THE PRINCIPAL MAGNETIC SUSCEPTIBILITIES OF BISMUTH SINGLE CRYSTALS

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(Received June 9, 1930)

Abstract

The Gouy method is used to determine the principal magnetic susceptibilities of bismuth single crystals grown by the method developed by Goetz. The specific susceptibility is shown to be a constant in all directions perpendicular to the principal crystallographic axis with a value of -1.487×10^{-6}. Parallel to the main axis the specific susceptibility is a minimum and has the value -1.046×10^{-6}. The mean is given then as -1.340×10^{-6}

I. Historical

Recent investigations by Goetz,[1] Goetz and Hasler,[2] and Goetz and Focke,[3] concerning the effects of allowing one half of a single crystal of bismuth to crystallize in a strong magnetic field, have shown that different orientations are affected in different ways. The effects are a maximum if the principal axis is perpendicular to the field and a minimum if the axis is parallel to the field. This at once leads to the conclusion that if the susceptibility is a contributing factor in the effect, it must also have a maximum perpendicular to the main axis and minimum parallel to it. This supposition is sustained by the results obtained by Nusbaum.[4] There is however an objection to accepting the absolute values of the results which he has reported since the value given by him as the maximum is still below that given in tables for the susceptibility of the heterogeneous crystal aggregate. Because of this, it was felt that a new determination of the principal susceptibilities of bismuth should be made.

II. Preparation of Crystals

The bismuth used in the crystals described in this paper was obtained from the Braun Corporation and subsequently vacuum distilled in an effort to purify it.

The single crystals were made by the method developed by Goetz.[1] Crystals are grown in this method, by moving a rod of the metal which is lying in a graphite trough through a furnace at such a speed that once a crystal begins to form, it can continue its growth throughout the entire rod, the orientation of the crystal being determined by innoculation. The orientations used were P_1 (principal axis perpendicular to the length of the rod) and P_3 (principal axis parallel to the length of the rod). The

[1] A. Goetz, Phys. Rev. **35**, 193 (1930).
[2] A. Goetz and M. Hasler, to appear in Phys. Rev.
[3] A. Goetz and A. Focke, to appear in Phys. Rev.
[4] C. Nusbaum, Phys. Rev. **29**, 905 (1927).

accuracy of the orientations could be checked after magnetic observations were taken by cleaving the crystal along the basal cleavage plane. In the case of the P_1 crystals, this plane was parallel to the rod making it possible to obtain a very accurate determination of the orientation. The crystals used were all correct to less than ten minutes of arc. The P_3 orientation could not be checked so accurately in this way since the cleavage plane is perpendicular to the rod but as no error could be noticed it was probably accurate to within two degrees.

The crystals were searched for strangers and twinning lamellae before and after the observations, by etching them in nitric acid. Only those crystals which were free from these imperfections were used as any such irregularities would have destroyed the accuracy of the results.

3. Method of Measurements

The Guoy method[5] was chosen for making these measurements. The principal requirement of this method is that the sample must have a uniform cross section over the entire section in which the field gradient is appreciable. The condition of the extreme ends is not important as they are both located in uniform fields. Both of these facts were advantageous because long uniform single crystals were available and also the twinning lamellae at the ends of the crystals, which invariably are formed when bismuth is cleaved, would be in such a position as to have no effect upon the results.

The arrangement is shown in Fig. 1.

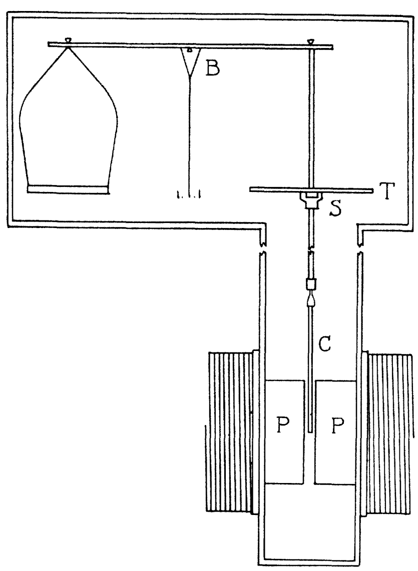

Fig. 1. Arrangement of apparatus.

An aluminum suspension S was designed to take the place of one of the regular pans of a sensitive analytical balance B. It was fitted with a divided

[5] E. C. Stoner, Magnetism and Atomic Structure, p. 40, (1926).

torsion head T so that the sample could be rotated about the vertical axis. The lower end of the suspension was about eight centimeters above the pole pieces P of the magnet. This distance was great enough so that the suspension was totally unaffected by the magnetic field. The pole pieces were flat faced and ten centimeters in diameter, giving a very uniform field between them with comparatively little stray field. The crystals C were thirteen centimeters in length, thus placing the lower end on the axis of the pole pieces while the other end was in a field of negligible intensity.

The balance, suspension, crystal and the ends of the pole pieces were enclosed in a cabinet to prevent air currents from disturbing the apparatus.

The greatest error to be expected between any two determinations of the susceptibilities was about 2 percent. The greatest single source of error was in the determination of the area of the cross section of the crystals. Due to the peculiar shape caused by the method of growth, the only practicable way of measuring the cross sectional area was by computing it from the weight, length and density of the sample under consideration. The error thus introduced was about 1 percent. The field strength was measured with a Grassot fluxmeter introducing a possible error of 0.5 percent, and the force exerted upon the crystal by the field was accurate to within 0.3 percent. The greatest deviation actually observed was less than 1.5 percent.

4. Theory of the Method

Let a crystal with principle susceptibilities k_1, k_2, k_3 be placed in a magnetic field as in Fig. 2.

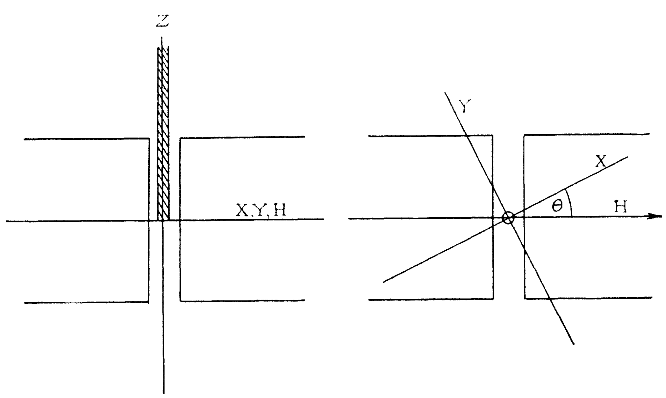

Fig. 2. Arrangement of crystal in magnetic field.

Take a set of axes parallel to the principal magnetic axes of the crystal with origin at its lower end.

Let z be parallel to the length of the crystal and let x make an angle θ with the magnetic lines of force. The boundary conditions will then be

$$H_x = H\cos\theta \qquad H_y = H\sin\theta \quad H_z = 0$$

$$\frac{\partial H_x}{\partial x} = \frac{\partial H_x}{\partial y} = \frac{\partial H_y}{\partial x} = \frac{\partial H_y}{\partial y} = \frac{\partial H_z}{\partial x} = \frac{\partial H_z}{\partial y} = \frac{\partial H_z}{\partial z} = 0$$

if the diameter of the crystal is small in comparison to that of the pole-pieces.

The equation for the energy of a magnetic field in an anisotropic medium is given as

$$E = \frac{1}{8\pi}(\mu_x H_x^2 + \mu_y H_y^2 + \mu_z H_z^2)$$

where

$$\mu_x = 4\pi k_x,\ \mu_y = 4\pi k_y,\ \mu_z = 4\pi k_z$$

thus

$$E = \tfrac{1}{2}(k_x H_x^2 + k_y H_y^2 + k_z H_z^2).$$

The force in directions parallel to the axes is

$$F_x = \frac{\partial E}{\partial x} = \frac{k_x}{2}\frac{\partial H_x^2}{\partial x} + \frac{k_y}{2}\frac{\partial H_y^2}{\partial x} + \frac{k_z}{2}\frac{\partial H_z^2}{\partial x}$$

$$F_y = \frac{\partial E}{\partial y} = \frac{k_x}{2}\frac{\partial H_x^2}{\partial y} + \frac{k_y}{2}\frac{\partial H_y^2}{\partial y} + \frac{k_z}{2}\frac{\partial H_z^2}{\partial y}$$

$$F_z = \frac{\partial E}{\partial z} = \frac{k_x}{2}\frac{\partial H_x^2}{\partial z} + \frac{k_y}{2}\frac{\partial H_y^2}{\partial z} + \frac{k_z}{2}\frac{\partial H_z^2}{\partial z}$$

or

$$F_x = k_x H_x \frac{\partial H_x}{\partial x} + k_y H_y \frac{\partial H_y}{\partial x} + k_z H_z \frac{\partial H_z}{\partial x}$$

$$F_y = k_x H_x \frac{\partial H_x}{\partial y} + k_y H_y \frac{\partial H_y}{\partial y} + k_z H_z \frac{\partial H_z}{\partial y}$$

$$F_z = k_x H_x \frac{\partial H_x}{\partial z} + k_y H_y \frac{\partial H_y}{\partial z} + k_z H_z \frac{\partial H_z}{\partial z}$$

Substituting the boundary condition these equations reduce to:

$$F_x = 0$$

$$F_y = 0$$

$$F_z = k_x H_x \frac{\partial H_x}{\partial z} + k_y H_y \frac{\partial H_y}{\partial z}$$

$$F_z = k_x \cos^2\theta H \frac{\partial H}{\partial z} + k_y \sin^2\theta H \frac{\partial H}{\partial z}$$

$$F_z = [k_y + (k_x - k_y)\cos^2\theta] H \frac{\partial H}{\partial z}$$

if $H = H$ when $z = 0$ and $H = H_0$ when $z = z$ integrating over the volume we get

$$F = \int F_z dv = \int [k_y + (k_x - k_y)\cos^2\theta] dx dy dz H \frac{dH}{dz}$$

or

$$k_y + (k_x - k_y)\cos^2\theta = \frac{2}{(H^2 - H_0^2)}\,\frac{F}{A}\,.$$

If F is given in grams we have

$$k_y + (k_x - k_y)\cos^2\theta = \frac{2gF}{A(H^2 - H_0^2)}\,.$$

In terms of the specific susceptibility x and if $H_0 < < H$, we have

$$x_y + (x_x - x_y)\cos^2\theta = \frac{2gF}{dAH^2}\,.$$

The similarity between this equation and that usually given for the Gouy method is seen at once by setting $x_x = x_y$ giving

$$x = \frac{2gF}{dAH^2}\,.$$

5. Discussion of Results

Fig. 3 shows the result obtained when the principal axis of the crystal is parallel to the rod (P_3). In this case the axis is always perpendicular to the

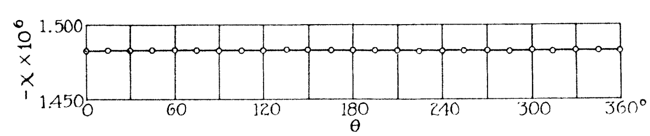

Fig. 3. Variation of specific susceptibility when major axis is parallel to length of the rod.

field and the distribution of the susceptibility in a plane perpendicular to the main axis is measured. It is seen at once that there is perfect circular symmetry about this axis.

Fig. 4 shows a compilation of five separate determinations of the variation of the susceptibility when the main axis is perpendicular to the rod, and a cosine square curve calculated from the average maximum and average minimum values of the susceptibility obtained in the cases of the five individual determinations. In the case of the maximum value the P_3 case is included. The agreement is evident showing that the variation definitely follows a cosine square law.

The principal susceptibilities of this bismuth may be given as $x_{min} - 1.046$ parallel to the main axis and $x_{max} - 1.487$ perpendicular to the main axis. The ratio of x_{max}/x_{min} is 1.425. This last point is particularly mentioned as it has been found that this ratio varies with varying amounts of impurities and an investigation of this effect is now in progress.

The susceptibility, calculated from these values, for a heterogeneous crystal aggregate is $-1.340, ({}^{2}x_{max} + x_{min})/3$ agreeing very well with the values given in various tables.

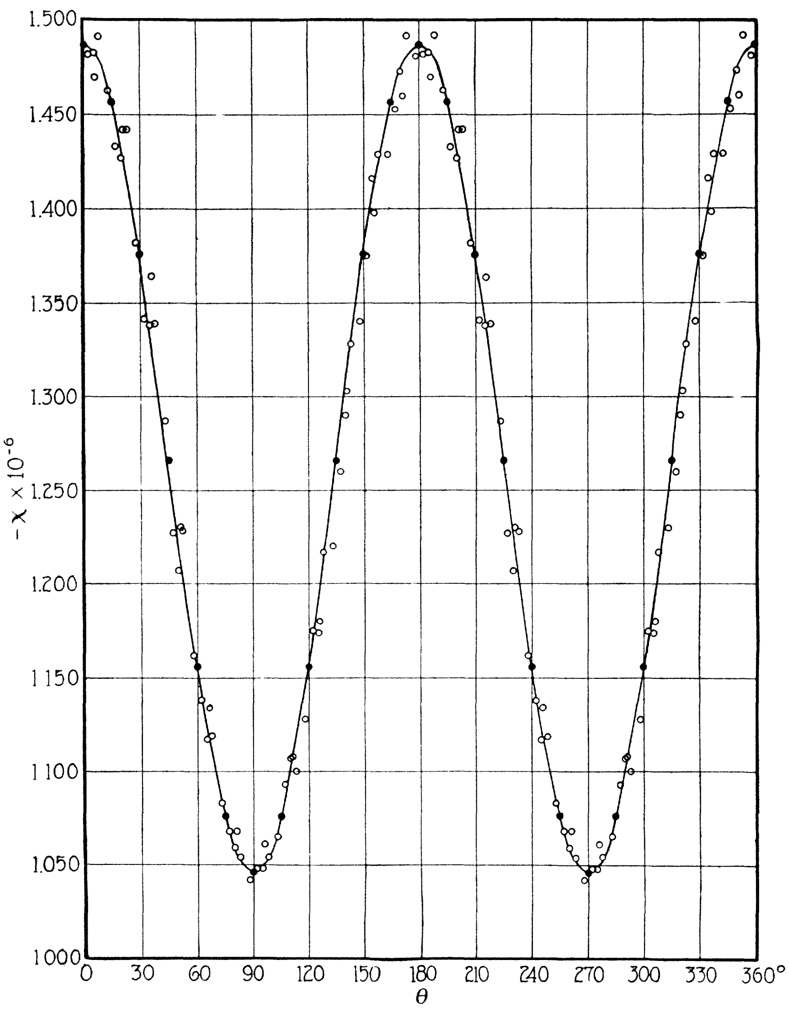

Fig. 4. Variation of specific susceptibility when major axis is perpendicular to length of the rod.

TABLE I. *Data for figure 4.*

Calculated Values		Crystal #120/11		Crystal #120/21	
θ	$-\chi\times10^6$	θ	$-\chi\times10^6$	θ	$-\chi\times10^6$
0	1.487	13	1.463	2	1.482
15	1.457	28	1.392	17	1.433
30	1.376	43	1.287	32	1.341
45	1.266	58	1.162	47	1.227
60	1.156	73	1.083	62	1.138
75	1.076	88	1.042	77	1.068
90	1.046	103	1.065	92	1.048
105	1.076	118	1.128	107	1.093
120	1.156	133	1.220	122	1.175
135	1.266	148	1.340	136	1.260
150	1.376	163	1.429	152	1.375
165	1.457	178	1.481	167	1.453
180	1.487	193	1.463	182	1.482
195	1.457	208	1.392	197	1.433
210	1.376	223	1.287	212	1.341
225	1.266	238	1.162	227	1.227
240	1.156	253	1.083	242	1.138
255	1.076	268	1.042	257	1.068
270	1.046	283	1.065	272	1.048
285	1.076	298	1.128	287	1.093
300	1.156	313	1.220	302	1.175
315	1.266	328	1.340	317	1.260
330	1.376	343	1.429	332	1.375
345	1.457	358	1.481	347	1.453
360	1.487	13	1.463	2	1.482

TABLE I. (*Continued*).

Crystal #120/51		Crystal #120/61		Crystal #120/71	
θ	$-\chi\times10^6$	θ	$-\chi\times10^6$	θ	$-\chi\times10^6$
8	1.492	5	1.483	6	1.470
23	1.442	20	1.427	21	1.442
38	1.339	35	1.338	36	1.364
53	1.228	50	1.207	51	1.230
68	1.119	65	1.117	66	1.134
83	1.054	80	1.059	81	1.068
98	1.054	95	1.048	96	1.061
113	1.100	110	1.107	111	1.108
128	1.217	125	1.174	126	1.180
143	1.328	140	1.290	141	1.303
158	1.429	155	1.416	156	1.398
173	1.492	170	1.473	171	1.460
188	1.492	185	1.483	186	1.470
203	1.442	200	1.427	201	1.442
218	1.339	215	1.338	216	1.364
233	1.228	230	1.207	231	1.230
248	1.119	245	1.117	246	1.134
263	1.054	260	1.059	261	1.068
278	1.054	275	1.048	276	1.061
293	1.100	290	1.107	291	1.108
308	1.217	305	1.174	306	1.180
323	1.328	320	1.290	321	1.303
338	1.429	335	1.416	336	1.398
353	1.492	350	1.473	351	1.460
8	1.492	5	1.483	6	1.470