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THE USE OF THE REFRACTION OF X-RAYS FOR THE DETERMINATION OF THE SPECIFIC CHARGE OF THE ELECTRON

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Abstract

The discrepancies in the determination of the value of the fundamental constant e/m by the two different methods developed so far make desirable the development of new methods for its evaluation. The index of refraction of x-rays offers one method. According to the theories developed so far, in the case of a dispersive medium with no critical frequencies near the frequency of the incident radiation, the value of e/m may be expressed in terms of well-known constants, the wave-length of the incident radiation, and δ (one minus the index of refraction).

Index of refraction of quartz for MoKα_1 and Kβ radiation. A new method of using the prism with x-rays has been developed and used to determine δ. For crystalline quartz of density 2.6480 gm/cm^3, δ was determined for the $K\alpha_1$ radiation of molybdenum as $1.804 \pm 0.001 \times 10^{-6}$ and for the $K\beta$ as $1.436 \pm 0.001 \times 10^{-6}$.

The calculation of e/m is complicated by the discrepancies that have arisen in the absolute determinations of x-ray wave-lengths. Using the absolute wave-lengths as determined from the results of Bäcklin, Bearden, and Cork, the values of e/m all lie between the values given by the spectroscopic and deflection methods, and they have nearly as great a range as the difference between the two older values. For this reason no definite conclusion can be made as to the most probable value of e/m.

Introduction

THE index of refraction of x-rays has, in addition to its intrinsic interest, an interest on account of the discrepancies in the evaluation of some of the fundamental constants existing at the present time. So far a number of different dispersion theories have been developed for the x-ray region, but they all agree in the limiting case when the frequency of the radiation is greatly different from the natural frequencies of the medium, when they all lead to

$$\delta = 1 - \mu = \frac{ne^2}{2\pi m\nu^2} \tag{1}$$

where μ is the index of refraction, n is the number of electrons per cm^3 in the refracting medium, e is the electronic charge, m the electronic mass, and ν the

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frequency of the radiation. If this formula is assumed correct, one of the constants can be evaluated in terms of the others and δ. Ordinarily it would not seem feasible to determine any of the constants in terms of δ; but in view of the great discrepancy in the value of e/m as determined by the two methods developed so far,[1] there is need of additional methods of determining e/m. An attempt to evaluate this fundamental constant from the refraction of x-rays thus seemed desirable.[2]

In Eq. (1) n may be replaced by Ndz/M, where N is Avagadro's number, d the density of the medium, z the "molecular number", and M the molecular weight; and ν may be replaced by c/λ, where c is the velocity of light and λ the wave-length of the radiation. Ne/c in turn is Faraday's constant in e.m.u. With these values, Eq. (1) may be solved for e/m as

$$\frac{e}{m} = \frac{2\pi M\delta}{Fdz\lambda^2} \text{ in e.m.u.} \tag{2}$$

Of the quantities on the right, all except λ and δ are known or can be determined with high precision. The value of λ is in doubt on account of the discordant results obtained in its absolute determination by the several observers. Whenever λ can be assumed to be known, δ is the limiting factor in the evaluation of e/m. It will be noticed that Eq. (2) is independent of e, the value of which is also in doubt.

For the determination of δ it was decided to use the deviation produced by a prism and to develop the method suggested in an earlier paper.[3] There it was shown that the maximum deviation of a beam of x-rays can be obtained either with the beam striking the first face of the prism at nearly the critical angle of reflection, the method used by Larson, Siegbahn, and Waller,[4] or with the beam striking the second face internally at nearly zero glancing angle of incidence and leaving at nearly the critical angle of reflection. In the second method the refracted beam is less divergent than the incident beam, and all wave-lengths suffer maximum deviation simultaneously. There is the disadvantage that an angular rotation must be used that must be measured with high precision.

Theory of Experiment

The experimental procedure adopted can be explained by reference to Figs. 1 and 2. Radiation from the slit S fell upon the prism P, which had its refracting edge A over the center of rotation of a spectrometer. Slightly to the front and to the side of the prism was a lead block B which acted with the prism as a sort of Seemann slit. The primary beam was defined by A and B. Refraction occurred in the neighborhood of A. After the primary and re-

[1] R. T. Birge: Phys. Rev. Supplement **1**, 43 (1929).

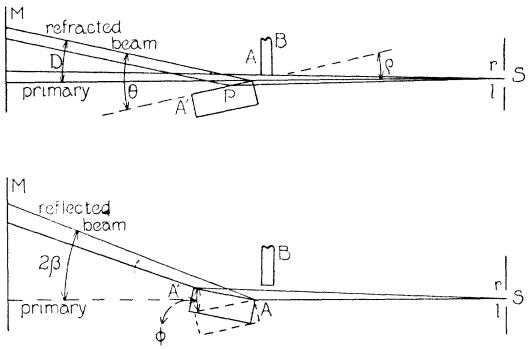
[2] It is a great pleasure to acknowledge that Professor A. H. Compton suggested the desirability of such a determination of e/m.

[3] H. E. Stauss, J. O. S. A. **19**, 167 (1929).

[4] A. Larson, M. Siegbahn, and T. Waller, Phys. Rev. **25**, 245 (1925).

fracted beams were registered on the photographic plate M, the prism was rotated through a small angle ϕ to the position of Fig. 2, and the beam was totally reflected from the face AA' at the angle β.

The value of δ is most readily calculated from $\mu = 1 - \delta = \cos \rho$, where ρ is the internal glancing angle with the face AA' and θ is the external angle. Assuming a single ray to be striking at A and assuming the ray to suffer no



Figs. 1 and 2. Diagrammatic sketch of experimental arrangement.

deviation at the first face, ρ and θ can be determined from ϕ, β, and D, where D is the angle of deviation.

$$\phi = \theta + \beta \quad \text{or} \quad \theta = \phi - \beta$$
$$\theta = D + \rho \quad \text{or} \quad \rho = \theta - D.$$

Substituting these values in the expression for μ, and solving for δ

$$\delta = D(D + 2\rho)/2. \tag{3}$$

The values of D and ρ need a slight correction $(1/3'')$ on account of the deviation in passing through the first face; but this can easily be calculated.

In practice the rays from the slit S are divergent. The simplest experimental procedure is to make all measurements from the edges produced by the radiation from the edge r (of the slit S) that passed A. The errors introduced by measuring edges will be discussed later. In the case of the refracted beam, the ray defined by A and r lies within the beam. Theoretically refraction occurs all along AA'. Practically absorption limits the effective length of the refracting face. For the experiments reported here, the intensity of the refracted radiation that emerged 1 mm from A was only 9 percent of that which emerged at A. The change in ρ produced by the penetration produces a negligible effect upon θ; so the refracted radiation from r may be considered a band of parallel radiation, of width about 0.002 mm. A correction of this amount can be made to the position of the refracted beam. It is evident that the width of the refracted beam is determined almost solely by the slit S.

In practice it was found advisable to use two different widths of the slit S for the refracted beam and for the primary and reflected rays. The former

varied from 0.1 to 0.3 mm in different trials; the latter was always 0.012 mm. The changes in width could be made accurately with a micrometer screw that formed part of the slit. On account of the changes in width, the position in the refracted beam corresponding to r had to be calculated in order to obtain D_r. This can be done from the relation $\cos\theta = \mu\cos\rho$ or $\theta^2 = \delta + \rho^2$, since $D = \theta - \rho$. In practice the refracted beam was not as sharp as was to be expected; so to diminish the chances of error, both edges of the refracted beams were measured and averaged. In the case where two strong beams of different wave-length overlap, as is the case for the two $K\alpha$ lines, another correction is necessary, which can be shown to be approximately

$$\Delta D = \theta\Delta\lambda/\lambda.$$

The position of D_r is then obtained from the average position by the formula

$$D = D'_{Av} - \frac{\Delta\theta_l' - \Delta\theta_r'}{2} + \theta\frac{\Delta\lambda}{2\lambda} \tag{4}$$

where the primes apply to the greater slit-width, and $\Delta\theta_l'$ and $\Delta\theta_r'$ are the corrections to be made in θ (and hence in D) on account of the changes of ρ produced by changing the slit-width. No correction was made for the γ line overlapping the β line because of the great difference in intensity. The corrections were always less than 1 percent.

The accuracy of the experiment depends to a very great extent upon the second or refracting face of the prism. It should be good up to the very edge A because half of the radiation is transmitted within 0.28 mm of A. The prism used in this experiment was a piece of crystalline quartz which had been cut from a larger pitch-polished surface. It was good to a quarter of a fringe and showed no peculiarities at the edges. Nevertheless, the refracted beam was more diffuse than was to be expected, and there was a band of radiation lying between the primary and refracted beams whose presence had not been foreseen. If in the preparation of the plate, even a slight bending occurred very near the edge A, it might affect the results by changing the inclination of parts of the refracting face. Moreover, a polished surface is never a geometrical plane, and the exact character of the surface is problematical. The observed abnormalities may in some way be due to surface effects. For this work it has been assumed, though it is difficult to prove, that the cause of the diffuseness does not displace the centers of the refracted beams.

Another factor affecting the accuracy of the experiment is the location of the edges of the beams. All the beams have penumbras within which the intensity falls from the maximum to zero. The calculations made above have all assumed that measurements can be made to the edge of zero intensity. This obviously is incorrect. On the other hand, the grains in an x-ray plate have considerable size, and the blackening may even run past the zero edge. A priori it is difficult to predict the positions that will be measured. It is possible to design more involved arrangements that permit

the centers of lines to be measured, and the results obtained by the present method seem to justify further work with such an arrangement.

Experimental Arrangement and Procedure

All the experimental work was done in a basement room. The plate-holder and the slit S, Fig. 1, were two meters apart, with a Geneva optical spectrometer midway between them. The telescope and collimator of the instrument were removed; and a special turntable was made that rotated with the scale of the instrument, so that its motion could be measured. The table was provided with perpendicular slides for centering the refracting edge of the prism. The slit, taken from a Geneva double x-ray spectrometer, was aligned by taking advantage of the striations present in a beam of x-rays from a narrow slit, the analogues of the spiral formed by a pinhole. An optical prism with its base 90° to the refracting edges was mounted on the table, and the slit adjusted until one of the striations and an edge of the prism were parallel. Other objects mounted on the table thereafter could be aligned by reference to the striations. That the method was satisfactory is shown by the fact that no systematic deviations were ever observed in reflection or refraction which could be attributed to inclination of the slit to the prism edge or reflector.

The plate-holder was made approximately perpendicular to the x-ray beam in both the vertical and horizontal directions. As the apparatus was used, the deviation from normal was too small to produce appreciable differences in the results. To obtain the distance from the refracting edge to the photographic plate, a piece of glass was substituted for the latter, a steel rod pressed against it, and the distance from the end of the rod to the refracting edge of the prism measured with a travelling microscope. The length of the bar was obtained by comparison with a standard meter. It is believed that this distance was determined with an error of less than 0.1 mm or 0.01 percent.

The prism, for most of the trials, had its refracting edge formed by the polished surface and a narrow bevel at 45° to it. The theory of the prism shows that the value of the prism angle is of minor importance in producing the deviation and this angle was assumed to be $135° \pm 15°$. To test the validity of this procedure the bevel was removed and a face was cloth-polished at 90° to the refracting surface. Later the prism was beveled again because, in the polishing, the tool had overlapped and injured the edge slightly. This was shown by the diffuseness produced at the edge of the reflected beam.

The $K\alpha_1$ and $K\beta$ radiations of molybdenum were used for the measurements. A water-cooled, self-rectified tube was run at 42 K.V. maximum and 20 ma. The times of exposure were adjusted to avoid over-exposure of the different lines. Eastman x-ray plates were used and the measurements made with a Gaertner comparator reading to 0.001 mm.

The experimental procedure finally adopted was as follows. The prism was set in position for refraction, the slit S opened to the desired width, and an exposure made for 1 to 2 hours, with the position of the direct beam cov-

ered with a lead sheet. The slit was then narrowed to 0.012 mm, the lead removed, and the primary beam registered for about four minutes. This position was read on the scale of the spectrometer, thirty settings of the cross-hair in the micrometer eye-piece of one of the reading microscopes being made on the edge of one mark on the scale. The prism was then rotated until the face AA' (Fig. 2) could reflect, a matter of 7′, and an exposure made for about ten minutes, the lead sheet again blocking out the primary beam. This position was read thirty times on the same edge of the same mark on the scale. Thus the measurement of the angular displacement was thrown entirely upon the micrometer screw of the reading microscope. The screw was calibrated at three greatly different positions on the spectrometer scale. The order of the procedure was changed in as many ways as possible in the different trials in order to eliminate any systematic error due to the order. Measurements of the photographic plates were made to both the right and the left to avoid errors in estimating the edge in coming to it from within or without the beam. Nine positions were measured on each plate, and five measurements, to both right and left, made at each position.

Results

After the experimental method had been developed to a satisfactory state, eleven trials were made. In one of the trials the plate became fogged and difficult to measure. It is listed in Table I, but is not used in the final average. On another plate, where the value of ρ was 17″, the $K\beta$ line was not sufficiently strong to be measured accurately under the same conditions as the $K\alpha$ line. The first two trials were made on the original prism; the next four on the right-angled prism; and the rest on the final bevelled prism (all being the same block of quartz). The probable error of a single determination of δ, as deduced from the deviations of the various readings and the arbitrarily assigned errors was much less than the differences between the various trials. In the experiment every factor that seemed to have any influence on the

Table I. *Values of* δ.

Trial	ϕ	$S(mm)$	ρ	$\delta K\alpha_1 \times 10^6$	$\delta K\beta \times 10^6$
1	6′10″	0.322	48″	1.818	1.447
2	4′56″	0.322	59″	1.819	1.447
3*	6′ 9″	0.322	28″	1.756	1.457
4	5′40″	0.322	51″	1.821	1.450
5	4′44″	0.222	17″	1.791	
6	6′11″	0.222	1′40″	1.794	1.424
7	7′37″	0.322	42″	1.783	1.425
8	7′10″	0.222	2′41″	1.795	1.421
9	7′ 7″	0.322	2′22″	1.805	1.448
10	7′ 6″	0.222	2′20″	1.793	1.428
11	6′56″	0.122	2′14″	1.819	1.431
Average (omitting 3)				1.804 ± .001	1.436 ± .00085

* Plate fogged.

result was varied, except the slit-width for the reflected and primary beams. The values of δ show no systematic variation with any of the quantities. The variations seem to be due to the measurements of the plates. If it can be assumed that the variations in δ are purely accidental in origin, the probable error of the average may be calculated. The results of the trials are shown in Table I, together with the variables entering into the experiment. The character of the refraction spectrum is shown in Fig. 3.

At the time this investigation was begun, there seemed to be one absolute determination of x-ray wave-lengths of outstanding accuracy, and the evaluation of e/m seemed to be very feasible. Since that time a new set of experi-

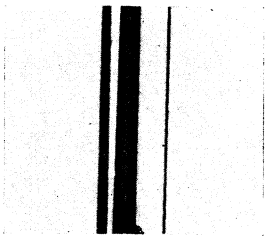


Fig. 3. A refraction spectrum of the molybdenum radiation formed by quartz. From right to left are the reflected beam, the $K\alpha$ line, and the $K\beta$ line. At the left is the primary beam. Between it and the $K\beta$ line can be seen the radiation refracted at small angles.

ments has been performed with is not in agreement with the others and the question of the accuracy of the grating measurements of x-rays has been raised. The question of the value of x-ray wavelengths is now about as uncertain as the problem of the value of e/m. Consequently the value of e/m has been calculated for the values of λ as determined by crystal measurements and by Bäcklin,[5] Bearden,[6] and Cork,[7] without passing any judgment upon the relative accuracies of the determinations. The absolute values for the molybdenum radiation were calculated from the proportion

$$\frac{\lambda_1 \text{ (corrected)}}{\lambda_1 \text{ (crystal)}} = \frac{\lambda_2 \text{ (grating)}}{\lambda_2 \text{ (crystal)}}$$

where λ_2 is the wave-length used in the absolute determinations, and the values of λ (crystal) have been corrected for refraction. If it is assumed that the differences between the observers are due to the experimental arrangements and the gratings, for the present purpose it is sufficient to use only one wave-length of each observer to calculate the molybdenum lines. The results are shown in Table II. The lines used as standards were chosen because they were single.

To obtain the values of e/m, the most probable values of the constants as given by Birge[1] were used. The density of the quartz was determined as

[5] E. Bäcklin, Dissertation, Upsala (1928).
[6] J. A. Bearden, Proc. Nat. Acad. Sci. **15**, 528 (1929)
[7] J. M. Cork, Phys. Rev. **35**, 1456 (1930).

TABLE II. *Absolute values of* λ *as determined by different observers.*

Observer	λ used	$\lambda\alpha_1(A\,U)$	$\lambda\beta(A\,U)$
Crystal Method		0.7078	0.6314
Bäcklin	Mo $L\beta$	.7087	.6322
Bearden	Cu $K\beta$	.7095	.6329
Cork	Mo $L\beta$	.7100	.6333

2.6480 ± 0.0003 gm/cm^3; the molecular weight was taken as $60.06 \pm .03$. The results are listed in Table III. The probable errors are calculated from those

TABLE III. *The values of* e/m.

Observer or method	$e/m \times 10^{-7}$ determined from $K\alpha_1$	$e/m \times 10^{-7}$ determined from $K\beta$
Crystal	1.773	1.773
Deflection	$1.769 \pm .002$	
Bäcklin	$1.768 \pm .007$	$1.769 \pm .007$
Bearden	$1.764 \pm .001$	$1.765 \pm .001$
Cork	$1.762 \pm .0013$	$1.763 \pm .001$
Spectroscopic	$1.761 \pm .001$	

given by Bäcklin and Bearden without evaluation, and from an arbitrary error of 0.01 percent assigned to Cork's results. This is not meant to imply any judgment as to the relative accuracy of the experiments.

It is clear that the determination of e/m from the refraction of x-rays must wait upon the solution of the absolute values of x-ray wave-lengths. The range of values of e/m, as determined from the values of the different investigators, is nearly as great as the original differences in e/m. If the average of all the experimenters is taken, the resulting value[8], 1.765×10^7, lies midway between the two older values. The two later experiments lead to a value closer to the spectroscopic value, but this does not warrant a definite conclusion, since one of them gives a result nearly midway between the older values. Whenever the absolute values of x-ray wave-lengths are known accurately, the method of this paper offers a way of evaluating e/m that is independent of the older methods and of the same order of accuracy.

I wish to thank the physics department of the University of Chicago for the apparatus placed at my disposal, and Professor A. H. Compton for his interest in all stages of the work.

[8] If the dispersion formula had been used in the form

$$\delta = \sum_i \frac{n_i e^2}{2\pi m(\nu^2 - \nu_i)^2}$$

where ν_i is a critical absorption frequency of the dispersive medium, the values of δ and e/m would be increased by unity in the last significant place.

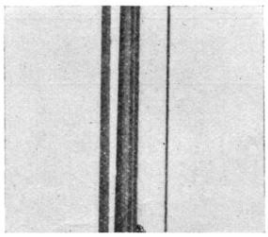


Fig. 3. A refraction spectrum of the molybdenum radiation formed by quartz. From right to left are the reflected beam, the $K\alpha$ line, and the $K\beta$ line. At the left is the primary beam. Between it and the $K\beta$ line can be seen the radiation refracted at small angles.