A DETERMINATION OF e/m FOR AN ELECTRON BY DIRECT MEASUREMENT OF THE VELOCITY OF CATHODE RAYS

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Abstract

The velocity of cathode rays, driven by potentials of from 10,000 to 20,000 volts, is measured directly by timing the passage of the electrons between two localized transverse high-frequency electric fields 75 cm apart. Those electrons which pass undeflected travel the distance between the deflecting fields in an even multiple of half a cycle of the oscillating fields. The velocity thus obtained, combined with the expression for the energy imparted to an electron in falling through a measured difference of potential, gives the value of e/m_0.

The mean value thus obtained is

$$e/m_0 = (1.761 \pm 0.001) \times 10^7 \text{ abs. em units}$$

This value is in agreement with the values obtained by spectroscopic methods and does not agree with the most accurate previous value obtained for free electrons. A possible explanation for the discordant value previously obtained for free electrons is given.

Introduction

The ratio of the charge to the mass of an electron has been measured by many investigators using various methods, a description of which is given in several places, among which may be mentioned the paper, "The Probable Values of the General Physical Constants," by R. T. Birge in the first issue of the Physical Review Supplement. It seems unnecessary here to review the previous work but it might be helpful to point out that the various methods of measurement may for convenience be classed in two general groups. The *first* group includes those experiments made with free electrons, as cathode rays, photo-electrons or β-particles. The *second* group of experiments involves spectroscopic measurements and hence deals with electrons within atoms.

Most of the methods of the first group involve the deflection of rapidly moving electrons by transverse magnetic or electrostatic fields or both. The most accurate work of this type is that of F. Wolf[1] using a method first suggested by H. Busch.[2] Briefly, his experiment consisted in projecting into a longitudinal magnetic field a diverging cone of cathode rays and adjusting the strength of the field to bring the rays to a focus. Wolf's result, corrected by Birge for the difference between the international and absolute volt, is

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[1] F. Wolf, Ann. d. Physik **83,** 849 (1927).

[2] H. Busch, Phys. Zeits. **23,** 438 (1922).

$$e/m_0 = (1.7689 \pm 0.002) \times 10^7 \text{ abs. em units}$$

where m_0 is the mass of the electron at rest.

A few of the methods of the first group are somewhat more direct than those to which reference has just been made. These methods involve a direct determination of the velocity of the electrons combined with the equation giving the energy imparted to an electron when accelerated by a longitudinal electric field. The relativity mass of an electron is

$$m = \frac{m_0}{(1 - (v^2/c^2))^{1/2}} \tag{1}$$

where m_0 is the mass at rest, v is the velocity of the electron at which it possesses mass m, and c is the velocity of light. The energy given to an electron in falling through a difference of potential E is

$$\int_0^v vd(mv) = m_0c^2\left[\frac{1}{(1 - v^2/c^2)^{1/2}} - 1\right] = eE. \tag{2}$$

Historically Wiechert[3] in 1899 was the first to make a direct measurement of the velocity of cathode rays by timing their passage between two points by means of damped high-frequency electric oscillations.

In 1912 one of the authors[4] reported at a meeting of the Physical Society some determinations of e/m_0 made by an improved direct method. This work was suspended, however, until better facilities were available.

The present work reported below was resumed about two years ago with further improvements in method and technique made possible by the development of vacuum-tube oscillators and the installation of the 100,000-volt storage battery in the Cruft Laboratory.

Kirchner[5] last November reported some preliminary measurements using practically the same method as that described in this paper. His results, obtained for accelerating voltages not exceeding 2500, agree approximately with Wolf's value given above.

Some of the experiments of the second group made upon bound electrons involve the measurement of the Zeeman separation in a known magnetic field. The most accurate work of this kind is that of Babcock.[6] With certain selected lines in the spectra of chromium, zinc, cadmium, and titanium he obtained a weighted mean value

$$e/m_0 = (1.7606 \pm 0.0012) \times 10^7 \text{ abs. em units}$$

Another method of obtaining the value of e/m_0 from spectroscopic measurements depends upon the Bohr-Sommerfeld theory for an atom consisting of a single electron moving around a positive nucleus. This method involves

[3] E. Wiechert, Wied. Ann. **69**, 739 (1899).
[4] E. L. Chaffee, Phys. Rev. **34**, 474 (1912).
[5] F. Kirchner, Phys. Zeits. **30**, 773 (1929).
[6] H. D. Babcock, Astrophys. J. **58**, 149 (1923); **69**, 43 (1929).

the Rydberg constant for hydrogen and ionized helium. Houston[7] using this method obtained a value which, corrected by Birge, is

$$e/m_0 = (1.7608 \pm 0.0008) \times 10^7 \text{ abs. em units.}$$

In view of the disagreement in the values of e/m_0 obtained by the experiments on free and on bound electrons, a disagreement greater than the probable errors of the experiments, Birge said "it seems to be necessary to assume two different values of e/m, one to be used in all cases involving atomic structure, and the other involving free electrons." The value of e/m_0 obtained in the present work using free electrons, i.e.,

$$e/m_0 = (1.761 \pm 0.001) \times 10^7 \text{ abs. em units,}$$

agrees well with the values obtained from spectroscopic data, and it is hoped will help to resolve the unpleasant suggestion made by Birge of the necessity of retaining two values for e/m_0.

Method and Apparatus

The value of e/m_0 was obtained through a direct measurement of the velocity of free electrons, and the use of the energy equation for a moving charge. The method of velocity measurement was briefly as follows.

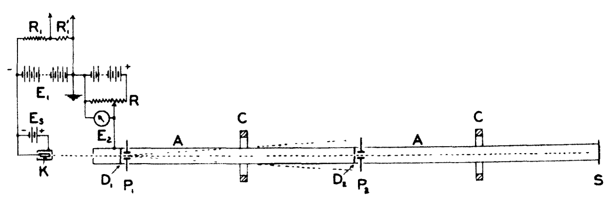

Fig. 1. Diagram of tube.

A highly evacuated cathode-ray tube was placed parallel to the earth's magnetic field, and a stream of electrons projected along its axis by means of a high potential. A diagram of the tube is shown in Fig. 1, in which K is the cathode, and the batteries supplying the driving potential are shown at E_1 and E_2. The anode, shown at A, was a long hollow metal cylinder, within which the electrons travelled with constant velocity. Placed in their path were two high-frequency electrostatic fields, furnished by small parallel plates activated by a high-frequency oscillator. The pairs of plates, shown at P_1 and P_2, were a known distance apart, and so connected that the two fields were 180° out of phase. A group of electrons passed P_1 undeflected each half cycle. If the time required for these electrons to travel the distance between P_1 and P_2 was a half cycle (or any multiple of a half cycle) of the oscillator, they were also undeflected by P_2 and so made a single central spot on the fluorescent screen S. But if their velocity was too great, they

[7] W. V. Houston, Phys. Rev. **30**, 608 (1927).

reached P_2 before the field there was zero, and were deflected. Alternate groups were deflected in opposite directions. Hence two spots appeared on the screen S. The same was true if the electron velocity was too small. For a fixed frequency of the oscillator, the potential on the tube was gradually increased until the two spots moved into one. If the voltage was increased still further two spots again appeared. The voltages causing similar patterns on both sides of the single spot were averaged to obtain the voltage corresponding to the correct velocity.

The cathode-ray tube was of Pyrex glass enclosing the metal parts, and was evacuated by a four-stage mercury diffusion pump, with an oil backing pump. The mercury pump was of steel, and was placed about two meters away from the tube to prevent its magnetic field from disturbing the electrons. The pump tube was 2.5 centimeters in diameter to allow rapid exhaustion, and near the cathode-ray tube it passed through a large trap cooled with liquid air.

The cathode of the tube, shown in cross-section in Fig. 2, was an indirectly-heated nickel thimble K, with the heating coil of tungsten wire located inside along the axis. A small spot of oxide on the end of the thimble gave

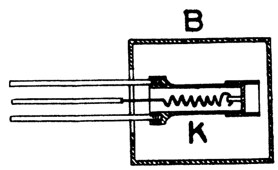

Fig. 2 Cross-section of cathode.

good emission when the nickel was only a dull red. A shallow open cylinder attached to and extending beyond the end of the thimble, caused the emitted electrons to take the form of a solid cone of rays. The negative terminal of the high-potential source was connected directly to the thimble.

A shielding cylinder of nickel, shown at B in Fig. 2, enclosed the cathode. This shield was pierced by a millimeter hole in front of the oxide coating, and was kept at a constant potential of about 20 volts positive with respect to the cathode by the battery E_s in Fig. 1. This small potential started the electrons away from the cathode, and those passing through the millimeter opening formed a narrow beam of rays falling upon the anode.

A water-cooled wax joint allowed the cathode to be removed for repairs without disturbing the shielding cylinder, whose millimeter opening was accurately aligned with two others in the anode.

The anode was a hollow aluminum cylinder 150 centimeters long, which was connected to the positive side of a 2,000-volt storage battery through a potential divider shown at R_2 in Fig. 1. The negative side of this battery was grounded, as was also the positive side of the high-potential battery E_1 connected to the cathode. The battery E_1 was a portion of the 100,000-volt storage battery in the Cruft laboratory, and could only be varied by steps of 1,600 volts. The use of the second battery permitted continuous variation of the driving potential.

The deflecting plates at P_1 and P_2, together with their neighboring diaphragms, were similar, and a cross section of the arrangement (labelled for P_1) is shown in Fig. 3. The deflecting plates were of aluminum, approximately 5 millimeters long and 3 millimeters wide, and were separated by

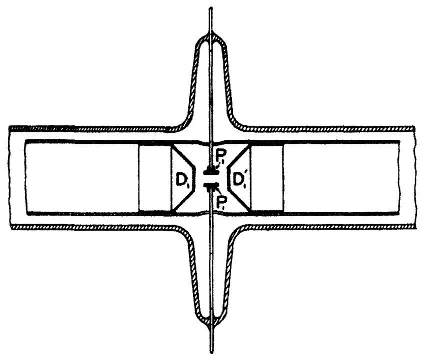

Fig. 3. Arrangement of deflecting plates and diaphragms.

about 3 millimeters. A diaphragm with a central millimeter hole, shown at D_1 in Figs. 1 and 3, was located above the plates P_1 to limit the electron stream to a narrow pencil of rays. A similar diaphragm D_2 was located above the plates P_2 to allow only the electrons that were undeflected by P_1 to pass through P_2. Diaphragms with 3 millimeter holes were placed below the plates, to prevent reflection of electrons from the inside of the tube and to balance geometrically the diaphragms D_1 and D_2. One of these is shown at D'_1 in Fig. 3.

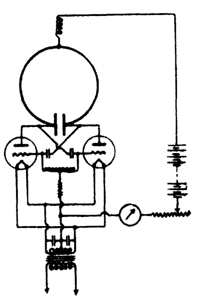

Fig. 4. Arrangement of connections.

A fluorescent screen S was placed at the lower end of the cylindrical anode, and a window was cut in the metal tube for observation of the fluorescent spot. The diaphragm D_2 was also covered with fluorescent material, and a similar window supplied. This was used to observe the amount of deflection caused by the plates P_1 and also for the adjustment of the focusing coils, the use of which is discussed farther on. Both windows in the tube were covered with fine copper gauze.

The distance between P_1 and P_2 was taken as the distance between the centers of the two pairs of plates, and was found to be 75.133 centimeters. This distance was accurately determined by means of a cathetometer and comparison with a standard meter rod checked by the Bureau of Standards.

The oscillator consisted of two 75-watt vacuum tubes, type 846, connected in push-pull arrangement to reduce second harmonics. The connections are shown in Fig. 4. The plate potential was 2000 volts. Small variations of wave-length were produced by means of a tuning condenser connected across the inductance in the oscillatory circuit. Larger variations were made by changing the inductance. In this manner the wave-length was varied from 3 to 6 meters.

The short-wave oscillator was inductively coupled at the center to a pair of parallel wires. These parallel wires were about 5 centimeters apart, approximately one-half wave-length long, and bent into a V with rounded vertex so that the ends could connect with the pairs of plates P_1 and P_2. The midpoints of the parallel wires were connected by a two-megohm resistance, the center of which was grounded to drain off any accumulated charge on the plates.

The success of this method of velocity measurement depends upon the accuracy to which the phase difference between the electric fields at P_1 and P_2 is 180°. Although theoretically the phase difference should be 180° if the node of the stationary wave system on the wires remains fixed, the system was constructed to be geometrically as symmetrical as possible to aid in securing this result. The position of the grounding resistance on the parallel wires could be varied considerably without any detectable effect upon the experimental results, showing that this resistance had no appreciable influence upon the phase difference at P_1 and P_2. In the course of the experiment different degrees of coupling to the oscillator were used, the oscillator was tuned above and below resonance with the wire system, the time of flight of the electrons between P_1 and P_2 was in some cases a whole cycle and in others a half cycle of the oscillations and yet all results were in close agreement. This consistency seems to allay all doubt as to the correctness of the assumption of the 180° phase relation.

For any one reading, the frequency was kept constant as indicated by beats with a harmonic of an oscillating crystal. Since the harmonics used ranged from the 28th to the 49th, it was impractical to make direct use of the beats between the crystal and the short-wave oscillator. Hence an intermediate oscillator of wave-length about 28 meters was used to produce beats with both. During each reading the frequency was checked several times by listening to the two sets of beats. The harmonics used were the 4th to the 7th of the crystal, and the 4th to the 8th of the intermediate oscillator and were identified by means of a calibrated wavemeter loosely coupled to the intermediate-frequency oscillator.

The frequency of the crystal was measured with the aid of Professor G. W. Pierce, in terms of a 1000-cycle clock, driven by a magnetostriction rod. The frequency of the crystal was found to be

$$n = 1{,}680{,}890 \text{ cycles per second,}$$

correct to 1 part in 40,000.

Allowing liberally for slight variations from zero beats in the case of both oscillators, (200 cycles for the intermediate, and 1000 cycles for the power oscillator), the error in frequency was less than 1 part in 20,000.

The voltage between ground and cathode (E_1 in Fig. 1) was measured by means of two resistances R_1 and R'_1 and a potentiometer. R'_1 was an accurately-known resistance of 89.958 ohms, and the potential drop across it was measured with the potentiometer and a standard cell. For potentials less than 12,000 volts, R_1 was a manganin wire resistance of 895,130 ohms, while for higher potentials a similar resistance of 6,936,600 ohms was used.

The high resistances were measured on a Wheatstone bridge, carefully insulated for leakage. The other members of the bridge were composed of two resistances of 500,000 ohms, and two of 100,000 ohms, which were carefully measured by building up from a 10,000-ohm Leeds-Northrup sealed standard. A substitution method was used throughout this work. The Wheatstone bridge was balanced by the addition in one arm of a decade box with a maximum resistance of 99,999 ohms. Each coil of this box was measured in terms of a sealed standard of the same size, and here again the substitution method was employed. The resistance of 6,936,600 ohms was measured in sections of about 1,000,000 ohms each for greater accuracy.

The 10,000-ohm Leeds-Northrup standard was checked against two others of the same type, and all three agreed to better than 1 part in 10,000. The one with the most recent certificate from the Bureau of Standards was taken as correct.

The resistance of 895,130 ohms was tested for changes in its value due to the current carried, and corrections of 3 to 5 parts in 10,000 were made according to the load. The resistance of 6,936,600 ohms carried so little current that no correction was required.

The 89.958-ohm resistance was measured in terms of a 100-ohm sealed standard, which had recently been checked.

The potentiometer was a new one, and was checked for equality of intervals. The standard cell was checked against three others, one of them a new one, and all four agreed to better than 1 part in 10,000. The new cell was taken as correct.

The variable voltage applied to the anode was measured by a General Electric voltmeter, which had been calibrated by a potentiometer for division errors. Allowance was made for a slight irregularity at the beginning and end of the scale.

The adjustment of the voltage until the two electron spots just came together, and again, until they just separated, was the most uncertain part of the experiment. A single spot appeared throughout a voltage change of 400 or 600 volts, and separate settings on either side of this range could not be made with more accuracy than one division on the voltmeter, corresponding to 20 volts. Hence many pairs of settings were made for one determination, and the midpoints of these pairs averaged. All settings were made in the dark, and so should be truly independent, and for reasons of symmetry, the voltage was always changed in such a manner that the two spots were brought together for each setting.

The use of concentrating coils to focus the electron stream, which was mentioned above, was necessary and yet undesirable. They were coils of about 2,000 turns, 15 centimeters in diameter, encircling the tube, and giving magnetic fields symmetric with the axis of the tube. One was placed midway between the deflecting plates at P_1 and P_2, and the other an equal distance below P_2. These are shown at C in Fig. 1. Without the coils, the electron stream spread out, giving a spot about a centimeter in diameter on the screen just above P_2. This was too large and also too faint for accurate work. Furthermore, the axis of the tube was not placed exactly parallel to the earth's field, and hence the electron spot was not central. The first concentrating coil focused the electrons in a small, intense spot, and brought them over to a central position. The second coil merely made the final adjustment more accurate by increasing the intensity and decreasing the size of the two spots on the final screen.

The use of the first coil was, however, undesirable, for it imparted a spiral motion to the electrons, thus increasing slightly the distance they travelled in the observed time. Also, if the strength of the magnetic field was such that the electron stream was accurately focused, the transverse electric field at P_1 had no effect on the position of the spot. In order to get a deflection, the coil had to be used off focus, and the amount of deflection depended on how much the focus was displaced. As the voltage range between each pair of settings depended upon the amount of deflection obtained, varying the current through the coils greatly varied the individual settings. However, readings were made under many different conditions of focusing, and the midpoints of the voltage pairs were in close agreement as shown by the following tables.

As to the increase in path due to the spiral motion of the electrons, the following estimate was made. By gradually increasing the current in the coil, the amount of rotation of the spot was found to be less than 180°. The distance from the center of the screen to that of the undeflected spot was about 1 centimeter, which would make the electron stream about 0.5 centimeter from the axis of the tube in the plane of the concentrating coil. With the magnetic field acting, however, the electron path would be concave toward the axis and so this distance would be decreased. From symmetry the maximum departure occurs in the plane of the coil. Assuming the path to be a helix on a cylinder of diameter 0.5 centimeter, the increase in path was found to be 1 part in 4500. This makes a decrease in e/m_0 of 1 part in 2,250, or 0.0008.

As the voltage on the tube was increased to 20,000, the undeflected spot became more central, and so this error should have been reduced. Therefore, a definite upward trend in e/m_0 with increasing voltages would have indicated an appreciable error due to this cause; however, no such trend was observed. Hence it seems reasonable to state that the error in e/m_0 from the use of the coils was not more than 0.0008.

RESULTS

Voltage measurements were made for six frequencies of the oscillator, and in order to average out unknown errors as much as possible, the settings for each frequency were made in groups over a period of several days. A sample group of such measurements is shown in Table I, where the actual voltmeter settings are given, with the midpoint of each pair, and the corresponding voltage. The pairs of settings were influenced by the intensity of the spots, the power of the oscillator, and how close together the spots were brought, and so varied considerably. However, their midpoints were in good agreement, each group being very consistent within itself, and all the groups for any one frequency agreeing well with each other.

TABLE I. *Sample group of settings.* 48th harmonic of the crystal beating with the fundamental of the oscillator. Frequency = 80,683,000 cycles/sec. Times of passage between plates = 1 cycle. Velocity = 0.60619×10^{10} cm/sec.

Voltmeter settings (factor = 20)	Midpoint	E_1+E_2 Total voltage for midpoint	δ from the average voltage
63–79	71	10776	10
61–80	70.5	10766	0
62–79	70.5	10766	0
62–80	71	10776	10
63–77	70	10756	10
64–76	70	10756	10
65–76	70.5	10766	0
61–79	70	10756	10
62–79	70.5	10766	0
61–80	70.5	10766	0
62–79	70.5	10766	0
63–79	71	10776	10
		Av. 10766	Av. 5

Tables II to VII give all the settings made at each frequency, arranged in groups as they were taken. For each group, the settings giving the same midpoint are combined, and the corresponding voltage listed. The average voltage for this frequency is also given, and the deviation of each setting from this average. The value of e/m_0 corresponding to the average voltage is computed from the energy equation

$$\frac{e}{m_0}=\frac{c^2}{E}\left[\frac{1}{(1-v^2/c^2)^{1/2}}-1\right]$$

The average deviation and probable error of e/m_0 are calculated from the voltage deviations. In these tables, all voltages have been changed to absolute volts by use of the conversion factor 1.00046 given by Birge.[8]

TABLE II. *Summary of observations with frequency of 80,683,000 cycles/sec.* 48th harmonic of the crystal beating with the fundamental of the oscillator. Times of passage between plates = 1 cycle. Velocity = 0.60619×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
11	10759	4
6	10769	6
1	10749	14
1	10739	24
1	10729	34
10	10757	6
7	10767	4
1	10777	14
10	10760	3
4	10770	7
2	10790	27
8	10769	6
2	10759	4
Table I { 6	10766	3
Table I { 3	10776	13
Table I { 3	10756	7
6	10764	1
6	10754	9
Total No. 88	Av. 10763	Av. 6.2

Av. $e/m_0 = 1.7613 \times 10^7$ abs. em units. Av. deviation = 0.0010
Max. $e/m_0 = 1.7668 \times 10^7$ Prob. error = 0.0001
Min. $e/m_0 = 1.7568 \times 10^7$

TABLE III. *Summary of observations with frequency 82,364,000 cycles/sec.* 49th harmonic of the crystal beating with the fundamental of the oscillator. Time of passage between plates = 1 cycle. Velocity = 0.61882×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
7	11235	3
4	11245	13
1	11215	17
1	11255	23
5	11236	4
4	11226	6
2	11246	14
7	11220	12
6	11230	2
Total No. 37	Av. 11232	Av. 7.9

Av. $e/m_0 = 1.7612 \times 10^7$ abs. em units Av. deviation = 0.0012
Max. $e/m_0 = 1.7639 \times 10^7$ Prob. error = 0.0002
Min. $e/m_0 = 1.7576 \times 10^7$

[8] R. T. Birge, Phys. Rev. Supp. 1, 1 (1929).

The error introduced by assuming that the midpoint of the two voltage settings agrees with the midpoint of the two corresponding velocities, is

TABLE IV. *Summary of observations with frequency 47,065,000 cycles/sec.* 28th harmonic of the crystal beating with the fundamental of the oscillator. Time of passage between plates = ½ cycle. Velocity = 0.70723×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
5	14853	28
1	14843	18
3	14824	1
2	14814	11
2	14804	21
1	14834	9
4	14817	8
3	14827	2
3	14837	12
8	14818	7
2	14828	3
2	14808	17
1	14798	27
Total No. 37	Av. 14825	Av. 11.6

Av. $e/m_0=1.7608\times10^7$ abs. em units
Max. $e/m_0=1.7640\times10^7$
Min. $e/m_0=1.7575\times10^7$

Av. deviation = 0.0014
Prob. error = 0.0002

that of substituting a linear for a square root relation. For voltage settings differing by 400 volts, this error for a total potential of 10,000 volts amounts to 1 part in 10,000. For higher voltages it is less than this, becoming 1 part

TABLE V. *Summary of observations with frequency 50,427,000 cycles/sec.* 30th harmonic of the crystal beating with the fundamental of the oscillator. Time of passage between plates = ½ cycle. Velocity = 0.75774×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
6	17117	19
2	17127	9
1	17137	1
4	17157	21
4	17147	11
2	17167	31
6	17127	9
6	17137	1
Total No. 31	Av. 17136	Av. 12.4

Av. $e/m_0=1.7601\times10^7$ abs. em units
Max. $e/m_0=1.7622\times10^7$
Min. $e/m_0=1.7569\times10^7$

Av. deviation = 0.0013
Prob. error = 0.0002

in 40,000 at 20,000 volts. This enters directly in e/m_0, and is such as to increase the tabulated values, but is negligible in comparison with other errors.

TABLE VI. *Summary of observations with frequency of 52,948,000 cycles/sec.* 63d harmonic of the crystal beating with the second harmonic of the oscillator. Time of passage between plates = ½ cycle. Velocity = 0.79563×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
8	18993	0
5	19003	10
2	18983	10
9	18986	7
2	18996	3
1	18976	17
5	18996	3
3	19006	13
1	18986	7
Total No. 36	Av. 18993	Av. 6.0

Av. $e/m_0=1.7600\times10^7$ abs. em units
Max. $e/m_0=1.7616\times10^7$
Min. $e/m_0=1.7588\times10^7$

Av. deviation = 0.0006
Prob. error = 0.0001

TABLE VII. *Summary of observations with frequency of 53,789,000 cycles/sec.* 32d harmonic of the crystal beating with the fundamental of the oscillator. Time of passage between plates = ½ cycle. Velocity = 0.80826×10^{10} cm/sec.

No. of settings	Voltage E_1+E_2	δ from the av. voltage
7	19622	4
5	19612	14
8	19625	1
5	19635	9
1	19615	11
6	19637	11
4	19627	1
2	19617	9
Total No. 38	Av. 19626	Av. 6.6

Av. $e/m_0=1.7610\times10^7$ abs. em units
Max. $e/m_0=1.7622\times10^7$
Min. $e/m_0=1.7599\times10^7$

Av. deviation = 0.0006
Prob. error = 0.0001

TABLE VIII. *Summary of results.*

	Velocity ($\times10^{-10}$) cm/sec.	No. of settings (weighting factors)	Av. e/m_0 ($\times10^{-7}$) abs. em units	δ from 1.7609	Av. δ of settings from 1.7609
Table II	0.60619	88	1.7613	0.0004	0.0011
Table III	0.61882	37	1.7612	0.0003	0.0012
Table IV	0.70723	37	1.7608	0.0001	0.0014
Table V	0.75774	31	1.7601	0.0008	0.0013
Table VI	0.79563	36	1.7600	0.0009	0.0010
Table VII	0.80826	38	1.7610	0.0001	0.0006

Weighted average $e/m_0=1.7609\times10^7$ abs. em units
Weighted average deviation of the six values of e/m_0 from 1.7609×10^7............0.0004
Probable error (obtained by weighting the deviations from 1.7609×10^7)...........0.0002

Table VIII summarizes these results, giving the six average values of e/m_0 for the six frequencies used. The values of e/m_0 for each frequency are more consistent with each other than with those obtained for other frequencies, and for this reason the six average values of e/m_0 are treated as six single determinations, weighted proportionally to the number of settings from which they are derived. The weighted average e/m_0 so obtained is

$$e/m_0 = (1.7609 \pm 0.0002) \times 10^7 \text{ abs. em units,}$$

with an average deviation of 0.0004, and a probable error of 0.0002, both similarly computed.

In view of the foregoing errors the final value of e/m_0 from the present work is conservatively written

$$e/m_0 = (1.761 \pm 0.001) \times 10^7 \text{ abs. em units.}$$

This method is, however, capable of much more accuracy, and the work is being continued with some improvements in technique.

Conclusion

Because of the difference between the present value

$$e/m_0 = (1.761 \pm 0.001) \times 10^7$$

and that obtained by Wolf,

$$e/m_0 = (1.769 \pm 0.002) \times 10^7$$

it may be noted that in one particular the two experiments differed quite markedly, that is, in their sensitiveness to the presence of residual gas. The effect of gas in the path of electrons is to retard them by an amount dependent upon their speed. J. J. Thomson[9] deduced the relation

$$v_0^4 - v_x^4 = ax$$

for the slowing down of electrons in metals. Here v_0 is the initial velocity, v_x the velocity after travelling a distance x in the substance, and a is a constant dependent upon the metal.

Widdington[10] tested this formula for metals, and also for air, obtaining for the constant in the latter case

$$a = 2 \times 10^{40}$$

when $p = 760$ millimeters. Assuming that a is directly proportional to the pressure of the gas, the equation becomes

$$v_0^4 - v_x^4 = \frac{2(10^{40})px}{760}$$

[9] J. J. Thomson, Conduction of Electricity through Gases, 2nd. edition, p. 378.

[10] R. Whiddington, Proc. Roy. Soc. **A86**, 360 (1912).

where p is the air pressure in millimeters of mercury. If the change in velocity is small,

$$v_0^4 - v_x^4 = 4v^3 dv$$

or

$$\frac{dv}{v} = \frac{2(10^{40})px}{4(760)v^4}.$$

Thus the percentage change in v is much less at higher speeds.

To compute a few cases actually involved, take first the present experiment. Here the distance x was 75 centimeters, and the voltage varied from 10,000 volts to 20,000 volts.

$$E = 10{,}000 \text{ volts} \qquad v = 0.60 \times 10^{10} \text{ cm/sec} \qquad \frac{dv}{v} = 0.39p.$$

$$E = 20{,}000 \text{ volts} \qquad v = 0.80 \times 10^{10} \text{ cm/sec} \qquad \frac{dv}{v} = 0.12p.$$

In Wolf's experiment, $x = 30$ centimeters, and the voltage varied from 3,500 volts to 4,500 volts.

$$E = 3{,}500 \text{ volts} \qquad v = 0.35 \times 10^{10} \text{ cm/sec} \qquad \frac{dv}{v} = 1.31p$$

$$E = 4{,}500 \text{ volts} \qquad v = 0.37 \times 10^{10} \text{ cm/sec} \qquad \frac{dv}{v} = 1.05p$$

Thus the percentage change in electron velocity introduced by the same gas pressure is much less for the present work than for Wolf's experiment.

To consider the effect of such a change in velocity upon the calculated values of e/m_0 take first the present work. The energy equation gives

$$\frac{e}{m_0} = (\text{const})\frac{v^2}{E}$$

approximately, and hence, if the measured velocity is too small for the applied potential, the calculated value of e/m_0 is too small. However, the change in velocity increases threefold as the voltage is reduced from 20,000 to 10,000 volts. Hence, if this error were appreciable, the calculated values of e/m_0 should show a definite trend with voltage. The absence of such a trend seems to show that this error was negligible in the present work.

In Wolf's experiment,

$$\frac{e}{m_0} = (\text{const})\frac{E}{H^2}$$

approximately, and the energy equation is assumed as the relation between the velocity and potential. If the velocity is decreased due to the presence of gas, the potential measured is larger than that actually corresponding to the average velocity, and so the calculated value of e/m_0 is too large. Moreover, no trend in his results can be expected, as the error does not change appreciably over the voltage range used. Wolf made no estimate of the gas pressure in his apparatus. If Whiddington's formula is assumed, a pressure of 0.004 mm would be sufficient to explain the discrepancy between his value of e/m_0 and the present one.

It is fully realized that in applying Whiddington's formula to the present case, a very great extrapolation is made. The value of a was obtained by measuring the distance required to halt an electron whose path was through air at atmospheric pressure. Yet here it is used to calculate a small percentage loss in the velocity of electrons passing through gases at extremely low pressures. Therefore, no great confidence can be placed in the resulting numerical values. In fact for electron velocities approximating those used in the present experiment Bohr[11] obtained from theoretical considerations a value of the constant a equal to about one-half that given by Whiddington.