accurately measuring the separation of the lines at the top and bottom of the plate, the perfection of the adjustment was determined. If the lines were not parallel, S_1 was readjusted, and the above process repeated. In this way the slits were made parallel to each other and parallel to the axis of rotation of the grating within $10''$ of arc.

The lines on the grating were aligned parallel to the axis of rotation of the grating in the following manner. A vertical slit about 50 cm from the grating was illuminated by a mercury arc. A double cross hair in the viewing telescope was rotated until the spectral lines formed by the grating were parallel to the cross hairs. The grating was rotated 180° and the spectral lines observed again. When the lines of the grating were parallel to the axis of rotation, the spectral lines formed in this position were parallel to the cross hairs. Since the grating space varies as the cosine of the angle of tilt this adjustment was only made to within 0.5° .

The distance between the plate holders was measured by placing an iron rod $3/4''$ square between the two plate holders, and then measuring with

inside micrometers the distance between the parallel ends of the iron rod and the plate holders. The length of the rod was determined by comparison with the laboratory standard, which had been calibrated by the Bureau of Standards, Washington, D. C. Independent measurements by different observers agreed to within 0.02 mm, so that the error in the final value was probably less than this. The comparator on which the separations of the lines on the plates were measured was carefully adjusted and calibrated. Six gratings have been used in the present work. Four of these gratings (1-4) were ruled by Professor Wood in this laboratory, and the other two were ruled by Mr. Pearson under the direction of Professor Michelson of the University of Chicago.* All the gratings were ruled on glass. Larger angles could have been used by using a heavier material for the ruled surface, but glass gratings give sharper and more intense spectra than the metallic surfaces. Also, the glass gratings can be spluttered with gold and used at large angles. In this case the intensity of the spectra is greatly increased as compared to the unspluttered glass surface and the sharpness of the spectral lines remains unchanged. The characteristics of the gratings are as follows.

Number 1 was ruled with 287 lines per mm on a plane optical glass surface with a very light ruling. The ruling was uniform and no lines were missing. The surface ruled was approximately 25 mm square. In order to examine the grating with optical light the top and bottom of the grating was etched, and then spluttered with gold. The intensity of the Rowland ghost lines in the 11th order was about 1/3 the intensity of the main line, thus m [from Eq. (8)] would be 0.00006 mm, and for $x = 4$ mm, Eq. (7) gives $\delta\lambda = 0.00003\lambda$. It was found that the intensities of the first and second orders of the x-ray spectra, from the etched portion of the grating, were increased 5 or 10 times as compared to the unetched part. The higher orders were not improved. Spluttering the grating with gold improved the intensity of the unetched portion but did not improve the etched part. The error of run was estimated by the focusing effect in various orders and was less than $10^{-5}d$.

Number 2 was ruled with 143 lines per mm on a plane parallel plate made from good quality plate glass. The ruling was heavier than on number 1 and it was not necessary to etch part of the grating for observation of the optical spectra. The ghost lines were weaker in this grating in the 22nd order than in the 11th order of number 1. Etching did not improve the intensity of the x-ray spectra even though the intensity of the optical spectra was increased several times. Spluttering with gold increased the x-ray spectra from the whole grating by a factor of three or four. The error of run was estimated as above and found to be of no importance. This grating had some missing lines near the middle of the ruling, so it was only used for testing the effect of etching and no results are listed from it.

* The results from these two gratings were obtained at the University of Chicago. The apparatus was essentially the same as described above. Through the courtesy of Professor Michelson the experiment was carried out in one of the constant temperature ruling engine vaults. The writer also wishes to express his appreciation to the members of the Physics Department for placing at his disposal the facilities for carrying out this part of the experiment.

Number 3 was ruled with 287 lines/mm on a plane parallel plate similar to number 2. This grating was not etched and was very similar to number 1. On this grating every 7th line was made longer than the others so that direct measurement of the distance between these 7th lines could be made. Also the number of lines on the whole grating could be easily determined, and by measuring accurately the distance between the extreme lines the grating space d was obtained. The error of run was less than $10^{-5}d$, and the intensity of the ghost lines was about the same as in grating 1.

Number 4 was a duplication of number 2 except the 4th lines were ruled longer as in number 3. This grating was not etched and possessed characteristics very similar to number 2 and 3.

Number 5 was a glass grating ruled with 50 lines per mm on the ruling engines at the University of Chicago. This grating seemed to be very free of ghost lines, as no ghost lines could be observed even in the very high orders. The first 1/4 of the ruled surface, however, possessed a very bad error of run, probably due to not allowing the ruling engine to run a sufficient length of time before the ruling was started. The grating space of this grating was obtained by carefully determining the pitch of the ruling engine screw. This grating was not carefully examined by the writer until after the x-ray spectra had been taken. The erratic results obtained were completely explained by this error of run in the grating. By eliminating all exposures, which were taken on the bad part of the grating the results were consistent with the results from the other gratings.

Number 6 was a grating with 600 lines per mm, ruled on the same engine as number 5. One part of this grating, which was lightly ruled and gave faint optical spectra, gave x-ray spectra about twice as intense as the more heavily ruled portion. The resulting wave-lengths were the same, within experimental error, from both parts of the grating.

Two types of photographic plates were used. The results with gratings 5 and 6 were obtained with selected plate glass plates coated with Eastman x-ray emulsion. These plates were developed, washed, and dried in the ordinary manner. It has been stated¹⁰ that certain precautions used in the manufacture of commercial plates are omitted in preparing special plates. Thus the plates used in the experiments with gratings 1-4 were the regular commercial x-ray plates. About one half of the plates were "treated" as described by D. Cooksey and C. D. Cooksey,¹⁶ and the remaining plates were not previously treated, but were dried in alcohol and allowed to attain equilibrium in a humid atmosphere. No difference in the consistency of the results was noted between the last two methods. A considerable difference was noted however between these latter methods and the regular developing methods used with the special plates. A test was also made using a standard which was made by silvering a piece of glass the size of the plates used, then making four diamond scratches at equal intervals on the silvered surface. This surface was put in contact with the emulsion and a flash of light from above the plate recorded four very

¹⁶ D. Cooksey and C. D. Cooksey, Phys. Rev. **36**, 80 (1930).

fine lines on the photographic plate. The plate was then developed, and by comparing the distance between these lines with those on the standard, a test could be made for contraction or expansion of the plate due to the different methods of developing. The results obtained were essentially in agreement with those of D. Cooksey and C. D. Cooksey¹⁶ except the variations were not quite as great.

METHOD OF TAKING EXPOSURES

In exposing a series of plates, the grating was first withdrawn from the path of the x-ray beam and a short exposure given to record the direct beam on the second plate P_2 at the position 1 in Fig. 1. A plate was put in the first plate holder P_1 , and the direct beam recorded at 2. The grating was next moved into position for reflection of the x-ray beam, and after allowing 15 to 30 minutes for the slide to attain an equilibrium position, the reflected beam was registered at 3. This plate was then removed and a short exposure given to record the reflected beam at 4. The plate P_2 was then covered with a lead screen almost to the position where the first order would appear, and a long exposure (6 to 24 hours) given to record the various orders of diffraction. At the end of the long exposure another set of plates was taken to make sure that nothing had moved during the exposure. A new plate was put in P_2 , and the reflected beam recorded at 4. Likewise a plate was put in P_1 and the reflected beam recorded at 3. The grating was then removed from the path of the x-ray beam and the two positions, 2 and 1 recorded. Since the wave-length depends so much on the position of the reflected beam, another method of taking the exposures was used on about 40 sets of plates. Exposures 1, 2, 3 and 4 were taken as above. A new plate (or the same plate with ends reversed) was placed in P_2 . The position of the reflected beam was covered with a screen. After about half the long exposure had been given the screen was removed for a short time to record the reflected beam. At the end of the exposure the grating was removed from the path of the x-rays and the direct beam recorded. Thus if there was any slow change in the position of the apparatus, the reflected beam was recorded in the mean position. The results were in good agreement with the above method. At first it was difficult to get the initial and final exposures to agree as closely as desired. In the latter exposures, however, when the temperature was constant and the vibration from the building damped out, the plates checked as accurately as comparator measurements could be made, which was to about 1 part in 10,000.

The separations of the lines on the plates were measured with an accurately calibrated comparator. In making the measurements 5 or 10 settings were made on each line on the plate. The plate was then reversed and a similar set of measurements made. If a difference of more than 0.004 mm existed between the distances as measured in the two positions, the plate was remeasured. Several sets of plates were arbitrarily remeasured after several weeks, but no appreciable differences were detected. The wave-lengths were calculated from Eq. (1) using Andoyer's 15 place trigonometric tables.

RESULTS

Typical results are shown in Fig. 2. It will be seen that the two α lines are not resolved on any of the plates, but this is not to be expected, as a calculation shows that the separation would only be a few hundredths of a mm. In order to compare these lines with the crystal measurements, one must take a weighted mean of the α_1 and α_2 lines as given by the crystal measurements. If we assume a form of the lines of the type $y = e^{-x^2}$ it can be shown that the maximum intensity of the unresolved doublet should occur very near the "center of gravity" of the two lines. Siegbahn's value of the copper $K\alpha$ line

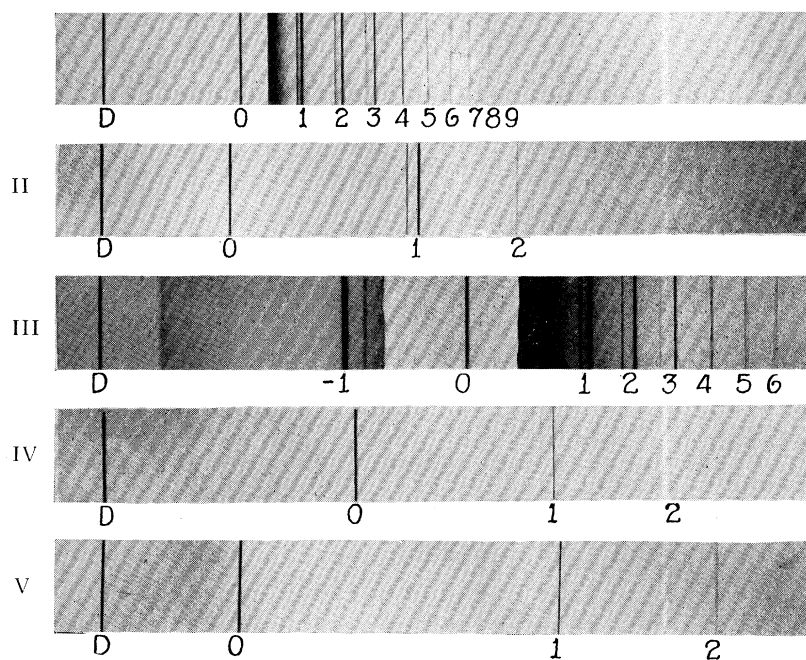


Fig. 2. I copper target, grating 50 lines per mm. II copper target, grating 287 lines per mm. III copper target, gold sputtered grating 143 lines per mm. IV chromium target, grating 143 lines per mm. V copper target, grating 600 lines per mm.

thus weighted is 1.5386Å, and for the chromium $K\alpha$ line is 2.2862Å. The measurements on the β lines do not involve this difficulty of weighing as the separation of the doublet is so small that it is of no significance in the present work. These crystal values must be corrected for the index of refraction of the x-rays in the crystal. The corrected crystal wave-lengths which are compared with the grating measurements are shown in Table IV.

The average results obtained from each grating are given in Table I. The column headed "gratings" gives the number of the grating as listed in the section on "alignment and calibration of apparatus." The prime numbers are the same as the unprimed numbers except the grating was spluttered with gold. In the experiment 172 sets of plates were taken under various condi-

TABLE I.

Grating	Cu	Cu	Cr	Cr
1 and 3	1.39210	1.54150	2.08463	2.29088
4'	1.39242	1.54193	2.08492	2.29109
4	1.39215	1.54183	2.08486	2.29111
5	1.39217	1.54158	2.08455	2.29075
5'	1.39235	1.54183	2.08499	2.29111
6	1.39207	1.54173	2.08465	2.29077
Weighted average	1.39225	1.54172	2.08478	2.29097

tions, such as different angles of reflection θ , different slit widths and length of exposure. The measurements from 107 plates have been retained and the other discarded because the initial and final exposures failed to check or on account of other experimental difficulties.

In order to show the type of variation obtained with each grating Table II gives the copper series results for grating 4'. The lowest value recorded for any wave-length was +0.13 percent and the highest +0.31 percent greater than the corresponding crystal values. In obtaining the average for each grating each order was given equal weight. This method was used because in the low orders the lines were intense and easily measured, while in the higher orders the lines were more difficult to measure but the errors were of less importance.

TABLE II.

Plate	Distance y	Cu $K\beta$	Cu $K\alpha$	Plate	Distance y	Cu $K\beta$	Cu $K\alpha$
127	10.357 mm	1.3921A	1.5415A	132	10.032 mm	1.3929A	1.5426A
		1.3922	1.5422			1.3924	1.5423
		1.3923	1.5424			1.3926	1.5422
			1.5423				1.5415
			1.5424				1.5419
128	10.068	1.3921	1.5415	133	26.808	1.3924	1.5425
		1.3924	1.5420			1.3929	1.5418
		1.3922	1.5422			1.3926	1.5420
			1.5421			1.3928	1.5419
			1.5419				1.5423
129	10.069	1.3926	1.5422	134	26.805	1.3928	1.5415
		1.3924	1.5425			1.3921	1.5421
		1.3926	1.5421			1.3924	1.5418
		1.3925	1.5425				1.5421
			1.5425				1.5419
130	10.066	1.3925	1.5422	135	26.807	1.3921	1.5416
		1.3925	1.5414			1.3922	1.5418
		1.3926	1.5417			1.3924	1.5413
			1.5417			1.3919	1.5414
			1.5420				1.5417
131	10.029	1.3925	1.5421				1.5411
		1.3922	1.5417				
		1.3924	1.5419				
			1.5420				
			1.5417				
		Average				1.39242	1.54193

In any case it was found that almost any consistent method of weighing gave practically the same result. The probable error has not been given for each set

of measurements as in almost every case it was much less than the probable consistent errors in the experiment. It is believed that the differences between the results for various gratings form the best method of determining the probable error in the experiment. This may be seen more easily from Table III

TABLE III.

Grating	Cu $K\beta$	Cu $K\alpha$	Cr $K\beta$	Cr $K\alpha$
1 and 3	0.213 (26)	0.203 (46)	0.215 (16)	0.218 (28)
4'	0.236 (30)	0.231 (49)	0.229 (3)	0.227 (5)
4	0.217 (4)	0.224 (11)	0.226 (15)	0.228 (27)
5	0.218 (41)	0.208 (73)	0.211 (32)	0.212 (51)
5'	0.232 (49)	0.224 (82)	0.232 (44)	0.228 (67)
6	0.211 (11)	0.218 (16)	0.216 (3)	0.213 (4)
Weighted Average	0.224 (161)	0.217 (277)	0.222 (113)	0.222 (182)

where the percent difference is given between wave-lengths as measured by grating and crystal methods. The number in parenthesis is the number of orders of each wave-length which were measured for each grating. Thus line 5', means that grating number 5 was spluttered with gold, 49 orders of the copper $K\beta$ line and 82 orders of the copper $K\alpha$ line were measured, and with the chromium target 44 orders of the $K\beta$ line and 67 orders of the $K\alpha$ line were measured. The weighted average was obtained by multiplying each value by the number of measurements (in parenthesis) and dividing the sum by the total number of measurements.

The final results are given in Table IV. The crystal wave-lengths are those given by Siegbahn¹⁷ which have been corrected for the index of refraction of the x-rays in the crystal. The limiting error has been estimated by the following method.

TABLE IV.

Spectral Line	Crystal λ	Grating λ	Limiting Error	Grating λ - Crystal λ
Cu $K\alpha$	1.38914A	1.39225A	± 0.00014	+0.224%
Cu $K\alpha$	1.53838	1.54172	± 0.00015	+0.217%
Cr $K\beta$	2.08017	2.08478	± 0.00021	+0.222%
Cr $K\alpha$	2.28590	2.29097	± 0.00023	+0.222%

In addition to the probable error calculated from the variation of the results, there are three possible sources of consistent error that have to be taken into consideration. They are: the measurement of the grating constant, the distance between the plate holders, and the precision of the comparator on which the plates were measured.

The grating spaces of gratings 1 to 4 were measured directly as described above. This method essentially determines the pitch of the ruling engine screw. The pitch of the screw as determined from these gratings differed by less than 1 part in 20,000. The grating space of gratings 5 and 6 was deter-

¹⁷ M. Siegbahn, "The Spectroscopy of x-rays" (1925).

mined by measuring directly the pitch of the ruling engine screw. It appears to the writer that the maximum error in the grating space of the gratings should not exceed 0.005 percent.

The distance between the plate holders (184.280 cm) was measured by two observers and the results agreed to within 0.002 cm. The iron bar was compared with the laboratory standard and also measured on the comparator on which the plates were measured. Thus the maximum error in the distance between the two plate holders must have been less than 0.004 percent.

In addition to the calibration of the comparator as already described, it was further checked for erratic or periodic error by measuring the distance between the lines on a coarsely ruled grating. From these results it is concluded that the absolute error in the comparator was not greater than 0.001 mm over the range used. Since the distances on the plates varied from 4 mm to 50 mm, the average error must have been less than 0.008 percent.

It will be seen from Table III that the differences between the results from different gratings and different wave-lengths are of negligible importance as compared to the possible consistent errors. Thus the limiting error has been taken as 0.01 percent.

TABLE V.

Investigator	Spectra line	λ	Grating λ - Crystal λ	Probable error	Date
Compton-Doan	Mo $K\alpha$	0.7078A	-0.1%	$\pm 0.4\%$	1925
Thibaud	Cu $K\alpha$	1.540	+0.1	$\pm 1.$	1926
Hunt	Pt $M\alpha$	6.1	+1.6		1927
	Al $K\alpha$	8.5	+2.0		
Bäcklin	Mo $L\alpha$	5.402	+0.11	± 0.2	1928
	Mo $L\beta$	5.174	+0.14	± 0.2	
	Al $K\alpha$	8.333	+0.12	± 0.1	
	Mg $K\alpha$	9.883	+0.09	± 0.2	
	Fe $L\alpha$	17.61	+0.17	± 0.2	
Wadlund	Mo $K\alpha_1$.708	+0.1	± 0.1	1928
	Cu $K\alpha_1$	1.537	0.	± 0.08	
	Fe $K\alpha_1$	1.938	+0.3	± 0.2	
Howe	Cu $L\alpha$	13.37	+0.6	± 0.3	1929
	Fe $L\alpha$	17.66	+0.5	± 0.3	
Bearden*	Cu $K\alpha$	1.5418	+0.22	± 0.02	1929
	Cu $K\beta$	1.3922	+0.22	± 0.02	
Cork	Mo $L\alpha_1$	5.4116	+0.29		1930
	Mo $L\beta_1$	5.1832	+0.30		
Bearden	Cu $K\alpha$	1.54172	+0.217	± 0.01	1931
	Cu $K\beta$	1.39225	+0.224	± 0.01	
	Cr $K\alpha$	2.29097	+0.222	± 0.01	
	Cr $K\beta$	2.08478	+0.222	± 0.01	

* These results are not the same as those published but they have been corrected for the difference between Eq. (9) which was used for calculating the wave-lengths and the correct Eq. (1). Also the index of refraction of the x-rays in the crystal was neglected in this preliminary report.

DISCUSSION

It is interesting to compare the absolute measurements of the x-ray wavelengths obtained by various investigators using ruled gratings. Table V gives a summary of these results, where the probable errors are approximately those given by the authors. It will be noticed that all the results are high except the pioneering results of Compton and Doan and the copper $K\alpha_1$ result of Wadlund.

The copper $K\alpha_1$ line was recorded by Wadlund on only three plates, and on the plate showing the most measurable orders a difference of 0.75 percent in wave-length was observed in different orders. Measurements made by the writer using the same grating gave results about 0.2 percent higher than the crystal values. The results obtained by Backlin and Cork on the molybdenum $L\alpha$ and $L\beta$ lines differ by almost 0.2 percent. Howe and Backlin differ by more than 0.3 percent on the iron $L\alpha$ line. These differences observed by the various investigators are rather difficult to understand. However, it has been observed by the writer that an error of 0.2 percent can easily be made unless all adjustments are precisely made. It is very essential that the angle θ be checked at the beginning and end of an exposure as this forms a satisfactory criterion for accepting or rejecting a given set of plates. The satisfactory agreement between the present results for different gratings and also those obtained by the writer in 1929 seem to support this criterion.

In Table III it will be noted that the results with the gold spluttered gratings 4' and 5' are higher than the results obtained with the unspluttered gratings. The difference appears to be greater than the experimental error but probably is not of sufficient magnitude to indicate a fundamental difficulty with the grating method. The spluttered layers were very thin (absorbed about 95 percent of optical light) and it is conceivable that there might have been some interaction between the top of the gold surface and the glass surface. However, the difference was so small that it was not thought worth while to plan a detailed experiment to test the question.

The great difference between the wave-lengths thus measured and those obtained from crystal measurements indicates some fundamental difficulty in the two methods. As was seen in the earlier part of this paper no suggestion has been supported which would influence the grating results. It is known, however, that crystals such as rocksalt are much less perfect than crystals of calcite, but even calcite cannot be considered a perfect crystal. In order to account for the mechanical properties of crystals, Zwicky¹⁸ has developed a theory which strongly suggests that even in crystals of calcite there might be a concentration of atoms at rather regularly spaced intervals. This would make the average density of the crystal greater than the density of the small elements of the crystal which are effective in diffracting x-rays. Such an imperfection in the crystal would account for the observed difference in wave-length as measured by ruled gratings and crystals. In any case it appears that the difficulty must be either in the crystal or in the constants used to calculate the grating space of the crystal from chemical data.

¹⁸ F. Zwicky, *Helvetica Physica Acta* **3**, 269 (1930).

DETERMINATION OF THE FUNDAMENTAL CONSTANTS

The absolute wave-length measurements obtained above allow us to determine directly the fundamental grating constant of crystal gratings. The complete Bragg law of diffraction may be written

$$n\lambda = 2d \sin \theta \left(1 - \frac{1 - \mu}{\sin^2 \theta} \right) \quad (10)$$

where n is the order of diffraction, λ the wave-length, d the grating space of the crystal, θ the angle of diffraction, and μ the index of refraction of the x-rays in the crystal. Sine θ and μ have been precisely measured by many investigators. It is thus possible to determine the distance d between the layers of atoms in the crystal almost as accurately as the wave-length λ of the x-rays is known. Siegbahn and Dolejsek¹⁹ have found for the copper $K\beta$ line $\sin \theta = 0.229334$. Larsson²⁰ has found $(1 - \mu)/\lambda^2 = 3.72 \times 10^{10}$ or $(1 - \mu)/\sin^2 \theta = 1.36 \times 10^{-4}$. Substituting these values and the wave-length of the copper $K\beta$ line from Table IV in Eq. (10) one finds

$$d = 3.0359 \pm 0.0003A$$

One of the most precise methods of measuring Planck's constant h is by determining the high frequency limit of the continuous x-ray spectrum. The quantum relation

$$Ve = h\nu$$

may be written in the form

$$h = Ve \left(\frac{2d \sin \theta}{c} \right)^2 \quad (11)$$

where v is the potential applied to the x-ray tube, d is the grating space of the crystal used and θ the minimum angle of diffraction. The experiment of Duane, Palmer and Yeh²¹ gives directly the value of $v \sin \theta$ as 2039.9 ± 0.9 volts. Using the value of d obtained from Eq. (10), $e = 4.77 \times 10^{-10}$ e.s.u. and $c = 2.99796 \times 10^{10}$ cm/sec we obtain

$$h = 6.573 \pm 0.007 \times 10^{-27} \text{ erg} \cdot \text{sec.}$$

All different theories of the dispersion of x-rays agree in the limiting case when the frequency of the radiation is much greater than the natural frequency of the electrons in the dispersing medium. The relation is

$$1 - \mu = \frac{ne^2}{2\pi m\nu^2} \quad (12)$$

where μ is the index of refraction, n the number of electrons per cm^3 in the

¹⁹ M. Siegbahn and V. Dolejsek, *Zeits. f. Physik* **10**, 160 (1922).

²⁰ Alex. Larsson, Inaugural Dissertation, Uppsala, 1929.

²¹ Duane, Plamer and Yeh, *Opt. Soc. Amer.* **5**, 376 (1921).

dispersing medium, e the charge on the electron, m the electronic mass, and ν the frequency of the radiation. This equation can be rewritten in the form

$$\frac{e}{m} = \frac{2\pi Mc^2}{F\rho Z} \times \frac{(1-\mu)}{\lambda^2} \quad (13)$$

where M is the molecular weight, F the Faraday constant, ρ the density, z the molecular number, λ the wave-length of the x-rays and c the velocity of light. Stauss²² has recently measured $(1-\mu)$ for the molybdenum $K\alpha_1$ and $K\beta$ lines using quartz as the refracting medium. His value of $(1-\mu)$ is

$$(1-\mu)_{K\alpha_1} \times 10^6 = 1.804 \pm 0.001.$$

If we increase the crystal value of the molybdenum wave-length by 0.221 percent, which seem justifiable from the agreement of the writer's measurements of the copper and chromium K -series, we obtain for the molybdenum line the wave-length

$$\text{Mo}K\alpha_1 = 0.7094\text{A}$$

Substituting this wave-length in Eq. (13) and the other constants from Birge's²³ tables one obtains

$$\left(\frac{e}{m}\right)_{K\alpha_1} = 1.765 \pm 0.001 \times 10^7 \text{ e.s.u.}$$

This value of e/m is very unsatisfactory because it does not agree with either the usual spectroscopic value or the deflection method value. However, it should be pointed out that Larsson's²⁰ value of $(1-\mu)$ is much higher than this value from Stauss's experiment. If one uses an average of the values obtained by Larsson and Stauss we have

$$\frac{e}{m} = 1.769 \times 10^7 \text{ e.s.u.}$$

Thus it appears that further experiments on the dispersion of x-rays will have to be made before confidence can be placed in the value of e/m obtained in this manner.

The above constants as determined from the grating measurements of x-ray wave-lengths are unaffected by the possible imperfections of the crystal. It may be interesting, however, to calculate other fundamental constants assuming that the crystal grating is perfect.

From fundamental considerations of crystal structure it can be shown that the grating space d of a rhombohedral crystal is given by

$$d = \left(\frac{nM}{N\rho\phi}\right)^{1/3} \quad (14)$$

²² H. E. Stauss, Phys. Rev. **36**, 1101 (1930).

²³ R. T. Birge, Phys. Rev. Sup. **1**, 1 (1929).

where n is the number of molecules in each elementary rhombohedron, M the molecular weight of the crystal, N is Avogadro's number, ρ the density of the crystal, and ϕ is the volume of a rhombohedron, the perpendicular distance between whose opposite faces is unity. It can also be shown that

$$\phi = \frac{(1 + \cos \beta)^2}{(1 + 2 \cos \beta) \sin \beta} \quad (15)$$

where β is the angle between the axis of the crystal. All the quantities in Eq. (14) can be precisely measured except N . Thus if the crystal is perfect, an independent determination of d should make possible a precise measurement of Avogadro's number N . Using the value of d found from Eq. (14) and the other constants from Birge's²⁴ tables one finds

$$N = 6.019 \pm 0.003 \times 10^{23} \text{ mol. per mole}$$

If Avogadro's number N can be accurately determined in this manner the charge on the electron e can be obtained from the relation

$$F = Ne \quad (16)$$

where F is the Faraday constant. Thus

$$e = 4.806 \pm 0.003 \times 10^{-10} \text{ e.s.u.}$$

Planck's constant h determined from Eq. (11) using $e = 4.806 \times 10^{-10}$ and the other constants the same as above, gives

$$h = 6.623 \pm 0.004 \times 10^{-27} \text{ erg}\cdot\text{sec.}$$

The values of N and e obtained from Eqs. (14) and (16), and the value of h using e from Eq. (16) appear to be entirely too high. The numerous ways in which these constants enter into theoretical calculations make the acceptance of these high values almost impossible. This may be interpreted as a support of the theory of mosaic structure of crystals. If the entire difference, observed in the x-ray wave-lengths as measured by the two methods, be attributed to the mosaic structure, then we now have a precise method for determining quantitatively the magnitude of this effect in any crystal. An independent experimental determination of the mosaic structure would of course allow us to calculate N and e as above, with higher precision than has been attained by other methods.

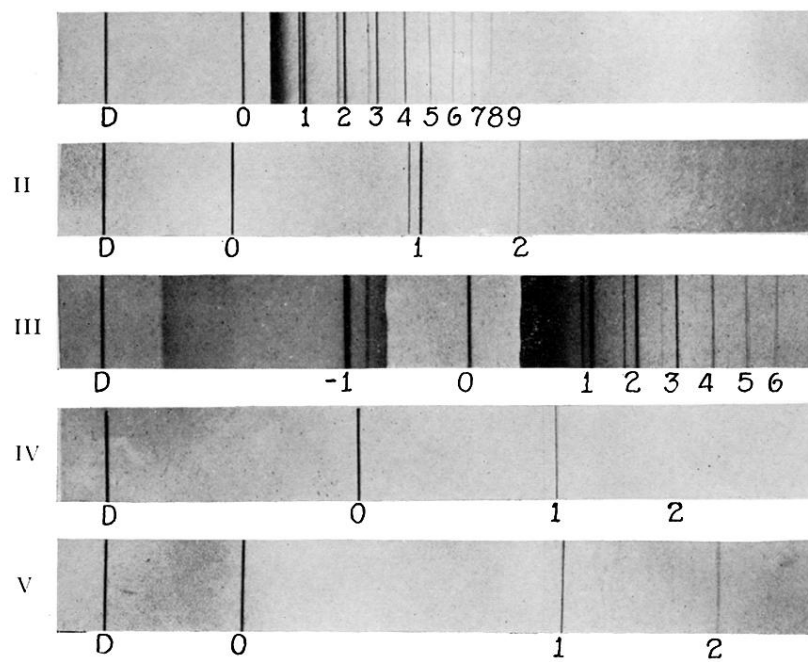


Fig. 2. I copper target, grating 50 lines per mm. II copper target, grating 287 lines per mm. III copper target, gold sputtered grating 143 lines per mm. IV chromium target, grating 143 lines per mm. V copper target, grating 600 lines per mm.