THE MAGNETIC ISOTROPY OF A PARAMAGNETIC ALUM

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ABSTRACT

It is shown that cubic crystals of potassium chromium alum are magnetically isotropic.

J FORREST,[1] in 1926, was led from both theoretical and qualitative experimental grounds to the conclusion that a number of crystals had variations in the magnetic susceptibility with direction of the applied field which were not in strict accord with the Thomson-Voigt symmetry relations. In order to test this conclusion, measurements were made by the author on large single crystals of copper,[2] and it was found that there was no variation in the susceptibility greater than 1 percent. Copper is a diamagnetic material, and its magnetic properties are supposed to be mostly due to the Larmor precession of its closed shell of electron orbits. The theoretical basis on which Forrest was led to expect the anisotropies in question was derived from the use of a model consisting of elementary magnets fixed to points of a space lattice. It therefore seemed desirable further to test Forrest's conclusions using some paramagnetic substance, whose magnetic properties would be determined chiefly by incompleted shells of electrons, and would thus more closely approximate the theoretical model.

There are several paramagnetic alums with a cubic structure which can easily be obtained in large crystals suitable for this. Potassium chromium alum—$K_2SO_4 \cdot Cr_2(SO_4)_3 \cdot 24H_2O$—was chosen for this purpose. Crystals were grown from saturated solutions of the pure salt by suspending small seed crystals in them. Two of these were ground into cylindrical shape for measurement, one having as the cylinder axis a $<100>$ axis of the crystal, the other a $<111>$ axis. The following were their dimensions.

Axis of cylinder	length	diameter
$<111>$	0.610 cm	0.686 cm
$<100>$	0.444	0.549

The structure of chrome alum has been determined by Vegard and Schjelderup,[3] who place the chromium atoms on a face-centered cubic lattice. The magnetic susceptibility per unit mass is given in the International Critical Tables[4] as 11.5×10^{-6}.

Measurements of the variation of the magnetic susceptibility were made with the apparatus previously described by the author.[2] Values of the susceptibility in arbitrary units were obtained for every 15° in the plane perpendic-

[1] J. Forrest, Trans. Roy. Soc. Edinburgh 54, 601–701 (1926).

[2] C. G. Montgomery, Phys. Rev. [2] 36, 498–505 (1930).

[3] L. Vegard and H. Schjelderup, Ann. d. Physik [4] 54, 146–164 (1917).

[4] International Critical Tables VI, p. 360.

ular to the axis of the cylinder. These relative values of the susceptibility were analyzed in a Fourier series in terms of the azimuth ϕ, the angle between the field and an arbitrary polar axis in the plane of the base of the crystal cylinder. This gave the following results:—

$$x<_{100}> = 85.85 + 2.32 \cos\phi - 0.34 \cos 2\phi - 1.36 \cos 3\phi + 0.12 \cos 4\phi$$
$$+ 9.05 \sin\phi + 2.71 \sin 2\phi + 1.29 \sin 3\phi - 0.45 \sin 4\phi$$
$$x<_{111}> = 166.68 + 1.34 \cos\phi + 0.18 \cos 2\phi + 0.11 \cos 3\phi + 0.42 \cos 4\phi$$
$$+ 3.81 \sin\phi + 0.27 \sin 2\phi + 0.72 \sin 3\phi - 0.38 \sin 4\phi$$

These two runs were not made at the same sensitivity of the apparatus, and hence corresponding coefficients in the two series should not be compared. As in the previous work, the terms with the 360° period are to be ascribed to an instrumental error and should be ignored in the interpretation of the data. About a $<100>$ axis, we should expect, if the crystal were other than isotropic, at least one large coefficient in the terms in 4ϕ. It is seen that the coefficients of these terms are much smaller than the coefficients of the 2ϕ and 3ϕ terms. These latter terms can have no physical significance, in view of the symmetry of the crystal. Hence we can assign none to the terms in 4ϕ. In a similar way, for the $<111>$ axis, the 3ϕ term is small compared with the 2ϕ and 4ϕ terms. Thus we must conclude that chrome alum, as well as copper, forms magnetically isotropic crystals.

It remains to give an explanation, if possible, for the fact that Forrest obtained experimental evidence of some anisotropy in his crystals. Forrest gives a curve[5] showing the dependence of the intensity of magnetization parallel to the field upon the direction of the field in the {100} plane, for ammonium iron alum—$(NH_4)_2SO_4 \cdot Fe_2(SO_4)_3 \cdot 24H_2O$. The curve drawn shows a four-fold symmetry. This substance is identical in structure with chrome alum, and would be expected to behave magnetically in the same way. If we measure off in this figure the radius vectors of the experimental points, and subject them to a Fourier analysis, as above, we obtain the following result in centimeters:—

$$I_{11} = 2.469 - 0.033 \cos\phi - 0.018 \cos 2\phi - 0.089 \cos 3\phi$$
$$+ 0.072 \cos 4\phi + 0.000 \sin\phi - 0.048 \sin 2\phi + 0.041 \sin 3\phi$$
$$+ 0.029 \sin 4\phi.$$

We see that here the terms in 3ϕ have a larger amplitude than the terms in 4ϕ, and we conclude that to draw a curve through these points which shows a four-fold symmetry is not justifiable. The explanation of Forrest's curves for the transverse component of the intensity of magnetization is not evident. It should be pointed out, however, that if these curves represent real effects, there should be a dependence of the susceptibility of the material upon the strength of the field, an effect which has not been observed.

[5] J. Forrest, reference 1, p. 615, outer curve, Fig. 1.