ELECTRO-OPTICAL MODIFICATION OF LIGHT WAVES

By L. H. Stauffer
Department of Physics, University of California

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Abstract

Broadening of the satellites of the Hg green line λ5461 was observed when the light passed between the plates of a Kerr cell on which was impressed a varying E.M.F. having a frequency of 2×10^7 sec^{-1}. The E.M.F. was obtained by superposing the output of a vacuum tube oscillator upon a steady potential of about 7000 volts. The maximum oscillator voltage was about 5000 volts. With this voltage the fine structure of the Hg green line became so diffuse that two satellites having a separation of 0.045A were scarcely resolved by the Lummer-plate. The broadening was observed to increase rapidly with the oscillator voltage. The observed effect is predicted by the classical electromagnetic theory and the quantum theories of dispersion of Kramers-Heisenburg, Schrödinger, and Dirac. The high frequency voltage effects a rapid variation of the refractive index of the nitrobenzene in the Kerr cell which in turn produces a corresponding variation in optical path, giving the source a virtual velocity, and thereby producing a Doppler effect. The magnitude of the observed broadening is in agreement with the predictions of theory.

Introduction

It has been shown experimentally by E. Rupp[1] that monochromatic light suffers a slight change in wave-length when sent through an electro-optical shutter actuated by a high frequency oscillator. Rupp's shutter consisted of a Kerr condenser with nitrobenzene as a dielectric, placed between crossed Nicol prisms. When an E.M.F. is applied to the Kerr cell plates, the dielectric becomes doubly refracting and the second Nicol allows some of the light to pass through it. When an alternating E.M.F. is applied, the beam is interrupted with a frequency dependent on the frequency of the applied field. If the shutter opens and closes n times per second, theory predicts that the emergent beam should contain the original frequency and two "side frequencies" differing by $\pm n$ from the original frequency. These beat frequencies, of course, are due entirely to the rapid interruption of the beam and should be present even if the beam were interrupted by a mechanical shutter.

Shortly after Rupp's results were published, Wawilow[2] and others pointed out another phenomenon which must be considered when an electro-optical shutter is used. A variation in field strength produces a corresponding change in the refractive index of the liquid in the Kerr cell which, in turn, effects a change in optical path. This variation in optical path would give the source a virtual velocity and, in accordance with classical electromagnetic theory (Doppler's principle) a change in frequency would result.

[1] E. Rupp, Zeits. f. Physik **47,** 72 (1928).
[2] Wawilow, Zeits. f. Physik **48,** 600 (1929).

The experiment here reported was undertaken with the hope of testing the adequacy of the classical theory in explaining the above mentioned effect. Though to be sure, the quantum theories of dispersion of Kramers and Heisenburg[3] Schrödinger,[4] and Dirac[5] are in agreement with the classical theory in this regard, there remains a reasonable suspicion that quantum phenomena might enter to modify the expected results. Indeed, such a possibility is suggested by a curious anomaly reported by A. Bramley.[6] He photographed the arc spectrum of iron through a water Kerr cell on which was impressed the output of a high frequency oscillator. With relatively low voltages, he was able to detect shifts of certain lines toward the red end of the spectrum—shifts in magnitude very many fold greater than expected from theory.

General Theory

Assuming the results of the classical electromagnetic theory, it is not difficult to deduce the frequency modification undergone by a train of light waves when it traverses a medium subjected to an electric field of varying intensity.

W. A. Michelson[7] has generalized Doppler's principle to cover the case of a rapid variation of the refractive indices of the media traversed by a light ray. If ν is the observed frequency of the light then according to Michelson's formulation

$$\nu = \nu_0\left[1 \pm \frac{1}{c} \sum_i \frac{d}{dt}(\mu_i l_i)\right] \tag{1}$$

where ν_0 is the frequency of the radiation as it leaves the source, c is the velocity of light in vacuum, l_i the distance traversed in a medium having an index of refraction μ_i and t the time. For the case of a single medium of fixed extent (1) reduces to

$$\nu = \nu_0\left(1 \pm \frac{l}{c}\frac{d\mu}{dt}\right) = \nu_0 \pm \Delta\nu. \tag{2}$$

If a change in refractive index is effected by applying an electric field of strength E_1, P. Langevin[8] has shown theoretically that

$$\frac{\mu_p - \mu_0}{\mu_s - \mu_0} = -2 \tag{3}$$

where μ_0 is the refractive index of the medium for zero field strength and μ_p and μ_s the indices for light polarized with its electric vector parallel and perpendicular respectively to the direction of the field. This relation has been

[3] H. A. Kramers and W. Heisenberg, Zeits. f. Physik **31,** 681 (1925).

[4] E. Schrödinger, Ann. d. Physik **81,** 108 (1926).

[5] P. A. M. Dirac, Proc. Roy. Soc. **A114,** 710 (1927).

[6] A. Bramley, Jour. Franklin Inst. p. 315, March 1929.

[7] W. A Michelson, Astrophys. Jour. **13,** 192 (1901).

[8] P. Langevin, Le Radium **7,** 249 (1910).

experimentally verified by Pauthenier[9] in his measurement of absolute retardation in the Kerr effect. It is also well established that the relative retardation in phase of the components of the ray vibrating parallel and perpendicular to the field is given by

$$\phi = 2\pi B l E^2 \tag{4}$$

in which B is the Kerr constant and ϕ is the retardation in radians. The difference in the optical paths of the two components is

$$\frac{\phi\lambda}{2\pi} = (\mu_p - \mu_s)l = BlE^2\lambda \tag{5}$$

in which λ is the wave-length in vacuum. This gives the relation

$$BE^2\lambda = \mu_p - \mu_s. \tag{6}$$

Expressions for μ_p and μ_s may now be obtained by combining (3) and (6), the result of which is

$$\mu_p = \mu_0 + 2BE^2\lambda/3 \tag{7}$$

and

$$\mu_s = \mu_0 - BE^2\lambda/3. \tag{8}$$

The frequency change $\Delta\nu$ for light polarized with its electric vector parallel to the applied field may now be obtained by taking account of equations (2) and (7). This yields.

$$\Delta\nu = \frac{\nu_0 l}{c}\frac{d\mu_p}{dt} \tag{8a}$$

or

$$\Delta\nu = \frac{2B\lambda_0\nu_0}{3c}l\frac{dE^2}{dt} \tag{8b}$$

but since $\nu_0\lambda_0 = c$

$$\Delta\nu = \frac{4}{3}BlE\frac{dE}{dt} \tag{8c}$$

Apparatus and Procedure

The experimental arrangement (see Fig. 1) consists essentially of a Kerr cell, K, coupled to a vacuum tube oscillator, a mercury arc, M, as a light source, and a Lummer-Gehrcke plate, L, as a spectroscope. The mercury arc was water cooled to reduce Doppler broadening and was operated at the lowest current density which would maintain a steady arc. Light from the arc passes between the plates of the Kerr cell and is polarized with its electric vector parallel to the field by the Nicol prism, N_3. A constant deviation prism D serves as a monochromator to separate the green line λ5461. The interference pattern produced by the Lummer plate may be observed

[9] M. Pauthenier, Comptes rendus **170**, 803 (1920).

visually or photographed at E. The retardation can be measured at any time during the course of the experiment by observing the deflection of the galvanometer, G, which is actuated by current from the photoelectric cell P. A beam of light from the incandescent lamp, S, is sent through the cell in a direction crosswise to the plates and is polarized by the Nicol, N_1 in a plane making an angle of 45° with the vertical. When a difference of potential is applied to the plates the vertical component is retarded and the horizontal component advanced so that the light leaves the cell elliptically polarized and part of it can pass the second Nicol, N_2, which is crossed with respect to N_1. As will be shown later, the arrangement may be used to measure the field applied to the cell.

The design of the apparatus is governed largely by equation (8c) which gives the frequency change in terms of the Kerr constant of the liquid used in the cell, the length of the plates, the field strength and its time derivative.

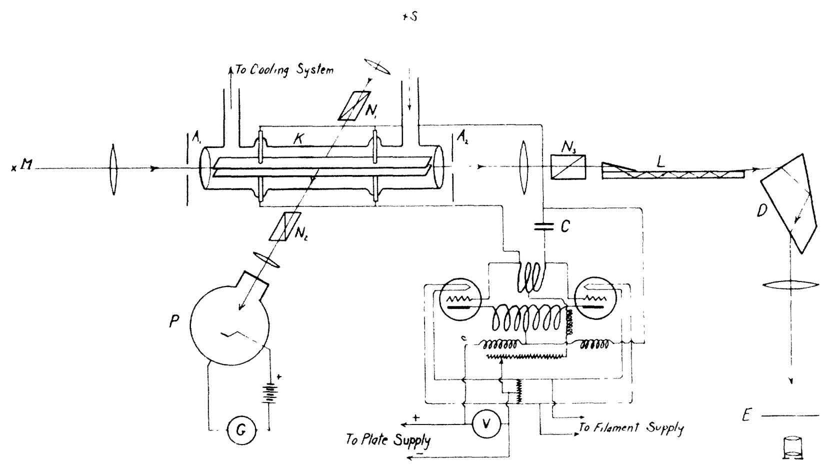


Fig. 1. Diagram of apparatus.

Nitrobenzene was chosen as the dielectric to be used in the cell for the reason that it has the largest Kerr constant of any known substance (3.5×10^{-5}). The quantity $E(dE/dt)$ may be made large by superimposing the oscillator output on a steady potential, in this case the plate potential of the oscillator.

The time rate of the field of course depends on the natural period of the oscillatory circuit, which is limited by the capacity of the Kerr cell. Since nitrobenzene has a specific inductive capacity in the neighborhood of 40, the capacity of even a small cell is comparatively high. The capacity was reduced as much as possible by making the plates narrow.

The Kerr cell used had nickel plates 10 cm by 2 mm with slightly rounded corners. The plate separation was 2 mm. Short tungsten leads of large diameter were used in order to reduce heating by the high frequency currents.

Considerable difficulty was at first encountered, due to the failure of the nitrobenzene to withstand the intense high frequency fields.

To reduce the conductivity to a minimum, the liquid was purified by treating with a weak base such as aluminum or calcium oxide to remove acids and then double distilling in vacuum during which process all water vapor was frozen out with liquid air. After such treatment, the liquid would

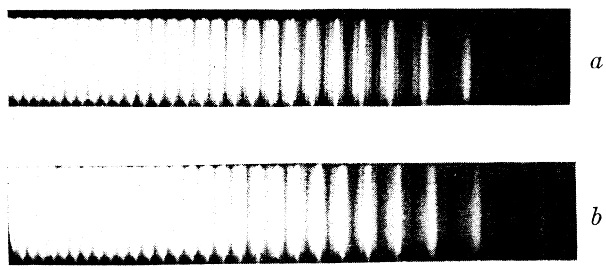


Fig. 2. Photographs of fine structure of Hg green line (a) Unmodified spectrum. (b) Modified spectrum.

stand very high D. C. potentials, but heated badly, and broke down when subjected to high frequency fields. To keep the nitrobenzene cool, a water cooled circulatory system was devised, which consisted of a small four

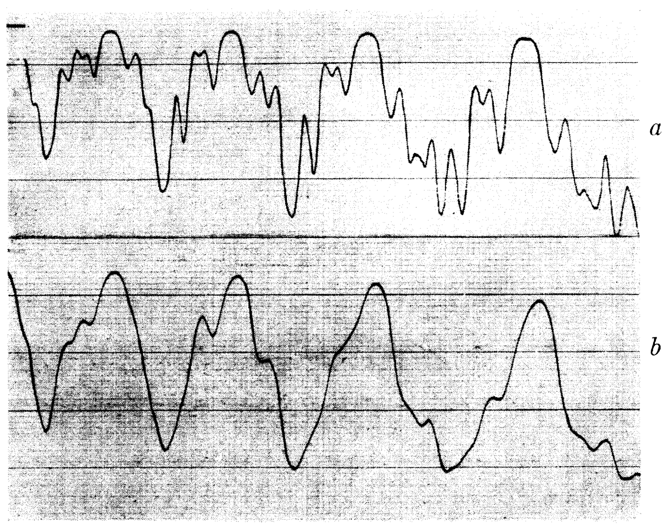


Fig. 3. Microphotometer curves. (a) Unmodified. (b) Modified.

bladed propeller rotating in a vertical chamber connected to the Kerr cell by tubes surrounded by water jackets. The whole was made of Pyrex glass to avoid contaminating the nitrobenzene. The liquid was found to withstand a much higher voltage when kept cool by this method.

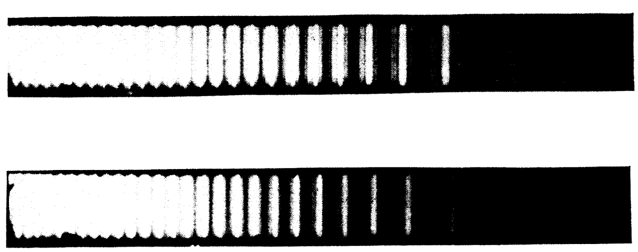
A diagram of the push-pull oscillator used is shown in Fig. 1. Two 100 watt Radiotrons of the type UX-852 were employed with between 7000 and 8000 volts on the plates. The highest frequency that could be attained with the Kerr cell used was about 2×10^7 sec.$^{-1}$. The plate voltage of the oscillator is applied to the cell by inserting a large condenser C, and connecting the isolated terminal of the cell through a choke to the positive side of the plate circuit.

Visual observations of the fine structure of the mercury green line revealed a slight broadening of the components when the oscillator was operating. The effect was repeatedly confirmed by photographing. Fig. 2a and b shows the interference pattern of the Lummer-Gerhcke plate without and with the oscillator in operation. Fig. 3 shows microphotometer curves of the unmodified and modified patterns.

The procedure followed in photographing was to take an exposure under no voltage conditions with the circulator operating and then directly underneath on the same plate an exposure with the oscillator in operation. The overloaded oscillator could only be operated over intervals of about 10 seconds without danger of overheating the tubes and Kerr cell. The second exposure was, therefore, broken up into a number of 10 second intervals.

Discussion of Results

The appearance of the fine structure pattern of the Hg green line is shown in Fig. 2a. A large number of orders are present and as the photograph shows, the dispersion diminishes toward the lower orders. The main line and five satellites are clearly resolved on the original plates but the enlarged reproduction shows less detail. The designation and wave-length separations of the satellites are taken from a paper by McLennan[10] reporting some work on the Hg green line in which he used a Lummer plate. In



a

b

Fig. 4. Photographs of fine structure with reduced oscillator voltage.

all the photographs shown the negative satellites are on the right. The satellite -6 is separated from the main line by an interval of -0.237A and overlaps into the succeeding order where it is seen between the main line and the satellite $+3$. The wave-length separations from the main line of the satellites -6, -5, -4, $+3$ and $+4$ are -0.237, -0.102, -0.0698, $+0.852$ and $+0.128$ respectively.

The fine structure of the line as modified by a high frequency field of approximately 22,500 volts per centimeter superimposed on a steady field

[10] J. C. McLennan, Proc. Roy. Soc. **102**, 33 (1922).

of 36,000 volts per centimeter is shown by Fig. 2b. This gives a calculated broadening of over 0.01 Angstroms.

Microphotometer curves of the unmodified and modified lines are shown in Fig. 3a and b. The results obtained in a later trial with an oscillating field of 19,600 volts per centimeter and a steady field of 40,000 volts per centimeter are shown in Figs. 4 and 5.

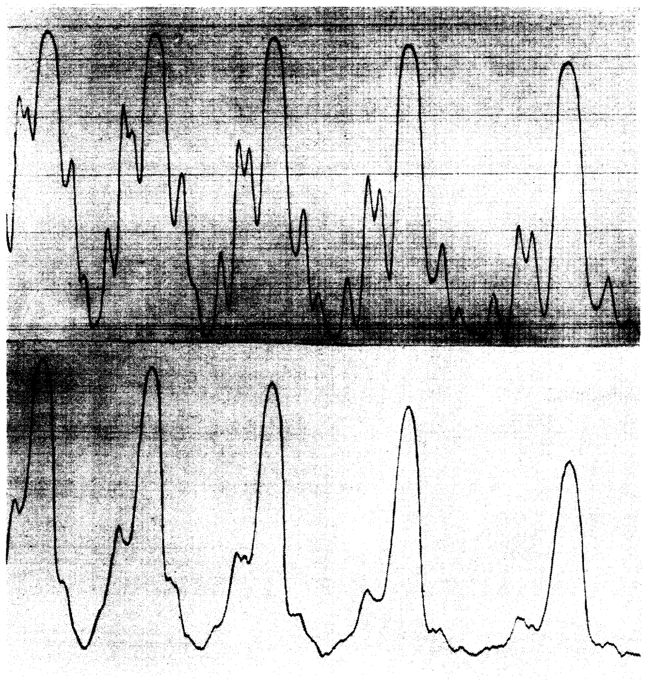


Fig. 5. Microphotometer curves. (a) Unmodified. (b) Modified.

In order to calculate the frequency change $\Delta\nu$ in terms of deflections of the galvanometer, G, and known constants of the apparatus it is necessary to modify equation (8b) by making the substitution

$$E = E_1 + E_0 \sin 2\pi nt$$

where E_1 is the steady field strength, E_0 the peak field strength of the oscillator, n the oscillator frequency and t the time. E_1 and E_0 are expressed in electrostatic volts per centimeter. Equation (8b) now becomes

$$\Delta\nu = \frac{2B\lambda_0\nu_0 l}{3c} \frac{d}{dt}(E_1 + E_0 \sin 2\pi nt)^2 \tag{9}$$

Taking the time average by integration over a quarter cycle gives

$$\frac{\overline{\Delta\nu}}{\nu_0} = \frac{8\lambda_0 Bnl}{3c}(E_0^2 + 2E_1E_0) \tag{10}$$

where $\overline{\Delta v}$ is the average value of the frequency change. Now if the intensity of the light reaching the photo-cell at any time t be represented by I we have in accordance with classical optics

$$I = I_0 \sin^2(\phi/2) \tag{11}$$

where I_0 is the intensity of light coming from the Nicol prism, N_1, and ϕ is the retardation. By substituting the value of ϕ given by equation (4) into (11) the expression for the intensity becomes

$$I = I_0 \sin^2 \pi B w(E_1 + E_0 \sin 2\pi nt)^2 \tag{12}$$

in which w is the width of the plates. In this experiment it can be shown that the average value of the retardation is small since w is only 2 millimeters. It is, therefore, permissible within the limits of error of the experiment to replace the sine by the angle itself. This is done to facilitate the integration of equation (12) to get the time average of the intensity I. The result of the integration is

$$\bar{I} = I_0\pi^2B^2w^2(E_1{}^4 + 3E_0{}^2E_1{}^2 + 3E_0{}^4/8) \tag{13}$$

Eliminating B between (10) and (13) the following expression is obtained

$$\frac{\overline{\Delta\nu}}{\nu_0} = \frac{8\lambda_0 nlI^{1/2}(E_0{}^2 + 2E_0E_1)}{3\pi cwI_0{}^{1/2}(E_1{}^4 + 3E_1{}^2E_0{}^2 + 3E_0{}^4/8)^{1/2}} \cdot \tag{14}$$

Since the galvanometer deflections are small and the photoelectric current is proportional to the intensity, the ratio of intensities may be replaced by the ratio of the corresponding deflections. It is also more convenient to deal with the ratio of E_0 to E_1 than to use the equation (14) as it stands. Designating the ratio E_0/E_1 by a, equation (14) becomes

$$\frac{\overline{\Delta\nu}}{\nu_0} = \frac{8\lambda_0 nlD^{1/2}a(a + 2)}{3\pi cwD_0{}^{1/2}(1 + 3a^2 + 3a^4/8)^{1/2}} \tag{15}$$

or in terms of λ_0 the average change in wave length $\overline{\Delta\lambda}$ is

$$\overline{\Delta\lambda} = \frac{8\lambda_0{}^2 nlD^{1/2}a(a + 2)}{3\pi cwD_0{}^{1/2}(1 + 3a^2 + 3a^4/8)^{1/2}} \cdot \tag{15a}$$

D_0, the deflection corresponding to the intensity I_0, may be obtained by uncrossing the Nicols by an angle θ and observing the deflection D_θ. D_0 is then given by the relation $D_\theta = D_0 \sin^2 \theta$.

The ratio, a may be obtained as follows. Let I_1 be the intensity of light reaching the photo-cell when a field E_1 is applied to the Kerr cell. Equation (13) then becomes

$$I_1 = I_0\pi^2B^2w^2E_1{}^4 \tag{16}$$

When the high frequency field is superimposed the intensity is given by equation (13). Dividing (16) by (13) and replacing the ratio of intensities by the ratio of the corresponding galvanometer deflections one gets

$$\frac{D_1}{D} = \frac{E_1^4}{E_1^4 + 3E_1^2E_0^2 + 3E_0^4/8} = \frac{1}{1 + 3a^2 + 3a^4/8} \,. \tag{17}$$

The value of a may now be obtained with sufficient accuracy by plotting a curve of D_1/D for values of a ranging from 0 to 2. Knowing E_1 from the reading of the voltmeter, V, the value of E_0 is available when a is known. The reason for not measuring E_0 by means of a voltmeter will be evident to those familiar with the difficulties in making accurate measurements of large currents or voltages at high frequencies.

The ratio of the length to the width of the plates was corrected for edge effect by taking measurements with the photo-cell and crossed Nicol arrangement, lengthwise as well as crosswise of the plates with the field on. As equation (13) shows the intensity is proportional to the square of the effective path traversed in the field, it was thus found that the value of $1/w$ must be reduced by 8.5 percent.

At first thought it might seem easy to measure the change in wave-length by taking measurements on the microphotometer curves of the fine-

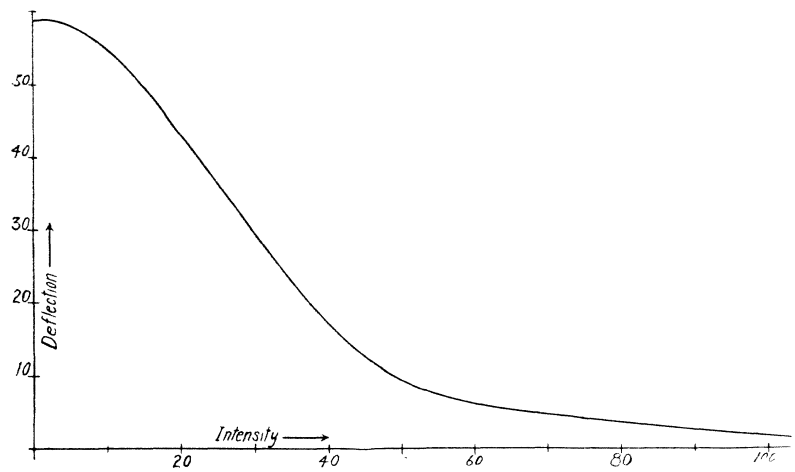


Fig. 6. Blackening curve for photographic plates.

structure patterns. The half-width of a satellite before the field was applied might be compared with the half-width of the modified satellite but this would probably not correspond to the average wave-length change given by equation (15a). It is also evident that the photographs of the modified and unmodified patterns are not comparable unless the exposures are the same since the width of the lines is influenced by the length of exposure. Since the ordinates of the microphotometer curves represent blackening and not intensity, it is necessary to know the blackening as a function of intensity for a given plate before two satellites can be compared. In view of these considerations one can only expect rough agreement between measured and calculated values of the wave-length change.

The procedure finally followed was to select a plate such as the one shown in Fig. 4 where the modification is not so great as to obliterate the structure and to measure the change in half-width of the main line. To facilitate intensity measurements the unmodified fine structure was photographed, the Lummer plate removed and a slit and a Nicol prism placed in the path of the beam. A series of exposures was then taken on the same plate, the intensity of the beam being varied by rotating one of the Nicols a known amount before each exposure. The plate was then microphotometered and a curve of intensity against deflection was plotted. The curve is shown in Fig. 6. The deflections (plotted as ordinates) are measured from the line of absolute blackening on the microphotometer curves. Thus a large deflection means a low intensity. The intensity relations between the fine structure components were found to vary slightly from order to order of the interference pattern. It was found that over the range of the five highest orders in which intensity measurements were made that the intensity of the satellite +3 remained very nearly half that of the main line. In no order was the variation greater than six percent. The peak of the satellite +3 was therefore taken as the ordinate representing half-intensity for the main line in any given order. The dispersion was obtained by measuring the distance between the peaks of the satellites +3 and +4 which have a wave-length separation of 0.043A. The results obtained from the curves shown in figure 4 are presented in Table I.

TABLE I. *Results obtained from the curves of Fig. 4.*

Order	Dispersion mm/A	Broadening mm	Half-width Change/2 A
I	100	2.40	0.012
II	82.3	1.80	0.011
III	71.2	1.40	0.0098
IV	66.7	1.40	0.010
			0.0107 Mean

The average change in wave-length as given by equation (15a) is 0.0078A. The maximum change in wave-length may be secured by calculating the maximum value of dE^2/dt and introducing it in place of the average value as has been done. The calculation is straightforward but rather long. It yields a result which is 1.4 times as large as the average value, 0.0078. The factor 1.4 is not a constant but varies with a, the ratio of E_0 to E_1. The maximum value of $\Delta\lambda$ is therefore about 0.011 Angstroms, slightly greater than the mean value given in the table.

Thus, within the experimental uncertainty (about 20 percent), the magnitude of the observed change in frequency is in agreement with the predictions of the classical theory and the several quantum theories of dispersion. No anomalous effects of the kind reported by Bramley were detected in the course of the experiment.

The author is indebted to Professor E. O. Lawrence, who suggested this experiment, for his constant assistance and many helpful suggestions.

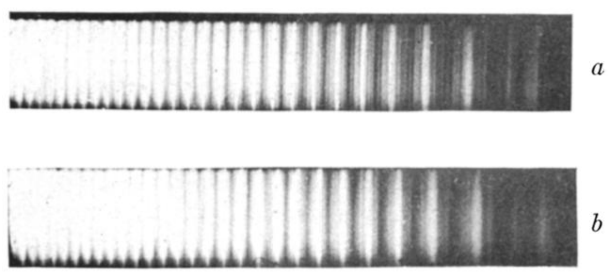


Fig. 2. Photographs of fine structure of Hg green line
(a) Unmodified spectrum. (b) Modified spectrum.

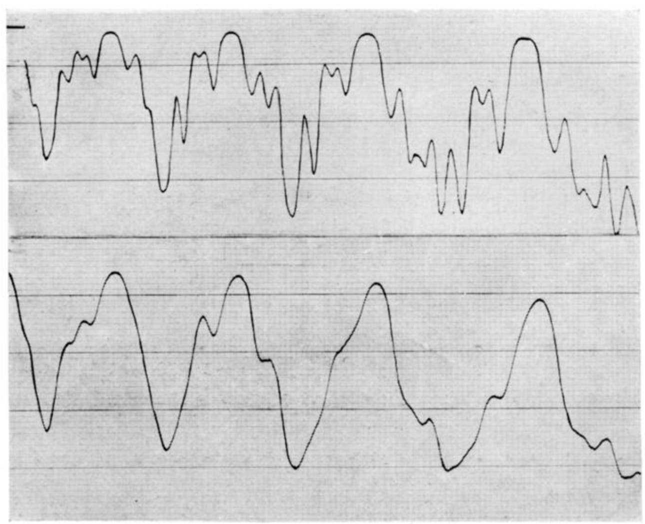


Fig. 3. Microphotometer curves. (a) Unmodified. (b) Modified.

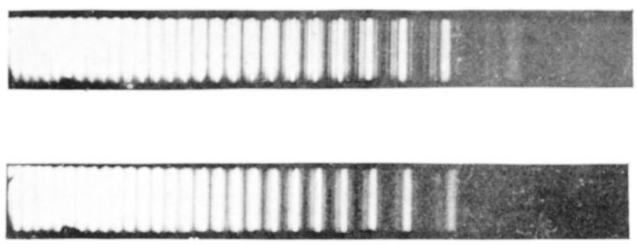


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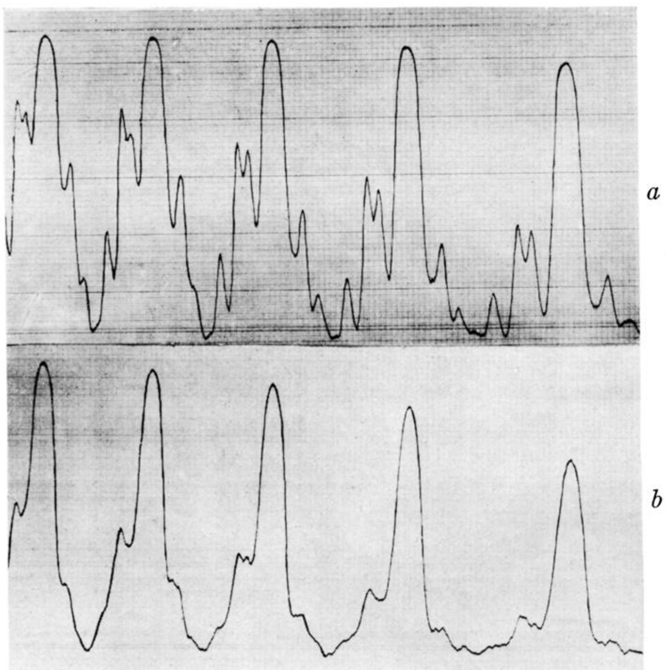
Fig. 4. Photographs of fine structure with reduced oscillator voltage.



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