THE INTENSITY OF X-RAY SATELLITES*

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Abstract

By means of the two-crystal ionization spectrometer measurements have been made of the wavelengths and intensities relative to $K\alpha_1$ of the satellites $K\alpha_{3,4}$ of Cu (29). $K\alpha_{3,4}$ has characteristics of a doublet and is separated, photographically, for elements below Sc(21) but is unresolved for elements of higher atomic number. The greater resolving power of the two-crystal spectrometer shows that this "doublet" contains, probably, more than the two components $K\alpha_3$ and $K\alpha_4$, the wavelengths of which latter were found, respectively, to be 1531.15 and 1530.15 X.U. The intensity of either of these components, at 40 K.V., is of the order of 0.25 percent of the intensity of $K\alpha_1$. A similar study of the $K\alpha_{3,4}$ doublet of Ni (28) suggests that this doublet may contain four components.

IT IS generally agreed that x-ray satellites have their origin in "doubly ionized atoms[1] for which reason these lines are sometimes called "spark lines." A knowledge of the intensities of the satellites relative to each other and to the intensities of the respective parent lines should therefore provide a basis for estimating the relative probabilities of single and double ionization in the target of the x-ray tube. Such information would materially assist in clarifying the present theories regarding the origin of these lines.

Up to the present only the roughest estimates of the intensities of satellites have been made. Practically all studies of satellites have been made by the photographic method. To bring out the satellites on a photographic plate requires that the parent lines should, usually, be very much over-exposed, so that direct comparison of intensities is difficult. Further, many of the satellites lie "in the shadow" of the parent line, the shape of which, were there no satellites, must be estimated (guessed!) in order to make even a semiquantitative determination of relative intensity.

According to such estimates as are available the relative intensities of satellites vary from a small fraction of one percent to several percent of the intensities of the respective parent lines. The several satellites of a given line vary among themselves. For example, the $K\alpha$ doublet has several satellites of which the doublet $K\alpha_{3,4}$ is much more intense than $K\alpha_{5,6}$. $K\alpha_4$ is more intense, perhaps by a factor of two, than $K\alpha_3$, for elements in the neighborhood of $Z=20$. But for Si(14) and below, $K\alpha_3$ is the more intense. The $L\alpha$ doublet, likewise, has several satellites[2], $L\alpha^{i}$, $L\alpha^{ii}$, $L\alpha^{iii}$ $\cdots$ $L\alpha^{vi}$ in order of

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[1] For a discussion of x-ray satellites and their origin see M. J. Druyvesteyn, Dissertation, Groningen, 1928. F. K. Richtmyer, Franklin Inst. Journal **208**, 337ff (1929).

[2] For terminology see F. K. Richtmyer, Jour. Franklin Inst., reference 1.

decreasing wave-length. Of these $L\alpha^{iv}$ and $L\alpha^{iii}$ are the most intense. $L\alpha^{v}$ and $L\alpha^{vi}$ are very faint.

The relative intensities vary, for any given satellite, with atomic number. The $K\alpha_{3,4}$ doublet is relatively strong for $Z=20$; it is very faint for $Z=30$ and practically disappears for $Z=33$. $K\alpha_{5,6}$ is easily observed from Na(11) to Si(14) but disappears above S(16).

It is also probable, since the excitation voltages of satellites are somewhat higher than those of the parent lines, that the law of increase of intensity with voltage differs from the corresponding law for the parent line, and hence, at least for voltages not too far above the excitation voltage, that the intensity of a satellite relative to parent line depends, for a given element, on the voltage.

The present investigation was undertaken to obtain information which might serve as a starting point for a systematic study of this general problem. The $K\alpha_{3,4}$ doublet of Cu(29), a (photographically unresolved) satellite of the $K\alpha_{1,2}$ doublet, was chosen as most suitable for an initial study. An x-ray tube with a copper target can be built to stand, with water cooling, an input of at least 2500 watts. And Davis and Slack have developed a window thin enough to transmit to the outside of the x-ray tube, a substantial proportion of Cu$K\alpha$ radiation so that it may be studied by an ionization spectrometer. The two-crystal x-ray spectrometer developed by Bergen Davis[3] and his collaborators is particularly suited for this purpose, partly because of its high resolving power and partly because of the intense beam of monochromatic radiation which it delivers.

With the two-crystal spectrometer and water-cooled, copper-target tube with thin window, observations were made of the apparent energy distribution through the line Cu$K\alpha_1$ and to a considerable distance toward shorter wave-lengths. The method of making observations requires no special comment, save only that every effort was made to maintain conditions constant during a run, by manually controlling voltage and tube current. The x-ray tube was run at approximately 40 kv and 50 ma.

The results are best shown by Fig. 1 which is the average of several runs. The abscissae represent Bragg reflection angles in seconds (starting from an arbitrary zero). The ordinate scale A gives the intensity distribution through the line $K\alpha_1$. The measurements below 300″ are plotted to ordinate scale B. A distinct "hump" is observed to extend from about 60″ to 200″. The observed points above and below this "hump" define a smooth line, *abc*. If the ordinates of this line be subtracted from the observations and these differences plotted on the abscissae scale there results the line *pqr*, which is the $K\alpha_{3,4}$ satellite of Cu(29). Photographic determination (by Siegbahn) of $K\alpha_{3,4}$ places the doublet (unresolved on the photographic plate) at 150″ as shown at "C" on the abscissae scale.

[3] A substantial proportion of the work herein described was done in the Physical Laboratory at Columbia University with the double x-ray spectrometer, x-ray plant and accessory equipment previously assembled by Professor Bergen Davis. To him and his colleagues, particularly Mr. Beller, the authors wish to express sincere thanks not only for the use of the apparatus, including the specially constructed x-ray tube, but for many other courtesies and suggestions during the progress of the work.

It is obvious that $K\alpha_{3,4}$ as here observed by the ionization spectrometer is at least a doublet, as Siegbahn found it to be by the photographic method for elements of atomic number below Sc(21). For, not only does the line have the appearance of a close doublet in spite of some scattering of the points, but, as shown on Fig. 1, the width at half-maximum of $K\alpha_1$ is 28 seconds of arc, while the corresponding width of $K\alpha_{3,4}$ is 83″. This suggests that $K\alpha_{3,4}$ may contain several components. The resolution is not sufficient to warrant an attempt accurately to determine these components. If, however, starting from the long-wave-length side, a line $rq'p'$, Fig. 1, be constructed with the same width at half-maximum as $K\alpha_1$, the long-wave-length component of $K\alpha_{3,4}$ can be approximately located. This long-wave-length component,

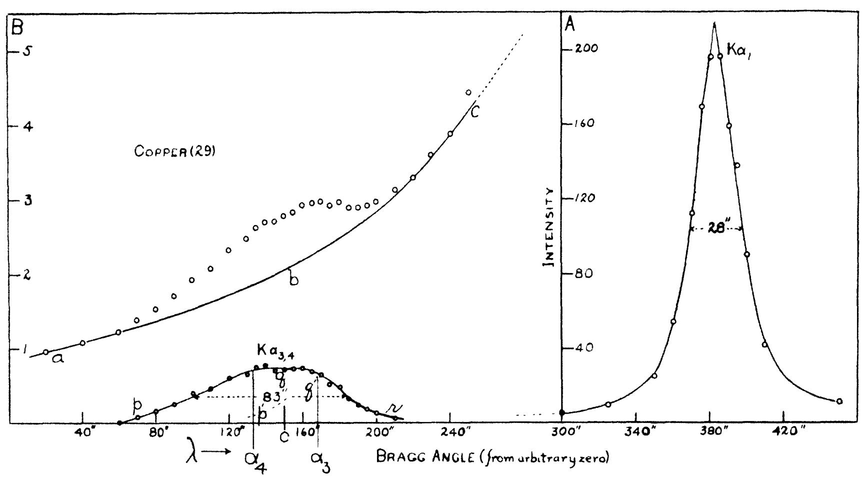

Fig. 1. Energy distribution through $K\alpha_1$ of Cu(29) including its satellites $K\alpha_{3,4}$. The abscissa scale represents Bragg reflection angles, in seconds, from an arbitrary zero. (The actual Bragg angle for Cu $K\alpha_1$ (calcite) is about 14°42′). Ordinate scale A gives the distribution through $K\alpha_1$. Ordinate scale B applies to the short wave-length "foot" of $K\alpha_1$ as shown by the line *abc*. The satellite doublet, $K\alpha_{3,4}$, shown by the line *pqr*, is the difference between the smooth line *abc* and the corresponding observed points. Two of the components of $K\alpha_{3,4}$ are estimated to be at "α_3" and "α_4", respectively, The photographic method locates the doublet at "C".

i.e. $K\alpha_3$, is thus located at "α_3" on the graph. Taking the maximum ordinate of the remainder as the next component, we locate $K\alpha_4$ at "α_4" on the graph. We are then able to compute the wave-lengths of $K\alpha_3$ and $K\alpha_4$ as follows:

	$K\alpha_3$	$K\alpha_4$
$\Delta\theta$ from $K\alpha_1$	217″	252″
λ from $K\alpha_1$	6.15 $X.U$	7.15 $X.U.$
λ ($\lambda K\alpha_1$ = 1537.30 $X.U.$)	1531.15″	1530.15″
$\Delta\nu/R$ from $K\alpha_2$	3.85	4.23
$(\Delta\nu/R)^{1/2}$ from $K\alpha_2$	1.96	2.06

Siegbahn gives 1530.7XU. as the wave-length of the (photographically unresolved) doublet $K\alpha_{3,4}$ of Cu (29). The mean of the wave-lengths given above is 1530.65XU.

One of us[4] has shown that a simple relation exists between the frequency ν of a satellite and that of the corresponding parent line; namely, $(\Delta\nu/R)^{1/2}$ is a linear function of atomic number, where R is the Rydberg constant and $\Delta\nu$ is the difference in frequency between the satellite and the parent line. For the $K\alpha_{3,4}$ satellites it was found that this linear relation held if $\Delta\nu$ was taken as the difference in frequency between $K\alpha_3$ or $K\alpha_4$ and the parent line $K\alpha_2$ (instead of $K\alpha_1$). The graph of this relation is shown in Fig. 2. The data for

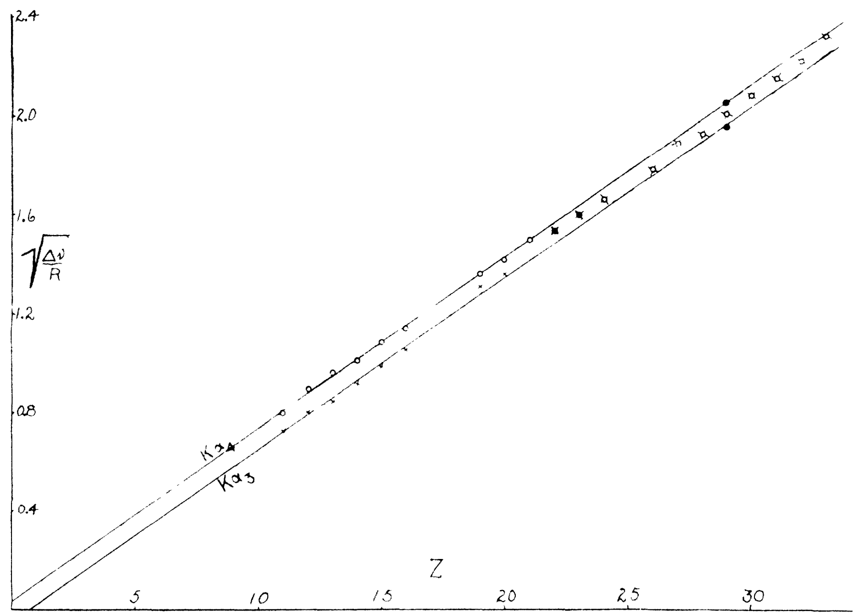

Fig. 2. The square root of the difference in frequency between a satellite and its parent line is a linear function of atomic number. Upper line: Square root of difference in frequency between $K\alpha_1$ and $K\alpha_2$; lower line, same for $K\alpha_3$. Source of data: Na(11) to Ni(28) from Siegbahn; Cu(29) to As(33) from F. K. R. at Cornell University). The solid circles are the present measurements for Cu(29) by the two-crystal ionization spectrometer.

the elements Na(11) to Ni(28) are taken from Siegbahn. Below Ca(20) the two components are resolved and are shown, respectively, by circles (for $K\alpha_4$) and crosses (for $K\alpha_3$). From Sc(21) to Ni(28) the "circle-cross" represents the unresolved doublet. The last five points (circle-cross), namely Cu(29) to As(33) are from unpublished data taken by one of us (F.K.R. at Cornell University). It is seen that these unresolved points fall between the two parallel lines defined by the elements, below Ca(20), for which the doublet is resolved.

[4] F. K. Richtmyer, Phil. Mag. **6**, 64 (July, 1928); Franklin Inst., Jour. **208**, 325 (Sept., (1929). Also F. K. Richtmyer and R. D. Richtmyer, Phys. Rev. **34**, 574 (1929)

The two solid circles at Cu(29) represent the doublet as resolved by the two-crystal spectrometer (See above data table). The closeness with which these two points fall on the respective lines seems to justify the conclusion that the difference in $(\Delta\nu/R)^{1/2}$ is constant and independent of atomic number, which empirical fact forms a basis for the term "doublet" as applied to $K\alpha_{3,4}$.

Referring now to Fig. 1 the maximum ordinate of $K\alpha_{3,4}$ is about 0.72; that of $K\alpha_1$ is about 215. The intensity of the components of $K\alpha_{3,4}$ is not greater than $0.72/215 = 0.0034$ of that of $K\alpha_1$, and in reality is somewhat less, since the line *pqr* has a higher maximum than any of its components. In round numbers, therefore, we may say that $K\alpha_1$ is some 400 times as intense as the $K\alpha_{3,4}$ satellites for the element Cu(29) at 40 kv.

A similar study was made of the $K\alpha_{3,4}$ of Ni(28). The results of two runs at two different voltages are shown at *A* and *B* of Fig. 3. The satellites are

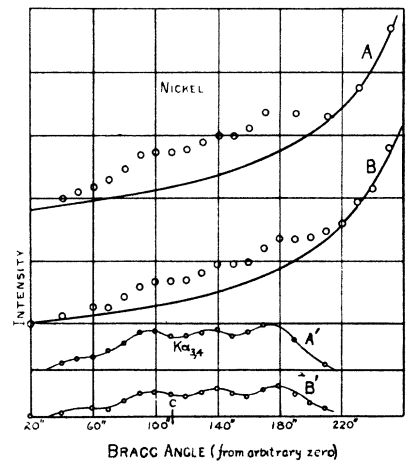

Fig. 3. The $K\alpha_{3,4}$ satellites of Ni(28). *A* and *B* represent observed energy distribution along the foot of $K\alpha_1$ of Ni, for two different tube voltages; *A'* and *B'*, the corresponding energy distribution through the satellites. The photographic method places the (unresolved) satellite at "*C*".

are shown at *A'* and *B'*. Photographic measurement places the doublet at "*C*." Further measurements are necessary to determine whether $K\alpha_{3,4}$ for Ni actually has three (or perhaps four) components as these data seem to indicate. Unpublished data[5] at Cornell University show that $K\alpha_3$ splits up into two components for elements below S(16).

No determination of the intensity of $K\alpha_{3,4}$ for Ni(28) relative to $K\alpha_{1,2}$ has been made since, as may be observed from Fig. 3, the absence of observations on the short-wave-length side makes it impossible to locate the base line accurately. However, the order of magnitude of relative intensity is the same as for Cu(29).

Since $K\alpha_1$ and $K\alpha_2$ originate from the same initial ionized state, namely single *K*-ionization, we may, by analogy, assume that the components of

[5] By O. Rex Ford.

$K\alpha_{3,4}$ originate in the same double ionized state which we may for the present call a KX-ionization, "X" standing for a missing electron the identity of which is as yet undetermined. Let P_K and P_{KX} be the relative probabilities of these single and double ionizations, respectively, under any given set of conditions—tube current, voltage, etc. Then, these data on relative intensities of one of the components of $K\alpha_{3,4}$ to $K\alpha_1$ make possible the estimate that (for Cu(29) at about 40 kv).

$$P_{KX}/P_K \geqq 1/400.$$

This estimate, of course, is very rough and gives only the order of magnitude of the ratio, as may be seen from the following: A K-ionized atom may return to the normal state by various routes. As a first step, it may emit any one of the diagram lines of the K series; or, as a result of autophotoelectric absorption, it may emit a photoelectron. We may expect, likewise, that a KX-ionized atom might return to the normal state in various ways other than by the emission of $K\alpha_{3,4}$. Indeed, there is some evidence[6] to indicate that, associated with some, at least, of the satellites, there is a continuous spectrum which may account for a considerable proportion of the doubly ionized atoms. The asymmetry of the satellite curves in Figs. 1 and 3—i.e., these curves are less steep on the short wavelength side—is suggestive of a continuous spectrum similar to that found with $L\alpha$ satellites in Fig. 1 of the article just cited[6].

It is hoped later to make a more extensive report on relative intensities of satellites on the basis of work now in progress at Cornell University.

[6] F. K. Richtmyer and R. D. Richtmyer, Phys. Rev., reference 4.