# THE MARGULES METHOD OF MEASURING VISCOSITIES MODIFIED TO GIVE ABSOLUTE VALUES

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Abstract

**Measurement of absolute viscosities.** A method for determining absolute viscosities in a Margules rotating cylinder type viscometer, without the aid of calibrating liquids of known viscosities, is described. This method involves the determination of true viscosity by extrapolating apparent viscosities for several lengths of inside cylinder to that viscosity corresponding to infinite length. By this method the viscosity of the commercial castor oil used is found to be 9.99 poises at 20°C, and 4.61 poises at 30°C, as compared with 9.86 and 4.51 poises quoted in the Smithsonian Tables for pure oil at corresponding temperatures.

**End corrections.** The additional length to be applied to the measured length to correct for finite dimensions is computed for several inside cylinders or spindles. When the radius of the outer containing cylinder is 3.2 cm, and the ends of the spindle are 1.5 cm from the upper and lower boundaries of the liquid, the corrections for spindle radii of 0.556 cm and 0.477 cm are found to be 0.62 cm and 0.52 cm respectively as long as the length of the spindle is 5 cm or greater. Accordingly, the end correction is apparently proportional to the 1.18 power of spindle radius.

**Constancy of calibrating factor.** Comparisons are also made between relative viscosities measured by this method and those for the same liquids measured by capillary flow. Results indicate that within experimental error these relative values are the same by both methods, for viscosities between 5 and 3500 poises, showing that the calibrating factor of the concentric cylinder system is constant over this range of viscosities.

IN A previous paper,[1] the apparatus and method were described for measuring viscosities of molten glasses ($10^2$ to $10^7$ poises) by the use of concentric cylinders. A simple way of determining the effect of the use of cylinders of finite length was also described. However, since this determination was made for only one viscosity—that of castor oil at room temperature—the question remained as to whether one can assume that the calibrating factor is entirely independent of viscosity when the ends of the cylinders are present. Measurements were accordingly undertaken with the object of comparing the relative viscosities of two liquids differing quite widely in this property, as found by the concentric cylinders and as given by the method of capillary flow, for which a constant calibrating factor is more generally accepted.

In a majority of cases, investigators of glass viscosities[2] have found, by more or less rigorous calibration with standard liquids, that the calibrating factor is apparently independent of viscosity. On the other hand, in an

[1] H. R. Lillie, J. Am. Ceramic Soc. **12**, 505–529 (1929).

[2] S. English, J. Soc. Glass Tech. **8**, 205 (1924). Stott, Irvine and Turner, Proc. Roy. Soc. **A108**, 154 (1925). Proctor and Douglas, Proc. Phys. Soc. (London) **41**, 500 (1929). M. Volarovich, J. App. Phys. Moscow **5**, 185 (1928).

investigation of the viscosities of soda-lime glasses over a large field of compositions, Washburn[3] found that his factor existing between force per unit shear and viscosity apparently varied by a factor of about three over the range of viscosities dealt with. Comparisons made by others have shown that agreement between their results and those of Washburn can be reached only if his calibrating factor be changed to a constant or their own made variable.

Since in any rotation method the torque is simply proportional to the viscosity and to the velocity gradient at the very surface of the cylinder upon which the torque is exerted, a variation in the factor connecting torque and viscosity for a given total relative angular motion between the two cylinders, would indicate that the velocity gradient at the surface of the cylinder is not proportional to the total relative velocity alone. In other words, the velocity distribution would have to depend on viscosity. The result of this would be that if the temperature of a liquid, contained between a stationary cylinder and one rotating with a constant speed, be changed so as to alter its viscosity, some parts of the liquid would be thereby slowed down while others would rotate faster in order to change the velocity gradient to the cylinders' surfaces. This is only conceivable if centrifugal or other forces tending to disturb the regular motion of the liquid have more effect in the less viscous liquid. This may be true in the case of cylinders of finite length.

## Measurement by the Concentric Cylinder Method

### Mathematical procedure.

As reported in the previous papers, viscosities have been measured without the aid of actual calibration with standard liquids, simply by determining the magnitude of the "end effect" at some arbitrary viscosity and assuming a constant calibrating factor for all viscosities. The end effect was determined by first finding the apparent viscosity of the liquid, using inside cylinders or spindles of various lengths, by using the formula

$$T = \frac{4\pi\eta\Omega R_1^2 R_2^2 l}{R_2^2 - R_1^2} \tag{1}$$

where $T$ is the torque on the inner cylinder, $\eta$ the viscosity, $\Omega$ the angular velocity of the outer cylinder, $R_1$ and $l$ the radius and length of the inner cylinder and $R_2$ the radius of the outer rotating cylinder. These apparent viscosities were then plotted against reciprocals of spindle length and the "true" value of $\eta$ corresponding to infinite length found by extrapolation. A correction to $l$ was then found for each spindle. This correction is not the same for all lengths of spindles of the same diameter, supposedly because the amount of deformation of the flow surfaces around the body of the spindle depends on the length of the spindle itself.

Some minor changes have been made in the method of computation of apparent viscosity. We have the relations

[3] Washburn, Libman and Shelton, Univ. of Illinois, Eng. Expt. Sta., Bull. No. 140 (1924).

$$T = K\theta = \frac{KZD}{2d}$$

and

$$\Omega = \frac{2\pi}{t},$$

where $K$ is the constant of the suspension, determined as described in the previous paper, $\theta$ its angular displacement, $D$ the deflection as read on a scale at a distance $d$ from the mirror, $Z$ the correction for straight scale and $t$ the time for one revolution of the containing cylinder. Eq. (1) becomes

$$\frac{KZD}{2d} = \eta \frac{8\pi}{t}(\pi R_1^2 l)\frac{R_2^2}{R_2^2 - R_1^2},$$

or, for apparent viscosity,

$$\eta = \frac{ZDt}{d}\left[\frac{K}{16\pi V}\left(1 - \frac{R_1^2}{R_2^2}\right)\right] \tag{2}$$

where $V$ is the total volume of the spindle. The quantity in the brackets is a constant for any one spindle since a constant value of $R_2 = 3.2$ cm was used during the whole investigation. All apparent viscosities were computed from this equation.

Fig. 1 represents diagrammatically the experimental viscometer used. A water bath is contained in a large cylindrical can placed on the rotating table $V$ of the viscometer frame described previously. This can is surrounded by a galvanized water jacket $J$, the purpose of which is to maintain an external temperature below that of the bath. The outer cylinder of the measuring viscometer, $C$, containing the liquid under investigation, is supported at the center of the bath by a hollow pedestal of brass and is closed by a threaded top carrying a small upright tube which extends through the surface of the water bath and allows the inner cylinder or spindle $E$ to hang into the liquid. This spindle is held by a brass connector $D$ upon which is fastened the mirror $M$, all of which hangs from the calibrated steel wire suspension $B$. With the exception of spindle No. 53, described later, the two ends of the spindle were 1.50 cm distant from the surface of the oil and the bottom of the container.

The temperature is maintained constant by means of a vapor pressure regulator $R$ constructed as shown. Methyl formate was found to be the most satisfactory liquid for actuating the mercury column. Various temperatures can be obtained by adjusting the pressure of air in the top part of the regulator. The contacts of the regulator are connected in series with a small flashlight dry cell and a telegraphic relay, both of which are fastened to the transite cover piece of the rotating can. This relay makes and breaks the circuit through the heater $H$ consisting of three units of nichrome wire which can be connected in various ways depending on the temperature desired. The current for the heater is supplied through mercury cups at the extreme bottom of the viscometer frame. Circulation of the water bath is maintained

by a small toy electric motor directly connected to the shaft of the stirrer $A$. This stirrer is so built as to take water in from top and bottom and eject it tangentially at the center. Current for the motor is supplied by a large 4 volt storage battery, through brushes acting on rings around the outside of the jacket $J$. It was found necessary to house the motor completely to prevent disturbance of the spindle due to air currents from the armature. At times

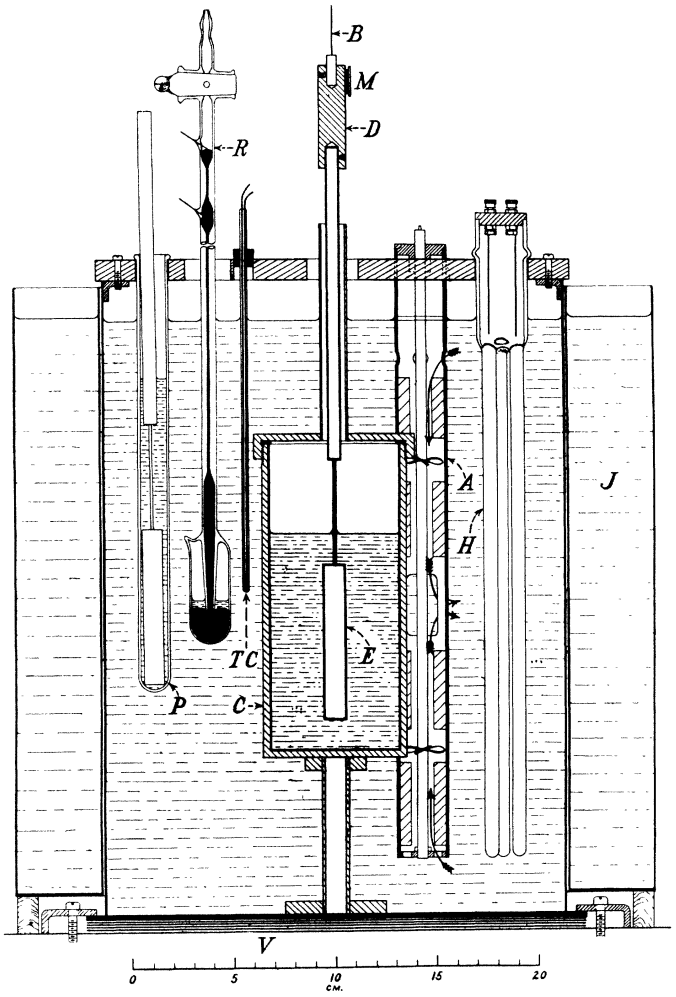


Fig. 1. Diagram of apparatus.

when various spindles were being used, one after another, a preheating tube $P$ was installed containing some of the same oil as was being used in $C$ so as to allow the next spindle to come to temperature before being placed in the central position. Temperatures were measured by means of three copper-constantan thermocouples placed as shown ($TC$) and connected in series through junctions kept at 0°C. A precision potentiometer accurate to less

than 1 microvolt was used for e.m.f. readings. These couples measured the average temperature of the bath to within 0.05°C and were sensitive to changes of less than 0.01°C. Their condition was checked periodically with a standard mercury thermometer to make sure that no short circuits were causing them to be far in error.

**Temperature control.**

Since all temperature readings made during actual viscosity measurements were for the water bath instead of for the liquid inside the viscometer proper, a few test runs were made to establish the relations between these temperatures. By placing a second set of couples in the center of the oil its temperature could be followed while the bath was cooling, heating, and just after it had come to equilibrium. In this way the following conditions were found:

Heater turned off: temperature lag 2.3°C, time lag 35 min.
Heater turned on: temperature lag 1.3°C, time lag 17 min.
(rate 0.08°C/min.)

Length of time after regulator began working, for oil to be
within 0.05°C of bath: 50 min.
within 0.01°C of bath: 70 min.

Accordingly, about two hours were always allowed, after the regulator began to function, before any viscosity readings were made. It was also observed that when the regulator had been in action for some time no periodic fluctuation of temperature in the oil could be detected.

**Effective spindle length.**

For determining "true" viscosities by extrapolating the apparent values for several spindles of equal radii but various lengths to the value corresponding to infinite length, five spindles of 3/8″ diameter and five of 7/16″ diameter were used. Their actual dimensions were:

| No. | $R_1$ | $l$ | $1/l$ | No. | $R_1$ | $l$ | $1/l$ |
|---|---|---|---|---|---|---|---|
| 101 | 0.4766 cm | 10.14 cm | 0.0986 | 106 | 0.5561 cm | 10.22 cm | 0.0979 |
| 102 | .4771 | 7.61 | .1314 | 107 | .5562 | 7.60 | .1315 |
| 103 | .4769 | 5.08 | .1969 | 108 | .5564 | 5.12 | .1953 |
| 104 | .4768 | 3.80 | .263 | 109 | .5561 | 3.82 | .262 |
| 105 | .4770 | 2.53 | .396 | 110 | .5561 | 2.54 | .394 |
| Mean | 0.4769 | | | Mean | 0.5562 | | |

The observed apparent viscosities found by the use of these spindles ($R_2$ constant and equal to 3.2 cm) are shown in Table I. The ratios of viscosity at 23.40°C and 28.20°C to that at 19.97°C, as given by each spindle, are also shown. It is quite evident that over this small range of viscosities relative results are independent of the spindle used. A mean of these ratios for the two higher temperatures was accordingly taken, weighting each value according to spindle length, and all the viscosities recomputed back to the basis

TABLE I

| Spindle No. | Observed Values | | | | | | | | | Mean $l'$ | Values read from curve (Fig. 2) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 19.97°C $\eta=10.020$ | | 23.40 $\eta=7.633$ | | | 28.20 $\eta=5.295$ | | | | | | | |
| | App. $\eta$ | $l'$ | App. $\eta$ | Ratio | $l'$ | App. $\eta$ | Ratio | $l'$ | | | App. $\eta$ | $l'$ | |
| 101 | 10.53 | 0.51 | 8.03 | 0.762 | 0.60 | 5.61 | 0.533 | 0.60 | | 0.55 | 10.53 | 0.51 | |
| 102 | 10.67 | .49 | 8.15 | .763 | .51 | 5.62 | .527 | .56 | | .52 | 10.70 | .52 | 0.517 |
| 103 | 10.99 | .49 | 8.41 | .766 | .52 | 5.86 | .534 | .54 | | .52 | 11.07 | .52 | |
| 104 | 11.29 | .48 | 8.57 | .760 | .47 | 5.92 | .525 | .45 | | .47 | 11.33 | .50 | |
| 105 | 11.72 | .43 | 9.10 | .776 | .49 | 6.17 | .526 | .41 | | .44 | 11.75 | .43 | |
| 106 | 10.63 | .63 | 8.09 | .761 | .61 | 5.63 | .529 | .63 | | .62 | 10.63 | .62 | |
| 107 | 10.85 | .63 | 8.28 | .762 | .64 | 5.75 | .529 | .66 | | .64 | 10.84 | .62 | 0.620 |
| 108 | 11.27 | .64 | 8.54 | .757 | .60 | 5.90 | .523 | .58 | | .61 | 11.24 | .62 | |
| 109 | 11.65 | .62 | 8.83 | .757 | .59 | 6.09 | .522 | .57 | | .59 | 11.61 | .60 | |
| 110 | 12.19 | .55 | 9.21 | .756 | .52 | 6.43 | .527 | .54 | | .54 | 12.16 | .53 | |
| | | | Mean ratios | .7618 | | | .5284 | | | | | | |

of 19.97°C. This process gave the points plotted in Fig. 2. Since for a given radius of spindle the relation between apparent viscosity and reciprocal of length appeared to be a linear one for values of $l$ greater than 5 cm, this was assumed to be true and the apparent viscosities as given by the six longest spindles were used for determining the best pair of straight lines intersecting

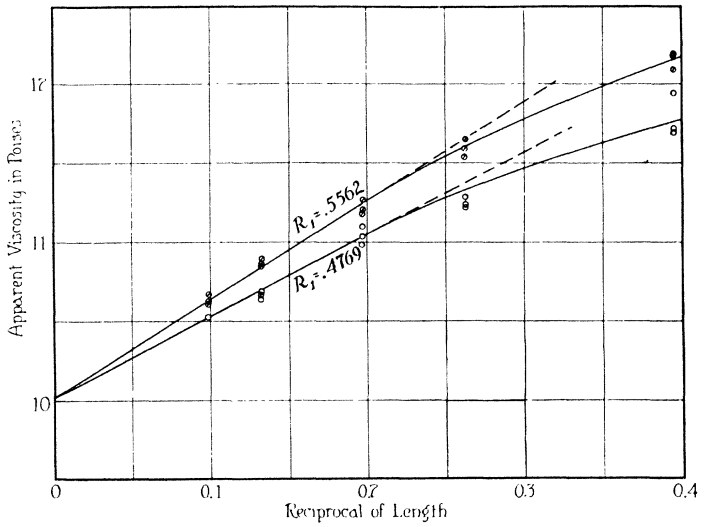


Fig. 2. Apparent viscosities of castor oil. Spindles 101–110 inclusive. Converted to 19.97°C

at $1/l=0$. In so doing it was discovered that the sum of the deviations become minimum for the two lines separately as well as collectively when the intercept was made at 10.020 poises. The average deviation in this case was about 0.3 percent, using only those spindles of length greater than 5 cm. The curved part of each line was drawn by estimate.

Using the extrapolated intercept as the true viscosity at 19.97°C and the mean ratios shown in Table I for computing the corresponding values at the

two higher temperatures, the following comparison is made with viscosities of pure castor oil as given by the Smithsonian Tables:

| Temp. | Observed | Sm. T. |
|---|---|---|
| 19.97°C | 10.020 poises | 9.92 poises |
| 23.40 | 7.633 | 7.42 |
| 28.20 | 5.295 | 5.14 |
| $\Delta \log \eta/\Delta 1/T$ | 297 | 302 |

Fig. 3 shows these two sets of data plotted as logarithms against the reciprocal of absolute temperature. This method of plotting was used in order to obtain a linear relation, thus facilitating interpolation. The figure includes also a similar curve for values proportional to viscosity as obtained later by capillary flow.

We now have sufficient means for computing some sort of correction to be applied to each spindle to account for the additional torque exerted by virtue of its finite length. Since others have assumed that this may be expressed in terms of an additional length of spindle which remains constant for a constant radius, it will be so computed in this case. If $l'$ is this additional length, we have

$$l' = l\left(\frac{\text{app. } \eta}{\text{true } \eta} - 1\right).$$

Values for this correction computed from observed apparent viscosities and for those read from the curves in Fig. 2 have been included in Table I. For lengths greater than 5 cm a constant correction is obtained for each radius as follows:

$$R_1 = 0.5562 \text{ cm}, \qquad l' = 0.620 \text{ cm}$$
$$R_1 = 0.4769 \text{ cm}, \qquad l' = 0.517 \text{ cm}$$

from which we have the relation:

$$l' \text{ proportional to } (R_1)^{1.18},$$

indicating that the end correction when stated in terms of additional spindle length varies approximately as the first power of radius.

**Temperature runs.**

In order to determine more accurately the temperature viscosity relations for the oil, a run was made with each of four spindles, in each case leaving the spindle undistrubed while the temperature was varied. For this purpose two spindles, No. 101 and No. 107, from the previous runs were used, together with two others of special sizes.

*Spindle No. 53.* In a previous trial of various spindles with specially shaped ends it was found that one with conical ends showed the least variation of torque with variation of the distance from the bottom of the oil to the end of the spindle. Many subsequent determinations of glass viscosities

were made using such a spindle, consisting of platinum-iridium and having the following dimensions:

$$R_1 = 0.477 \text{ cm}, \quad l = 3.82 \text{ cm}, \quad V = 3.010 \text{ cc},$$

where $l$ is the length of the cylindrical portion. Each end of the large portion of the spindle terminated in a 90° cone, while the whole was supported by a stem of about 0.16 cm diameter. A duplicate of this spindle was made of iron and called No. 53. Its distance of 1.5 cm from the upper and lower boundaries of the oil was measured to its cylindrical portion instead of to its pointed ends.

*Spindle No. 152.* Another spindle used considerably in glass is the one of "sillimanite" described in the previous paper on glass viscosities. It is nearly the same as spindle No. 101 but is held by a porcelain stem of 1/4″ double-bore thermocouple tubing. A similar one was made of brass of dimensions:

$$R_1 = 0.5151 \text{ cm}, \quad l = 10.18 \text{ cm}, \quad R' = 0.3174 \text{ cm}, \quad V = 8.960 \text{ cc},$$

where $R'$ is the radius of the stem. This was called spindle No. 152.

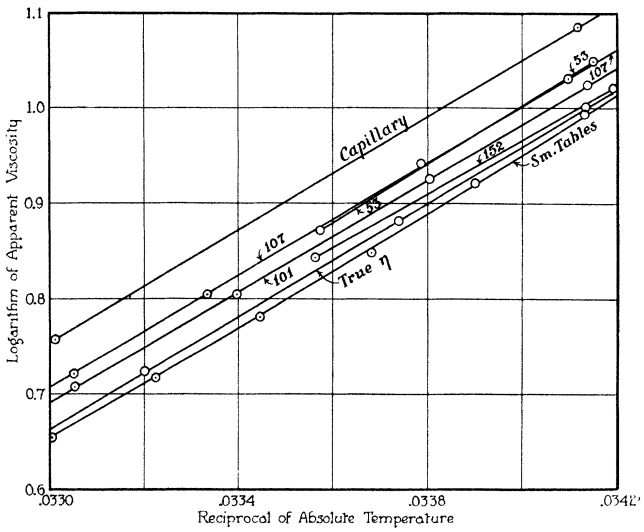


Fig. 3. Temperature runs in castor oil.

Table II gives the results for apparent viscosity of the castor oil obtained with these four spindles. The values for 20°C were interpolated on the curves in Fig. 3.

It is quite evident that the result obtained with spindle 152 at 19.47°C is in error since the slope of the curve for this spindle is less than for any of the others. Also, the similarity of No. 152 to No. 101 suggests an apparent viscosity at 20°C more nearly equal to 10.5. However, this trouble was not discovered until after the viscometer was dismantled and no redetermination was made.

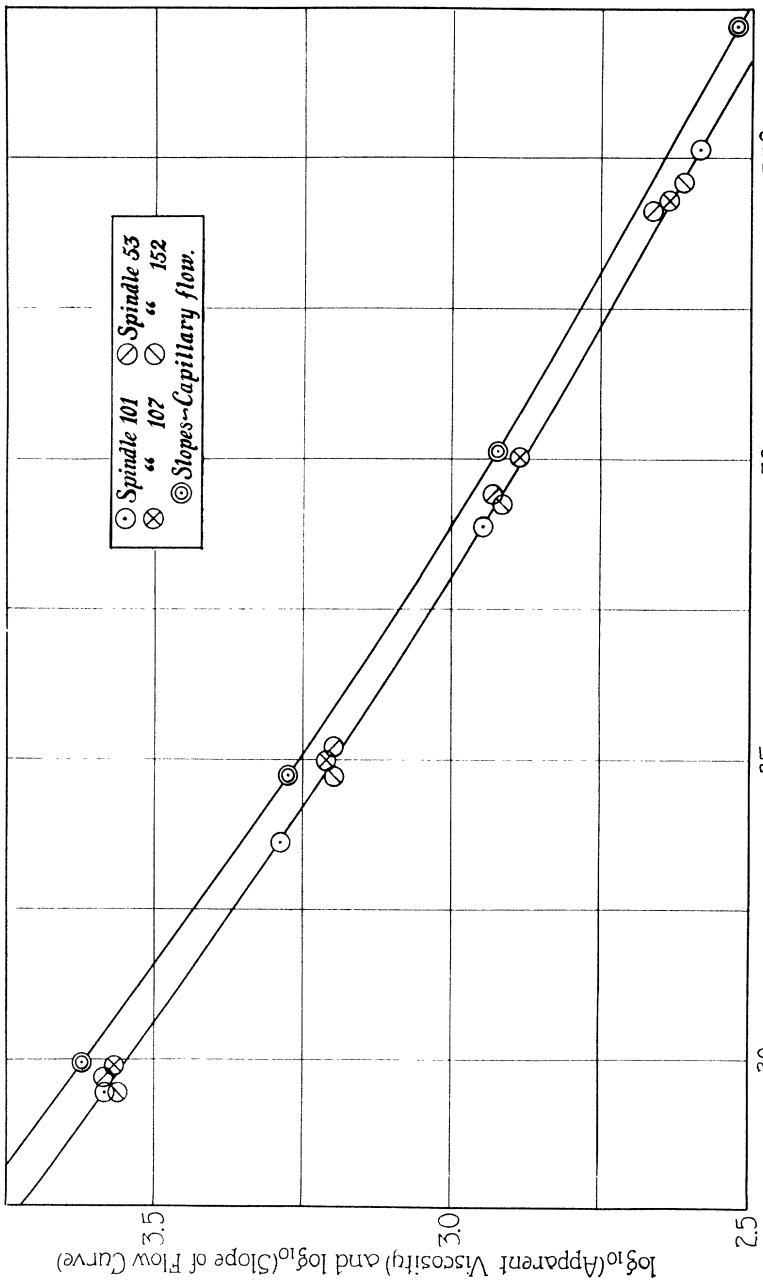


Fig. 4. Apparent viscosities of glucose solution.

TABLE II.

| No. 101 | | No. 107 | | No. 53 | | No. 152 | |
|---|---|---|---|---|---|---|---|
| Temp. | App. $\eta$ | Temp. | App. $\eta$ | Temp. | App. $\eta$ | Temp. | App. $\eta$ |
| 19.94 | 10.57 | 20.00 | 10.99 | 19.84 | 11.20 | 19.47 | 10.52 |
| 20.00 | 10.50 | 20.30 | 10.74 | 20.00 | 11.04 | 20.00 | 10.10 |
| 22.83 | 8.44 | 23.00 | 8.77 | 24.87 | 7.45 | 24.97 | 6.97 |
| 26.46 | 6.38 | 26.99 | 6.36 | | | | |
| 29.55 | 5.10 | 29.59 | 5.26 | | | | |

**Measurements in glucose solution.**

For the purpose of measuring relatively high viscosities, a supply of heavy glucose solution was obtained from a candy making concern. At the time it was drawn from the large supply tank this solution was placed in several pint jars and kept tightly sealed until used for the various runs. It was found in preliminary tests that evaporation could be completely stopped and surface film thereby avoided by covering the surface of the glucose with a thin layer of oil. Observation also showed that the oil had no tendency to creep between the glucose and its container, while a sharp surface always showed between the two liquids. Accordingly, as soon as the glucose had been placed in the cylindrical container of the viscometer and the spindle had been put in place, a small quantity of oil was added. As a further test of constant concentration of the solution, the first temperature used in each viscosity run was repeated at the end of the run. Good checks were consistently obtained.

TABLE III.

| No. 101 | | No. 107 | | No. 53 | | No. 152 | |
|---|---|---|---|---|---|---|---|
| Temp. | App. $\eta$ | Temp. | App. $\eta$ | Temp. | App. $\eta$ | Temp. | App. $\eta$ |
| 19.45°C | 3836 | 19.90 | 3707 | 19.70 | 3842 | 19.45 | 3650 |
| 23.64 | 1940 | 25.00 | 1627 | 25.24 | 1576 | 24.74 | 1581 |
| 28.88 | 885 | 30.04 | 771 | 29.43 | 854 | 29.25 | 823 |
| 35.15 | 385 | 34.30 | 436 | 34.20 | 447 | 34.60 | 411 |

Table III gives the observed apparent viscosities of the glucose as found by the four spindles used in the similar runs in castor oil. Fig. 4 shows these results plotted in terms of their logarithms against temperature. The line drawn through the points is for spindle No. 101. The other three spindles would give three curves parallel to and near this line.

TABLE IV

| Spindle No. | Apparent $\eta$ for glucose | | | | Oil at | $\eta$ glucose $\div \eta$ oil at 20° | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 20° | 25° | 30° | 35° | 20°C. | 20° | 25° | 30° | 35° |
| 101 | 3506 | 1574 | 760 | 392 | 10.50 | 334.0 | 150.0 | 72.4 | 37.3 |
| 107 | 3608 | 1630 | 777 | 396 | 10.99 | 328.2 | 148.3 | 70.7 | 36.0 |
| 53 | 3658 | 1637 | 789 | 410 | 11.04 | 331.0 | 148.1 | 71.4 | 37.7 |
| 152 | 3342 | 1521 | 744 | 385 | 10.10 | 331.0 | 150.7 | 73.6 | 38.1 |
| | | | | | Mean | 331.1 | 149.3 | 72.0 | 37.3 |
| | | | | | | ±0.5% | ±0.7% | ±1.4% | ±1.7% |

Table IV gives values read from this family of curves corresponding to the various temperatures stated. From these values have been computed the viscosities of the glucose solution relative to castor oil at 20°C, as determined with the same spindles (Table II).

## Measurement by Capillary Flow Method

The second part of the investigation consisted of measuring the relative viscosities of castor oil and glucose solution over the same range of temperatures by capillary flow. The same capillary viscometer was used for both liquids and the rates of flow so regulated as to make the maximum rates of shear as nearly as possible the same as those which prevailed in the cylinder method.

**Apparatus.**

The thermostat used was essentially the same as that described above for the rotating cylinder apparatus, with the exception of one slight change. A side tube was sealed into the regulator just above the top of the mercury column, this tube terminating in a large bulb immersed in the water bath. The purpose of this was to reduce further the effect of changing room temperature on the pressure over the column.

Fig. 5 is a sketch of the viscometer. Two similar reservoirs $r_1$ and $r_2$ are connected by the capillary $C$ selected for roundness and uniformity of bore. The dimensions of the bore were found to be:

$$R = 0.1175 \text{ cm}; \; R^4 = 1.905 \times 10^{-4} \text{ cm}^4; \; L = 9.992 \text{ cm}$$

The ends of the capillary were ground and polished perpendicular to the axis. DeKhotinsky cement was used for fastening into the reservoirs, care being taken to have the two sides as symmetrical as possible. To avoid slipping when the higher temperatures and pressures were used, the bottom part of the viscometer was buried in plaster of Paris as shown. The reservoirs are fitted with screw tops $A$ made of brass. The tubes $t_1$ and $t_2$ lead to a system of stopcocks which allow either tube to be connected to pressure or suction systems or to the volumetric manometer at will. This manometer is constructed as shown at the right of Fig. 5. The left-hand leg $a$ is graduated in cm and has been calibrated for volume. The liquid used in diphenyl oxide, chosen for its low viscosity and low vapor pressure. By keeping the levels equal in tubes $a$ and $b$ by means of the counter columns of mercury in $c$ and $d$, the volume of flow through the capillary $C$ can be followed and measured. This flow is corrected for temperature differences between the bath and the room by the following:

$$F = V(T_1/T_2)$$

where $F$ is the actual flow through $C$, $V$ the volume registered in the manometer, $T_1$ and $T_2$ the absolute temperatures of bath and room respectively.

The pressure system contains a large carboy for holding the pressures constant during measurements.

**Measurements for castor oil.**

*Technique:* For creating such a small rate of flow as was desired, very small pressures were necessary. This meant that a difference in level in the two reservoirs would have an effect on the rate of flow. The following method was used for avoiding error from this source: Both reservoirs were first opened to

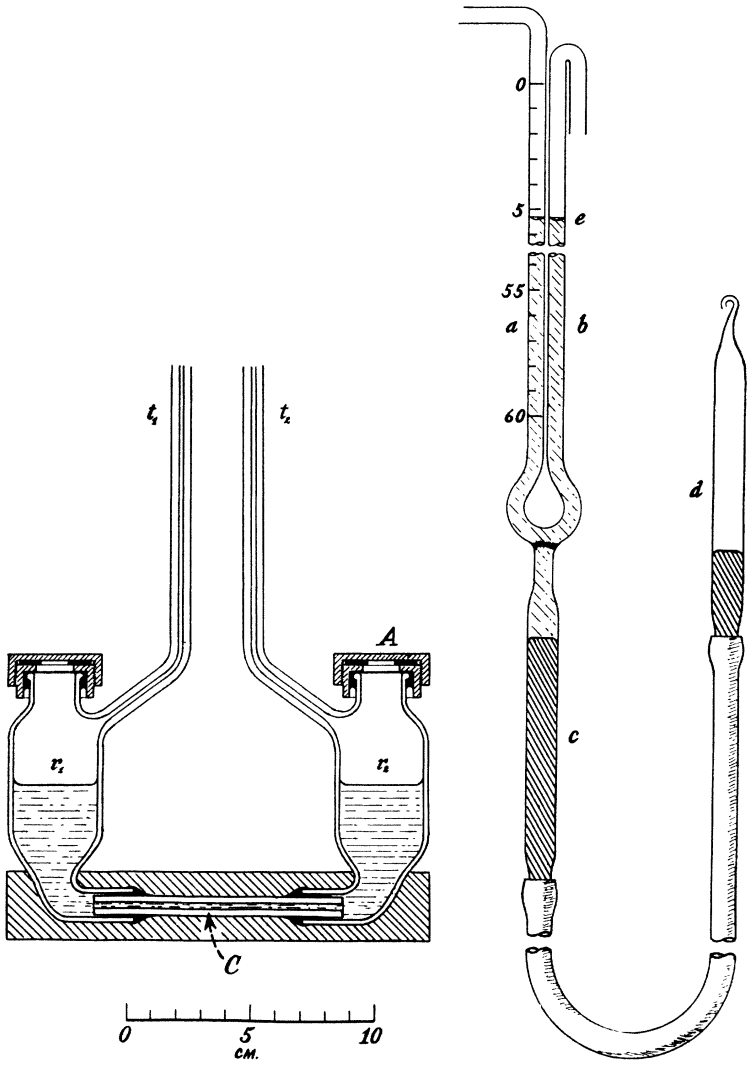


Fig. 5. Sketch of viscometer.

atmospheric pressure and the liquid allowed to come to rest. One tube, e.g. $t_1$, was then connected to the volumetric manometer, the columns $a$ and $b$ being first adjusted to about the 30 cm mark. A slight suction was then caused in $r_2$ and the oil made to flow back until the colunns $a$ and $b$ reached the 10 cm mark or thereabouts. At this time $r_2$ was connected to the pressure

system and, keeping atmospheric pressure in $r_1$ by lowering $d$ to keep $a$ and $b$ on a level, time readings were made for each 2 cm drop in $a$ by means of a split-time watch. These time readings were then plotted against the corresponding column positions, giving a slightly curved line whose slope at 30 cm was the rate of flow corresponding to zero hydrostatic pressure.

In the case of castor oil, pressures were measured by means of a Nujol manometer and later converted to the usual cm of mercury units.

*Results:* The actual numerical results will not be quoted here. Fig. 6 shows them graphically, the two kinds of points plotted representing the two

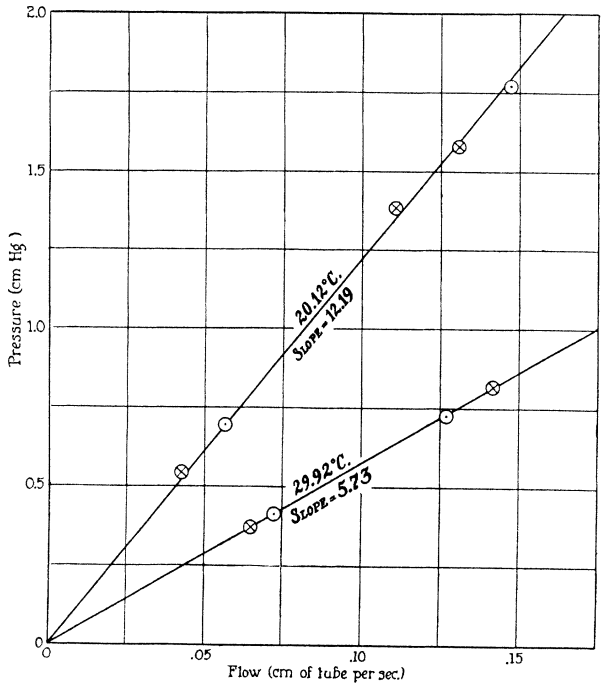


Fig. 6. Pressure-flow relations for new castor oil.

directions of flow through the capillary. It is evident that the system is quite nearly symmetrical and that the relation between rate of flow and pressure is a linear one. The slopes are:

| | |
|---|---|
| 29.92°C | 5.73 |
| 20.12 | 12.19 |
| 20.00 (extrapolated) | 12.31 |

These slopes, proportional to viscosity, have been plotted in Fig. 3. The value obtained in this case for $\Delta \log \eta/\Delta(1/T)$ is 297 as compared with 297 for the concentric cylinders and 302 for values from the Smithsonian Tables. This indicates that the oil itself differs slightly from that for which the viscosities in the Tables are quoted.

**Measurements for glucose solution.**

*Technique.* In order to have about the same rates of flow in the glucose solution as in the oil, very much higher pressures were used. This fact made the effect of difference of level between the two reservoirs negligible in comparison with the other forces acting. It was, therefore, unnecessary to carry out the procedures as outlined above, the flows being now measured simply by taking the total time for a certain flow—usually 10 cm in the volumetric manometer.

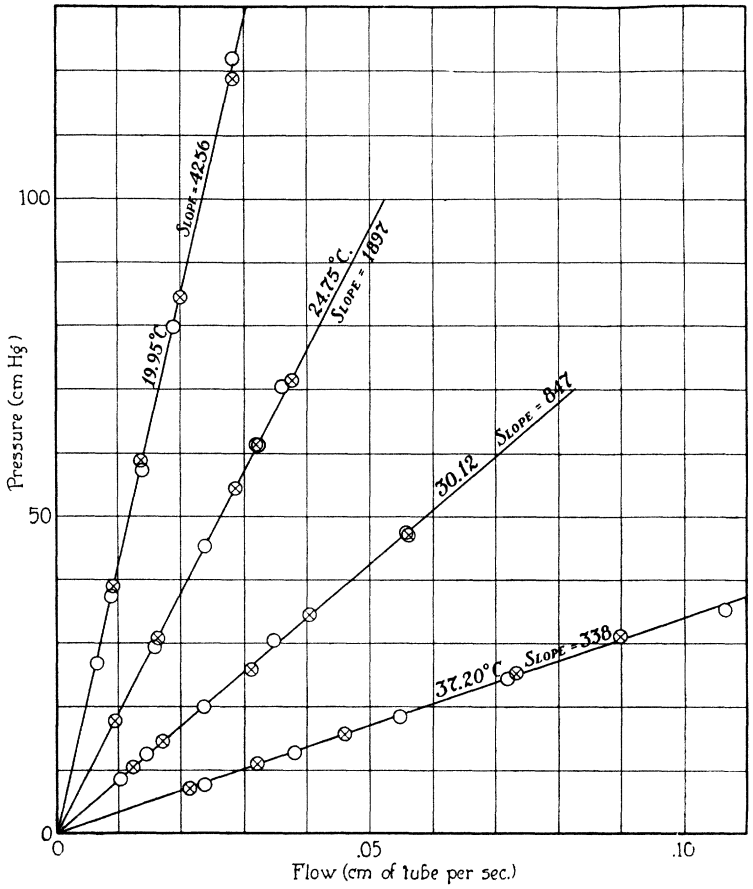


Fig. 7. Pressure-flow relations for glucose solution.

A covering of oil over the glucose was used as before and bubbles were eliminated by warming and slightly reducing the pressure for a very short time. In this case mercury was used in the pressure manometer. No suction was used over the glucose at any time during the measurements. It was found necessary to use a soft wax in the stopcocks of the pressure system. Pressure was obtained from a cylinder of nitrogen by the use of a reducing valve, a stopcock being closed between the pressure system and the reducing valve during the flow observations.

*Results.* Fig. 7 shows the pressure-flow relations for the glucose solution. As in the case of castor oil, it is evident that the two directions of flow agree and that the relations are represented by straight lines. It may be said in this connection that any small difference observed in apparent viscosity as the pressures were decreased seemed to be toward lower values instead of toward high ones as would be expected if plasticity were present in the glucose.

The slopes of the observed pressure-flow lines are:

| | | | |
|---|---|---|---|
| 37.20°C | $338 \pm 4$ | 24.75°C | $1897 \pm 21$ |
| 30.12 | $847 \pm 11$ | 19.95 | $4256 \pm 53$ |

These values, proportional to viscosity, are plotted logarithmically in Fig. 4. Values for the reference temperatures, interpolated by means of this graph, are shown in Table V, together with viscosities relative to castor oil at 20°C, as obtained by the two methods.

TABLE V.

| Temp. | Interpolated slope | Oil at 20°C | $\eta$ relative to castor oil at 20° | | Deviation of cylinders from capillary |
|---|---|---|---|---|---|
| | | | capillary | cylinders | |
| 20°C | $4220 \pm 53$ | 12.31 | $342.5 \pm 1.3\%$ | $331.1 \pm 0.5\%$ | $-3.4\%$ ($\pm 1.8\%$) |
| 25 | $1824 \pm 21$ | | $148.1 \pm 1.1\%$ | $149.3 \pm 0.7\%$ | $+0.8\%$ ($\pm 1.8\%$) |
| 30 | $865 \pm 11$ | | $70.3 \pm 1.3\%$ | $72.0 \pm 1.4\%$ | $+2.4\%$ ($\pm 2.7\%$) |
| 35 | $446 \pm 6$ | | $36.2 \pm 1.3\%$ | $37.3 \pm 1.7\%$ | $+3.1\%$ ($\pm 3.0\%$) |

The table also gives the deviations of values of relative viscosity as obtained by the cylinders from those by capillary flow. The figure in parentheses following each of these deviations is the sum of the estimated probable errors for the two methods taken to represent the probable error to which the final comparison is subject. The fact that this deviation is zero for about 2000 poises, while it must be zero also for the viscosity of the oil, indicates that errors in the measurements themselves, and in the various graphical interpolations made, are responsible for all the deviations. If this can be considered true it may be said that the calibrating factor of the concentric cylinders remains constant over the range of viscosities between 5 and 3500 poises. In any case the deviation from constancy is very small.

**Rates of shear.**

In the rotating cylinder system, the expression for rate of shear is:

$$s = \frac{2\Omega R_1^2 R_2^2}{r^2(R_2^2 - R_1^2)}.$$

This reaches a maximum when $r$ is its minimum value $R_1$, or maximum rate of shear

$$S = \frac{2\Omega R_2^2}{R_2^2 - R_1^2} = 2.06\Omega \text{ approximately.}$$

The values of $\Omega$ used were from 0.233 to 0.524 radians/sec. The maximum rates of shear, then, ranged from

$$S = 0.48 \text{ to } 1.08 \text{ cm/sec/cm.}$$

In the capillary flow sytem, rate of shear is expressed by

$$s = \frac{-4rQ}{\pi R^4}$$

which reaches a maximum numerical value at $r = R$, or

$$S = \frac{-4Q}{\pi R^3}.$$

In the experiments reported above, $Q$ was expressed in terms of cm of tube in the volumetric manometer. To convert the values to cc/sec. we must multiply the numbers by the volume of tube per cm (0.1131). This gives values of $Q$ from 0.0045 to 0.017 for oil and from 0.0011 to 0.0113 for the glucose. Using $R^3 = 1.62 \times 10^{-3}$ we have $S = 3.54$ to 13.4 for oil and 0.86 to 8.9 for glucose. Although these limits do not correspond exactly to those found for the cylinders, they are of the same order of magnitude and actually overlap to some extent.

**Capillary end correction.**

The equation for viscosity by the capillary flow method is, since the kinetic energy correction is negligible,

$$\eta = \frac{\pi P R^4}{8Q}\left(\frac{1}{L+\lambda}\right)$$

where $P$ is pressure in dynes/cm$^2$, $Q$ volume rate of flow and $R$ and $L$ the radius and length respectively of the capillary. $\lambda$ represents a length that must be added to $L$ to give the effective length of the constriction through which the liquid must flow.

The viscosity of the castor oil at 20°C, found by the concentric cylinders is 9.99 poises. The slope $P/Q$ found as 12.31 by capillary flow is in terms of cm of mercury per unit velocity of the column in the volumetric manometer. It is equivalent to $1.445 \times 10^6$ in c.g.s. units. Then we have

$$L + \lambda = \frac{\pi \times 1.905 \times 10^{-4} \times 1.445 \times 10^6}{8 \times 9.99}$$

$$= 10.83$$

But since $L = 9.99$ cm,

$$\lambda = 0.84 \text{ cm or about } 3.5 \text{ diameters.}$$

The experience of others has shown that $\lambda$ is usually of the order of a few diameters.

**Acknowledgment.**

In conclusion, the writer wishes to express his appreciation for the assistance rendered by Dr. G. S. Fulcher in the preparation of this paper.