TENSION COEFFICIENT OF RESISTANCE OF METALS*

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Abstract

A new method for measuring the change in resistance of a metal under tension has been developed and applied to fifteen metals. A direct current is sent through the wire under test while longitudinal vibrations set up in the wire cause the resistance and consequently the potential drop in the wire to fluctuate. These changes are amplified and measured. In this way it has been found that for bismuth, aluminium, manganin, and constantan the resistance decreases with tension while for all the other metals investigated the resistance increases with tension.

Introduction

THE change in electrical properties of metals under stress was first investigated by William Thomson, later Lord Kelvin, and described by him in a paper entitled "The Electrodynamical Qualities of Metals."[1] Due to the lack of sensitive apparatus, his experiments, which were conducted on copper and iron, gave untrustworthy results. Soon afterwards, H. Tomlinson,[2] at Lord Kelvin's suggestion, undertook to determine the exact change in resistance of wires caused by tension along their length. This remains the most extensive investigation on the subject. Other work has been done by W. E. Williams[3] on bismuth; by Donaldson and Wilson[4] on lead; and by N. F. Smith[5,6] on iron, steel, copper, and brass. The most complete study in recent years has been made by P. W. Bridgman[7,8,9] on many metals which he had at hand in the form of wires because of previous work on their pressure coefficients of resistance.

The method, a static one, used by all other investigators for studying these effects is to put a known load on the wires and then to measure the corresponding change in resistance. From this change is subtracted that produced by alteration in dimensions. The remainder is considered to be the change in resistance due to the stress.

The object of this research was the development of a dynamic method of

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[1] W. Thomson, later Lord Kelvin. Phil. Trans. **146,** 649–672 (1856).

[2] H. Tomlinson, Phil. Trans. **174,** 1–172 (1883).

[3] W. E. Williams, Phil. Mag. [6] **13,** 635–643 (1909).

[4] J. A. Donaldson, and R. Wilson, Proc. Roy. Soc. Edinb. **27,** 16–20 (1907).

[5] N. F. Smith, Phys. Rev. **28,** 107–121 (1909).

[6] N. F. Smith, Phys. Rev. **28,** 429–437 (1909).

[7] P. W. Bridgman, Proc. Am. Acad. **57,** 39–66 (1922).

[8] P. W. Bridgman, Proc. Am. Acad. **59,** 117–137 (1923).

[9] P. W. Bridgman, Proc. Am. Acad. **60,** 423–444 (1925).

measuring the tension coefficient of resistance and its application to metals whose coefficient is not easily obtained by static measurements. In the method finally adopted, the tension applied is alternating, in order to eliminate spurious effects due to thermoelectric forces and other effects of local or variable heating. A direct current is sent through the wire. As a result of the variations in resistance there are corresponding variations in the potential drop across the wire, and these are impressed on the grid of the first tube of calibrated amplifier, by means of which they are magnified and measured. From these data and the value of the direct current through the wire the change in resistivity due to tension is calculated.

Apparatus and Procedure

The wire to be stretched, Fig. I, is fastened at one end to a brass cylinder which is placed in a holder that can be moved by a screw in the direction of the wire axis to control the tension. The other end is wrapped around a hook. A glass rod, welded to the hook, connects it to the moving arm of a moving

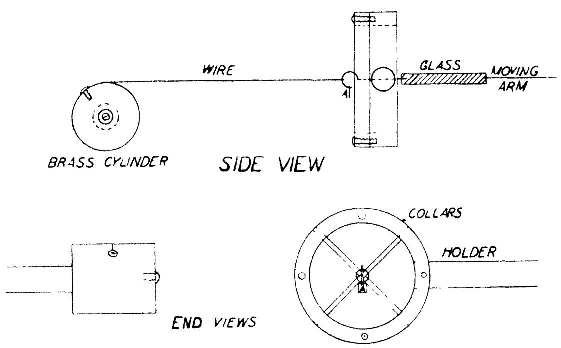

Fig. 1. Diagram of apparatus.

armature loud speaker unit. The hook is held in position by a cross-shaped piece of thin steel cut from an old telephone diaphragm. The ends of this cross are held fast between two brass collars. From them projects a rod which holds the collars securely. This arrangement keeps the motion of the hook axial.

The tension is applied by moving the brass cylinder back by means of the screw, thus stretching the wire. Care must be taken to avoid the production of standing transverse waves which will appear with amplitudes ranging from 1 mm to 5mm if the tension bears the appropriate relation to the length and density of the wire.

The glass rod is put in to insulate the wire from the loud speaker unit. Alternating current of 1025 cycles per second from a tube generator and amplifier is led into the loud speaker coil, and sets the wire into longitudinal vibration with a frequency equal to that of the alternating current. This frequency of current was chosen because it could be conveniently checked by a standard tuning fork making 1024 vibrations per second.

A battery, Fig. 2, sends a direct current through the wire. The change in resistance of the wire changes the potential drop across it, the alternating component is impressed on the grid of a vacuum tube of the amplifier. The choke coil, of 15 henrys inductance, makes the circuit impedance so high that the change in potential drop due to the current change is made negligible.

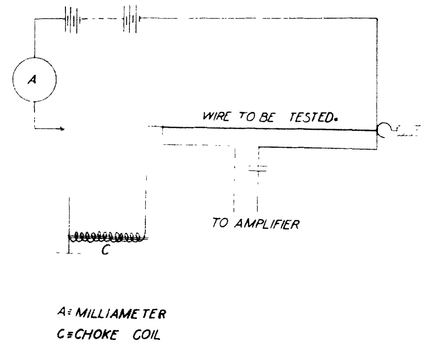

Fig. 2. Battery arrangement.

Fig. 3 is the wiring diagram of the six-tube resistance coupled amplifier. The amplifier is broken into two sets of three each, each part with its own batteries in a sheet tin box. Unless this is done the amplifier maintains continuous electrical oscillations. The connections to the first three tubes are arranged so that either one, two, or three may be used together with the last

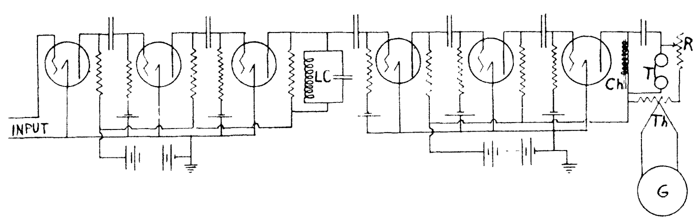

Fig. 3. Wiring diagram of the six tube resistance coupled amplifier.

three. It has been found difficult to get steady amplification without considerable noise with six tubes. Five tubes, with an amplification of about twenty-five thousand, gave very satisfactory performance and this number was used throughout the work. Between the third and fourth stages is fixed a circuit (*LC*) tuned to 1025 cycles. Its impedance to current of this frequency is five thousand ohms; to all other frequencies its impedance is considerably smaller. Thus it acts as a filter, being a low resistance shunt for all currents of frequencies other than 1025 cycles. Its use makes the amplifier very quiet. The negative side of the B battery and the metal shielding are grounded.

In the last stage is a power tube in the plate circuit of which is a hot-wire vacuum thermocouple (Western Electric No. 232) with an adjustable resistance in series with it. The condenser (C) allows only alternating current to go through the thermocouple, which is used in connection with a galvanometer. The phones are used to detect any extraneous disturbance. The amplifier is first calibrated by sending a known e.m.f. into the imput circuit and noting the deflections of the galvanometer in the output circuit.

The amplitude of vibration is measured with an optical lever. The back of a small light mirror about five millimeters in diameter is fastened to a tightly drawn fine wire, and the tip of the mirror rests against a small lip (A in Fig. 1) projecting from the hook to which the moving end of the vibrating wire is attached. Light from a slit is condensed on the mirror and focussed on a scale. As the lip moves back and forth the mirror is tipped about the wire holding it. The reflected image of the slit is drawn out into a band, whose width is proportional to the amplitude of vibration of the hook and the end of the wire.

One end of the specimen to be measured is fastened to the hook (Fig. 1) and the other end to the brass cylinder. A slight tension is put on the wire. The mirror is moved into such a position that an image of the slit is thrown on the scale. The oscillator is started and the wire forced into longitudinal vibrations, care being taken to avoid transverse vibrations. The amplifier is then switched on and the galvanometer reading noted. Then a small direct current is sent through the wire and the galvanometer reading again noted. This is repeated for different values of the direct current, which never exceeds a tenth of an ampere. The purpose of the several measurements is to decrease the effect of the errors from irregularities in the operation of the amplifier and those due to outside disturbances. Meanwhile the width of the band of light on the scale is observed from time to time. Galvanometer readings are taken only for equal amplitudes of vibration of the end of the wire, as shown by the width of the band of light. Some wires give considerable trouble in that the amplitude of vibration must be carefully adjusted to avoid breaking the wire in case transverse vibrations are set up.

After enough readings are taken the oscillator is stopped and the length of the wire and distance of the mirror axis from the wire are measured. The resistance of the wire is found with a wheatstone bridge.

Materials Tested and Their Method of Preparation

All the wires were either 0.004″ or 0.005″ in diameter, and all were obtained C.P. from Baker and Co., except molybdenum, tungsten, manganese, constantan, and nichrome. Molybdenum was from the Fansteel Products Co., tungsten from the Lamp Works of the General Electric Co. at Nela Park, manganin (84 Cu 4 Ni 12 Mn), constantan (60 Cu 40 Ni) and nichrome (85 Ni 15 Cr) from the Driver-Harris Co.

Annealing was effected by passing a current through the wire, except in the cases of aluminium and copper. Silver, gold, palladium, and platinum were annealed in air for an hour at a temperature about 300° below their melting points. Aluminium and copper were maintained for two hours at a temperature of 400°C in a steam bath. Molybdenum, tungsten, manganin, and constantan were annealed in an atmosphere of hydrogen for one hour; manganin and constantan at 700°C, molybdenum at 2000°C, tungsten at 2500°C.

Bismuth, lead, tin and nichrome were not annealed; the first three are self-annealing and nichrome has such a high resistance that a large enough current could not be obtained with the power supply available.

Tin and bismuth were the most difficult to measure both on account of their softness and because the change in potential drop would not remain constant. In the results for these two wires abnormally large errors may be found.

Measurements and Calculations

Length of wire, distance of scale from mirror, width of light band, resistance of wire, direct current through wire, and galvanometer deflections are the only measured quantities. The first three give the fractional change in length of the wire; the last two enter in the calculations of the change of resistance. From the galvanometer deflection the potential difference impressed on the input of the amplifier is obtained and this equals the product of the direct current by the change in resistance. The fractional change in resistivity is calculated from the following relation:

$$\Delta R/R = (1 + 2w - K)\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho} \cdot$$

where w = Poisson's ratio, K = a constant and l = length of wire.

The factor $(1+2w)\Delta l/l$ is the change in resistance due to alteration of dimensions of the wire In most cases the values of Poisson's ratio were taken from tables.[10] For some metals this ratio was computed from the compressibility (k) and Young's Modulus (E) for the material. The formula used in $w=0.5-Ek/6$. Table I indicates how close the calculated values of

Table I.

Metal	$E\times 10^{-11}$	$k\times 10^{11}$	$Ek/6$	w (calculated)	w (observed)
Silver	7.90**	1.02	0.134	0.366	0.38*
Aluminium	7.05**	1.38	.162	.338	.34*
Gold	8.0**	.603	.0805	.42	.42*
Cadmium	5.0	2.27	.203	.297	.30*
Copper	12.3	.756	.155	.345	.35*
Iron	21.3	.606	.215	.285	.28*
Nickel	20.2	.542	.182	.318	.31*
Lead	1.62	2.38	.064	.436	.45*
Palladium	11.3	.536	.101	.40	.40*
Platinum	16.8	.372	.104	.396	.39*
Tin	5.43	2.02	0.183	.317	.33*

* E. Grüneisen, Ann. d. Physik [4] **25,** 825–851 (1908).
** Kaye and Laby's Tables. E = Young's Modulus in c.g.s. units, k = compressibility in c.g.s. units, and w = Poisson's ratio.

the ratio are to observed values. All the values of the constants used were taken from the International Critical Tables unless otherwise noted. Young's

[10] E. Grüneisen, Ann. d. Physik [4] **22,** 801–851 (1907).

Moduli for molybdenum and nichrome were measured for this investigation with an apparatus constructed by Mr. L. R. Jackson of this laboratory.

The term $-K\Delta l/l$ is the change in resistance produced by a change in temperature of the wire as it is stretched and released. When a wire is suddenly stretched there will be a fall in temperature if the metal expands when heated and this temperature change is reversible.[11,12] The change in temperature is given by the formula:

$$\Delta T = -TaP/C_pd$$

The fractional change in resistance due to this change in temperature is

$$\Delta R/R = -K\Delta l/l$$

$$K = abTE/C_pd$$

In these formulae T = absolute temperature; a = thermal coefficient of expansion; P = applied tension; C_p = specific heat; d = density; b = temperature coefficient of resistance; and E = Young's Modulus.

Table II contains the values of w and K used, together with the computed values of $\Delta R/R$, $\Delta\rho/\rho$, and $\Delta\rho/\rho$ divided by $\Delta l/l$.

TABLE II.

Metal	w	K	$\frac{\Delta l}{l}\times 10^4$	$(1+2w-K)\frac{\Delta l}{l}\times 10^4$	$\frac{\Delta R}{R}\times 10^4$	$\frac{\Delta\rho}{\rho}\times 10^4$	$\frac{\Delta\rho}{\rho}\div\frac{\Delta l}{l}$
Silver	0.38	0.68	0.274	0.295	0.86	0.565	2.06
Aluminium	.34	.66	.80	.816	.274	−.542	−.68
Gold	.42	.563	.211	.27	.75	.48	2.27
Bismuth	.34	.40	.229	.294	−.50	−.794	−3.52
Copper	.35	.677	.629	.64	1.64	1.00	1.59
Molybdenum	.33	.402	.408	.514	.963	.449	1.10
Lead	.45	.424	.236	.349	.613	.264	1.12
Palladium	.40	.44	.327	.445	1.33	.885	2.70
Platinum	.39	.45	.395	.525	1.89	1.36	3.44
Tin	.33	.244	.216	.320	1.54	1.22	5.65
Tungsten	.32	.76	.827	.73	1.94	1.21	1.46
Zinc	.30	1.05	.172	.094	.638	.544	3.16
Manganin	.33	.00	.498	.83	.28	−.55	−1.10
Constantan	.33	.00	.517	.86	.735	−.125	−.246
Nichrome	.30	.02	.35	.55	.928	.378	1.08

The probable error in the resistance measurement, in the length of the wire, usually about sixty centimeters, in the distance of the mirror to the scale, and in the direct current is less than one-half of one percent in each case. The width of the light band, from one to four centimeters, was measured with a probable error of from one to three percent, being higher for smaller widths. The resultant error in the fractional change in length is about of the same magnitude. The probable error in measuring the alternating potential difference is dependent on the accuracy with which the amplifier has been calibrated and on the steadiness of the galvanometer deflections.

[11] W. V. Houston, Introduction to Mathematical Physics, p. 192.

[12] P. G. Nutting, Jour. Wash. Acad. of Sci. **19**, 109–115 (1929).

The calibration is accurate to one percent, and the precision of the galvanmeter reading is within two percent except in the measurements on bismuth and tin. For these metals, the galvanometer deflections were so unsteady as to cause a probable error of about ten percent in the potential difference measured.

Into the calculations there enter the thermal expansion coefficient, the temperature coefficient of resistance, the specific heat, Young's modulus and the density, all in correction terms, while Poisson's ratio enters directly. The uncertainty in the values of these constants varies considerably from metal to metal. The constants entering indirectly contribute about two percent to the uncertainty while Poisson's ratio can not be relied on within five percent. Thus the final results have errors of about seven percent due to constants used in calculating. Improved values of these constants will permit correspondingly greater accuracy in the final results. The measured quantities contribute about three percent more so that the possible error of the results is about ten percent. In the case of bismuth and tin the possible error is about 20 percent.

TABLE III. $\Delta\rho/\rho$ *for a tension of 1 kg/cm² by different observers.*

Metal	Bridgman	Tomlinson	Rolnick
Aluminium	$+1.8\times10^{-6}$	-0.63×10^{-6}	-0.96×10^{-6}
Silver	+3.4	+2.07	+2.51
Gold	+3.87		+2.86
Bismuth	−36.5		−32.2
Copper	+1.33	+.88	+1.32
Lead		+9.75	+7.0
Palladium	+1.66		+2.44
Platinum	+1.56	+2.25	+2.08
Tin		+5.89	+10.6
Zinc		+2.75	+3.26
Manganin	−.76		−.91

For bismuth, W. E. Williams gives $\Delta R/R=-53.5\times10^{-6}$ while the writer obtains $\Delta R/R=-28.1\times10^{-6}$.

In Table III is a comparison of my results with those of other investigators. These results have all been reduced to fractional changes of resistivity with a tension of 1 kg/cm², using the values of Young's Modulus mentioned above. Considering the wide variation between the results of other workers, the present results tally fairly well with theirs.

The writer is deeply indebted to Professor L. W. McKeehan for his suggestion of the problem and for his advice and encouragement during its progress.