WAVE-PARTICLES AS TRANSMITTED POSSIBILITIES: QUANTUM POSTULATES DEDUCED FROM LOGICAL RELATIVITY

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Abstract

General relativity is logically unsatisfactory because it bases measurement on infinitesimals, and absolute infinitesimals do not exist. A standard of comparison being a logical necessity, we propose to provide it by the new fundamental hypothesis; "The physical world is composed of elementary events of identical and definite non-zero four-dimensional extent." The photon is a possibility of absorption-event initiated by an emission event; it is exhibited as a set of superposed moving volumes or three-dimensional sections of the four-dimensional possibility. The electron is a possibility transmitted with smaller velocities. Mass or energy is proportional to the time-duration of the event-possibilities, and obeys thence, the relativity rules. A wave function is postulated for the possibility being realized, and from Doppler's principle the quantum rule necessarily follows: mass proportional to frequency, and group-velocity equal to particle velocity. The "particle" exists only when the transmission is parallel the time-edge of the possibility; in atomic orbits presumably the condition does not apply, and the parallel-displacement tracks of relativity which are shown to to be required outside, do not apply within the atom. The finite extent of the events which exhibit the particle necessitates an indeterminacy in the situation of the particle.

§ 1. Logical Weakness of the General Theory of Relativity

In unified field theory, particularly the form given by Eddington, tracks of all particles, and straight lines, are defined by means of infinitesimals, or infinitesimal displacements. Thus at the basis of the theory rests a set of equations of the form[1,6]

$$dA_i = (ij,\ k)A_k dx_j \cdots i,\ j,\ k = 1,\ 2,\ 3,\ 4.$$

The displacement dx_j is supposed to be infinitesimal, and the corresponding increment dA_i must also be infinitesimal. It is however a mathematical fact that there are no absolute infinitesimals, and that the property of being small is only relative. In applying the relativity to experience, we have to fix arbitrarily what shall be small. Thus there is nothing in the relativity theory of geodesics to say that the orbits shall not represent electron tracks round atoms rather than planetary tracks, and nothing but a pragmatic test to give the verdict. This elasticity may be satisfactory from the experimenter's point of view, but the theoretician cannot be satisfied until the theory can tell itself whether its findings are to apply to microscopic or macroscopic phenomena. Relativity at present finds that phenomena on the minute can be only a replica of phenomena on the grand scale; there is in its very fundamental constitution no absolute infinitesimal, it is the merely relative infinitesimal

[1] Eddington, Mathematical Relativity, p. 213, Eq. 91.1.

of mathematics in which any contortion of curvature may exist and which is merely a class of finites.[2]

Herein is not only a logical defect of the theory, but also the reason why the theory fails to include atomic phenomena within its unifying field. Classical physics I suppose would have unanimously supported the view that phenomena on the minute were replica of those on the large scale; but with the advent of atomic physics this attitude has died, and the principle of indeterminacy has introduced an absolute standard of minuteness into physics that is not in pure mathematics.

Thus, to give it logical completeness, relativity requires an added postulate over and above the postulates made by Einstein at the foundation of the theory. Einstein's postulates concerned with observers, the fact that all observers were on the same footing for erecting descriptions of the external world; but the new postulate must concern the means whereby the knowledge of the external world is derived by all observers. We postulate that "The observations of every observer can be analyzed into a complex of minute events each with the same four-dimensional extent." The old postulates asserted that differences between observers were only relative, while the new postulate asserts that the differences between the minute events observed are also only relative. It carries the relativity a step further into the heart of physics; and incidentally provides a standard of minuteness for relativity, raising that theory to a more completely logical status.

Other postulates, of course, suggest themselves as alternatives; for instance the postulate that space-time is itself discontinuous, or that there is a minimum quantum of action, where action is defined from the usual relativity equations. We may justify our actual choice by appealing to Whitehead's philosophy of space-time.[3] This philosophy, generally accepted by those who have taken the trouble to study it, at least in general idea, shows that space-time may be logically defined from the extensive properties of events. Thus the world of events is fundamental, and space-time comes out of it by an abstractive process actually carried through with some success by Whitehead. Thus we cannot lightly postulate a discontinous space-time after Whitehead has erected a continuous one, until someone succeeds in erecting a discontinuous one and elucidates what the meaning of such a construct really is. Again, accepting Whitehead's philosophy, we immediately see that our fundamental postulate must concern with events rather than with such entities as mass or action, or for that matter with space-time either. If there is a minimum action or mass, or a discontinuous space-time, it should come out of the theory since relativity defines mass and action from functions in space-time, and not need tacking on to the theory as an *ad hoc* assumption. The properties of space-time that give a minimum action must be derivable from the events in terms of which the space-time is defined; hence our fundamental hypothesis must concern with events.

[2] Whitehead, Process and Reality, p. 465.

[3] Whitehead, Process and Reality and Principles of Natural Knowledge.

We claim therefore that our postulate is logically required to make the theory satisfactory at its foundation, and does not add anything *ad hoc* to the after structure of the theory.

§ 2. Geometry of Four-Dimensional Regions

We propose to examine the representation of minute four-dimensional regions representing the minimum events of the world on the Minkowksi diagram. In drawing Minkowski diagrams the beginner generally has to remind himself that a point on the diagram represents a definite and unalterable event with definite coordinates in both space and time for each system concerned. But in representing the minute events postulated here on a scale that shows the boundaries of the events as lines in the diagram, we can no longer assume that each point on the diagram represents a definite event for every system; there are no events of such precise coordinates. This lends a certain elasticity to the diagrams that is not familiar, and that must be carefully remembered in following the argument given below; the same event may be represented, if convenience dictates it, by differently shaped areas in the different systems of reference.

In some particular system S' the simplest four-dimensional region is a volume stationary and existent for a definite time; suppose it represented by $AB'CD'$ where AB' is along X' axis, and AC along T' axis, giving a two-dimensional section. We shall suppose that the same event is in a system S represented by $ABCD$ where $B'D'$ cuts the X axis in B; this will be a moving volume, for S, existent for a definite time.

Let dt, dt' be the time durations of the event, and dV, dV' the two volumes proportional to the X, X' sections. Then simple relativity transformations show that, taking velocity of light unity for convenience,

$$dt' = (1 - v \cdot v)^{1/2} \cdot dt;\, dV' = dV \cdot (1 - v.v)^{-1/2}: \tag{1}$$

where v is the velocity of S' relative to S. Thus the four-dimensional extent $dt \cdot dV = dt' \cdot dV'$ is a constant or invariant for this transformation.

In the particular case when v is unity, these formulae are no longer of use, for there are no systems S, S' with this relative velocity. If we call the velocity given by the gradient of the edge AC of the event the "self-velocity" of the event, then there may be events with self-velocity unity but which can therefore never be exhibited as volumes stationary in any coordinate system. Such an event could be represented by $ABCD$ for S, where AC is along the light-track gradient unity, and AB along the X axis. This would transform to $AB'CD'$ for S' with C still on the light track and B' at the intersect of BD with the X' axis. Here we can easily see that

$$dV' = dV(1 + v)/(1 - v \cdot v)^{1/2} \tag{2}$$

$$dt' = dt(1 - v \cdot v)^{1/2}/(1 + v) \tag{3}$$

and again the four-dimensional extent of the event is invariant. Alternatively the formulae may be written

$$dV = dV'(1 - v)/(1 - v \cdot v)^{1/2} \tag{2'}$$

$$dt = dt'(1 + v)/(1 - v \cdot v)^{1/2} \tag{3'}$$

with

$$dt = k/dV, \;\; dt' = k/dV'. \tag{4}$$

These of course hold only when the self-velocity of the event is parallel the relative velocity of S, S'. If the relative velocity v is along the Y, Y' axes, and the self velocity of the event is along the X, axis, then by drawing the appropriate figure it can easily be verified that

$$dt' = dt/(1 - v \cdot v)^{1/2} \tag{5}$$

$$dV' = dV(1 - v \cdot v)^{1/2} \tag{6}$$

with again

$$dt' = k/dV', \;\; dt = k/dV; \tag{4'}$$

the most general case being easily deducible from these two.

§ 3. Transmission of Four-Dimensional Regions

(a). Regions with unit self-velocity. Photon theory.

Suppose that the region $ABCD$ of the previous article represents one stage in the process of transmission of a possible event, the transmission being with the same vector velocity as the self-velocity of the possible event. The region $ABCD$ is a section (X-axis) of the region that would be occupied by the event were it to be realized at that stage, and examined by the system S. The fact that it is being transmitted along the X-axis means that the boundary AB is moving along the X-axis with the velocity of transmission, and the boundary CD is also moving along the X-axis with the same velocity. Since this velocity is the same as the velocity given by AC or BD we see that the boundary AB reaches the stage CD after the time dt which is the duration of the possible event. Thus in the realized event CD is ahead in space, but behind in time compared with AB, but in the transmission AB and CD coincide, for the occurrence of CD is simultaneous with the arrival of the transmitted AB. Were the transmission to materialize into an event, the edge AB would be realized first, and leave the edge CD travelling on for the time dt before it was realized.

Thus the transmission looks like a simple moving volume travelling with the velocity of light; all the volumes which would occur in succession in the event appear as coincident in an instantaneous picture of the transmission, and if we had not started from the complex, we should have mistaken the process for a simple moving volume, albeit with something corresponding with a pseudo-density of the overlapping volumes.

To picture a region that has extension in space and in time that is itself being propagated with velocity through space, is a new and perhaps rather difficult concept, and the results of investigating it in detail seem to be of considerable interest.

Thus suppose that a quantum emission corresponds to a four-dimensional event with self-velocity unity, and that this originates a possible absorption transmitted with this self-velocity. This transmission will appear as a moving volume, or particle; in other words it is the photon of recent optical theory. But the photon and the transmitted possibility both differ from mere moving volumes: the one in having mass or energy, and the other in having the pseudo-density of the superposed volumes corresponding to different stages of the possible event. Identifying these differences we assume next that the energy of the photon is proportional to the time-duration of the possible event; this may be taken as a definition of the energy of a particle on the new theory.

$$E \text{ is } prop. \text{ to } dt$$

and hence from Eq. (3′)

$$E = E'(1 + v)/(1 - v \cdot v)^{1/2} \tag{8}$$

if E is the energy of the photon for S, and E' for S'.

The probability that the event will occur or that the possibility will be realized cannot be deduced *a priori;* but since the wave-theory of radiation has been interpreted statistically, we may assume that there will be a probability wave-function whose phase-velocity equals the velocity of transmission. From wave theory it then follows that in transforming between the frequencies, the generalized Doppler principle[4] will necessarily hold:

$$n = n'(1 + v)/(1 - v \cdot v)^{1/2} \tag{8'}$$

and comparison with Eq. (8) therefore shows that

$$E = hn \tag{9}$$

where h is a universal constant, the quantum theory rule. Eq. (4) shows that the volume dV is proportional to the period, and thence the wave-length of the waves; or the section of the volume in the direction of the transmission is proportional to the wave-length.

(b). Regions with any self-velocity. Electron theory.

From the results of the foregoing we are naturally lead to expect transmitted possibilities of normal velocities to have the same properties as material particles. Let an event possibility of self-velocity u be transmitted with that velocity, then the energy of the particle it appears as, is again defined as proportional to the time dt of the event-possibility, or the pseudo-density of the superposed volumes. Eqs. (1) then give

$$E' = E(1 - v \cdot v)^{1/2} \tag{10}$$

which is the usual relativity form for the energy, since here E' is the rest-energy of the particle. Translated into the usual notation it is of course

[4] Cunningham, Principle of Relativity.

$$mc^2 = m_0c^2(1 - v^2/c^2)^{-1/2}. \tag{10'}$$

If it is questioned whether the electron is but a transmitted possibility, we may point to the usual interpretation of the matrix mechanics; the electron connects quantum events, and is in effect a transmitted possibility of an absorption. From this point of view it is natural to seek for a wave-function that shall give the probability of the event being realized at any stage. But to obtain the frequency transformation of these waves we must pursue a slightly different track from that which gave us the formulae for light waves; here the velocity is not an invariant and the problem is less simple. The familiar reasoning[5] is however perfectly satisfactory and conclusive. A stationary particle must correspond with a stationary wave system, say of the form for S',

$$P = A \exp\,(2\pi n' i t'). \tag{11}$$

The Lorentz transformation gives

$$P = A \exp\,(2\pi n i t - x/w) \tag{12}$$

relative to the S system, where

$$n = n'(1 - v^2/c^2)^{-1/2} \tag{14a}$$

and

$$w = c^2/u. \tag{14b}$$

Comparison of Eq. (14a) with Eq. (10) again gives the quantum rule

$$E = hn \tag{9}$$

while combination of Eqs. (14a), (14b) shows in the usual way that u is the group velocity of the waves.

We thus reach a satisfactory interpretation of the wave theory of matter, of the nature of the electron, and the similarity, yet difference between the photon and the material particle.

§ 4. Equations of Motion of Particles

At the outset of this work we made the simplifying assumption that the transmission of the possibilities was parallel to the self-velocity of the event-possibility transmitted. Only with this assumption is the transmission exhibited as a particle, for only then will the various volume-sections of the event be superposed on one-another. When therefore the particle exists, their tracks are necessarily given by parallel displacement; for by our assumption the edges AC, CC' of consecutive possible events are parallel, and hence the edge, which is parallel to the velocity vector of the particle, is moved by parallel displacement. This leads immediately to the unified field theory already given mathematical expression by the present writer[6]; the most general

[5] Haas, Wave Mechanics and New Quantum Theory, Chap. 2.

[6] Band, Phys. Rev. **36**, 1405 (1930).

rule for the parallel displacement of the velocity vector gives the tracks of the particles actually agreeing with the experimental tracks. The small displacements of the theory must obviously be of the same order as the dimensions of the events, and the equations will be meaningless if we attempt to apply them to smaller orders of length.

But we have already seen that the existence of the particle is by no means a necessity; so far as our theory is concerned, there may be transmissions in directions other than the self-velocity of the events, and what we may vaguely call the track of the transmission will then have appreciable curvature within the extent of one possible event, and the parallel displacement rule no longer applies. These "tracks" are not then tracks of particles at all, and the usual wave-theory interpretation of quantized orbits seems to be the most natural one here.

In passing, we remark that on the present theory we have refused to regard the particle as fundamental; the whole structure of relativity is erected from events of finite four-dimensional extent, and a particle is an abstract or recognized permanency among the extensive relations between events. Thus our postulate of finite extent to the ultimate events at once forbids the accurate estimation of the position or motion of the particle when it actually exists; for the particle can only be observed through the events which exhibit it, and these do not give exactitude.

In conclusion, we cannot claim to have treated more than an elementary set of cases; but it seems sufficient to show that it is possible by recognizing a logical weakness of Relativity to deduce from the most natural additional hypothesis to remove this weakness, the essential basis of the new quantum theory; to illuminate the meaning of and deduce the assumptions of the unified field theory; and finally to suggest an interpretation of the physical world that will harmonize the two great fields of theoretical research of recent years, relativity, and atomic physics.