BARKHAUSEN EFFECT
II. DETERMINATION OF THE AVERAGE SIZE OF THE DISCONTINUITIES IN MAGNETIZATION

By Richard M. Bozorth and Joy F. Dillinger
Bell Telephone Laboratories, New York, N. Y.

(Received February 12, 1930)

Abstract

When the magnetic field-strength acting on a ferromagnetic material is changed, the magnetization changes discontinuously (Barkhausen effect). These discontinuous changes have been examined in 1 mm wires; an expression is derived and experimental arrangements are described for determining their average size for a given material in a given state of magnetization.

Experimental determinations of the average size have been made for iron (including a single crystal and a hard-drawn wire), nickel, and several iron-nickel alloys (permalloys). The average size is greatest on or near the steepest part of the hysteresis loop. The greatest average size, expressed as the volume of material the magnetization of which must be changed from saturation in one sense to saturation in the opposite sense to produce the same change in magnetization, is much the same for all of the materials examined, the extremes being 1.2×10^{-9} cm^3 for annealed iron and 45×10^{-9} cm^3 for 50 percent nickel permalloy. This shows that the sizes of the discontinuities do not depend to any considerable extent on the size or kind of crystals.

Criticism is made of previous work on the size of the coherence region, the region within which the change in magnetization is confined. Although the effect of a single discontinuity in magnetization may be detected as far as 10 cm from its source because of the eddy-currents induced, the experimental evidence is consistent with the view that the permanent change in magnetization is confined to the volume in terms of which the size of the discontinuity is measured as stated above, always less than 10^{-5} cm^3.

Introduction

It is a well-established fact that when a continuously-varying magnetic field is applied to a ferromagnetic substance, the induction varies not smoothly, but by sharp sudden jumps or "discontinuities." One may interpret it by supposing that the substance is made up of very small regions or "units," each of which is always magnetized to saturation, and that during the magnetization of the sample, the magnetic axes of half of these are reversed, one after another. The volume v of each unit may then be computed from the amount of the corresponding discontinuity. Of course, it may be that the change in the magnetic moment of a unit is less drastic than a complete reversal of all the atomic magnets; in that case, the value of v computed by our formula will be proportionately smaller than the actual volume of the region. However, it is simpler, and we believe also that for discontinuities on the steeper parts of the hysteresis loops it is correct, to interpret v in the first-mentioned manner.

We wish to know the average sizes of these units for as many ferromagnetic materials and as many parts of the magnetization curve as possible. Especially we wish to compare it with the sizes of the crystals on one hand, of the smallest magnetizable atom-groups on the other. For instance, one of us[1] has shown, for a variety of materials, that on the steeper parts of the hysteresis loop nearly all the discontinuities correspond to units larger than 10^{-13} cm^3, comprising more than 10^{10} atoms.

In the present work we have obtained data relevant to these questions by determining the average amount of the discontinuities on various segments of the hysteresis-loops of several substances: iron, nickel, several permalloys both annealed and hard-worked, and a single crystal of iron.

Theory

In our apparatus a slowly and uniformly changing magnetic field acts on the cylindrical specimen P, Fig. 1. When a sudden change, characteristic of the Barkhausen effect, occurs in the induction B in the sample, an e.m.f.

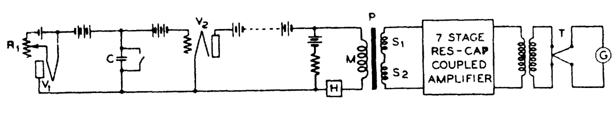

Fig. 1. Diagram of the apparatus.

is induced in one of the search coils S which are connected in series opposition to the input of an amplifier. At any instant the e.m.f. impressed on the amplifier is equal to the algebraic sum of the separate e.m.f.'s induced in the two coils. The e.m.f. in each coil is proportional to the dB/dt in the material inside of that coil, consequently when the coils are connected in opposition the e.m.f. acting on the amplier, and the output current, i, are proportional to the difference between the values of dB/dt within the two coils. Considering dB/dt in one coil to be positive and that in the other to be negative, the difference is $(dB/dt)_1+(dB/dt)_2$, and the mean value of this sum, or $\overline{dB/dt}$ for the combination, is zero. The thermocouple T measures $\overline{i^2}$ (the mean value of i^2) which is then proportional to $\overline{(dB/dt)^2}$. We can now use a statistical relation derived by T. C. Fry,[2] provided the following assumptions are valid:

(1) The law of superposition holds; that is, the e.m.f. impressed on the amplifier at any time is the algebraic sum of the separate e.m.f.'s induced by separate sudden changes in induction.

(2) The discontinuities are random in character both as to magnitude and time; that is, the magnitude and exact time of beginning of any sudden change in induction is independent of previous changes.

[1] R. M. Bozorth, Phys. Rev. **34**, 772–784 (1929). See also an abstract of the present paper, Phys. Rev. **33**, 1071 (1929).

[2] T. C. Fry, J. Frank. Inst. **199**, 203–220 (1925).

(3) The form of the induction-vs-time relation is the same for all the impulses produced by the various discontinuities; that is, any impulse is identical with the product of any other by some constant factor.

(4) The time required for the thermocouple reading to reach a steady value is large compared to the duration of a single current impulse but small compared to the gross changes in the output of the amplifier, corresponding to what may be considered changes in the magnetic state of the material.

These assumptions are valid except in relatively minor details. Assumption (2) is not exactly true unless the material inside one search coil has exactly the same magnetic characteristics as that inside the other. In reality materials are not quite homogeneous. Assumption (3) implies that the magnetization in a small volume changes very suddenly but that the e.mf. induced in the search coil is influenced by the rate of decay of the eddy-currents set up by the change in magnetization. If these small volumes are located at different distances from the axis of the sample, the eddy-currents will decay at slightly different rates and assumption (3) is not strictly correct.

The equation derived by Fry is

$$S=\bar{\nu}\bar{w}, \qquad (1)$$

where S is the power expended in the thermocouple, $\bar{\nu}$ the mean number of separate impulses occurring per second, and $\bar{w}$ is the average energy expended in the thermocouple by a single impulse. Obviously,

$$S=\overline{i^2}R, \qquad (2)$$

where R is the resistance of the thermocouple and $\overline{i^2}$ is the mean square current through it. The mean frequency of the impulses, $\bar{\nu}$, may be expressed in terms of the average volume of material, $\bar{v}$, the magnetization of which is assumed to change from saturation in one direction to saturation in the other:

$$dB/dt=8\pi I_s\overline{v\nu}/V, \qquad (3)$$

where I_s is the saturation value of magnetization and V is the total volume of the sample in which magnetic discontinuities affect the search coils.[3] The energy w for one discontinuity is

$$w=\int_0^\infty i_1{}^2R\,dt, \qquad (4)$$

where i_1 is the current (a function of time) which would flow in the thermocouple if a single isolated discontinuity occurred in the sample. This current is proportional to the rate of change of induction during this single event,

$$i_1=A\pi r^2 dB_1/dt, \qquad (5)$$

[3] B is used here as an approximation for $B-H$, since in these experiments H is always small compared with B. The apparatus measures changes in $B-H$ rather than in B.

where A is a constant determined by calibrating the apparatus and r is the radius of the sample. Also, for a single impulse the induction will be a function of time,

$$B_1=\frac{8\pi I_s v}{V}f(\tau), \tag{6}$$

where $f(\tau)$ is the same function of time for all impulses (assumption 3) and is determined by the decay of eddy-currents, and v is the volume which changes in magnetization.

Combining Eqs. (4), (5) and (6), and taking the mean value,

$$\overline{w}=64\pi^4 A^2 R I_s^2 r^4(\overline{v^2}/V^2)\int_0^\infty [f'(\tau)]^2 d\tau .$$

Using also Eqs. (1), (2) and (3), and putting $V/\pi r^2=l$, the length of the specimen which affects the search coils, we have the result

$$\frac{\overline{v^2}}{\bar{v}}=\frac{l\overline{i^2}}{8\pi^2 A^2 I_s r^2 dB/dt\int_0^\infty [f'(\tau)]^2 d\tau} . \tag{7}$$

The function $f(\tau)$ is subject to the condition that $f(\infty)=1$. Its time deriative, $f'(\tau)$, characterizing the decay of eddy-currents, might be determined experimentally as a function of μ, ρ and r by using a suitable oscillograph to record the wave-form of the current impulses, caused by single Barkhausen discontinuities, which pass through the thermocouple heater. Since the recording of these wave-forms puts a very severe requirement on the oscillograph, and since it is a laborious undertaking in any case to determine $f'(\tau)$ completely in terms of μ, ρ and r, we have used the expression derived by Wwedensky[4] and supported by experiment. For a spontaneous change in magnetization, we find

$$\int_0^\infty [f'(\tau)]^2 d\tau=\frac{2(10)^9\rho}{\pi r^2\mu}, \tag{8}$$

where ρ is the resistivity in ohm-cm, and μ is the permeability which controls the rate of decay of eddy-currents, in this case the reversible permeability or permeability for small alternating fields. Substituting Eq. (8) in Eq. (7),

$$\frac{\overline{v^2}}{\bar{v}}=\frac{l\mu\overline{i^2}}{16\pi(10)^9 A^2 I_s\rho dB/dt} . \tag{9}$$

Wwedensky's expression was derived for a long cylinder the magnetization of which is uniform, consequently Eq. (8) is not strictly applicable to the present problem and must be considered as an approximation. Even if the

[4] B. Wwedensky, Ann. d. Physik **64,** 609–620 (1921).

sudden change in magnetization occurs entirely within a volume small compared to the cube of the radius of the specimen, the disturbance will spread along the axis of the wire to an extent depending on the permeability, resistivity, and diameter of the wire. The extent of the disturbance, necessary for choosing the proper value of l to be used in equation (9), can be determined as described in the next section.

The ratio $\overline{v^2}/\bar{v}^2$ is very near unity for most distribution functions, but may in certain cases be very large. One of us[1] has shown that there are very few discontinuities corresponding to volumes several orders of magnitude smaller than the average. Taking this into consideration, we may with reasonable safety extrapolate to the origin the size distribution curve determined by Tyndall[5] for silicon steel. For the curve so extrapolated we find that $\overline{v^2}/\bar{v}^2 = 0.7$, a value sufficiently close to unity to make $\overline{v^2}/\bar{v}$, as given by Eq. (9), a reasonably good approximation to $\bar{v}$, the average volume in which we are interested.

All of the quantities on the right-hand side of Eq. (9) can be determined experimentally without unusual difficulties, as described below.

Application to Experiment

Fig. 1 shows the experimental arrangement for measuring $\overline{i^2}$ of Eq. (9). The magnetizing coil, M, is 60 cm long; each search coil is 3.8 cm long, 1.7 cm outside diameter, 0.4 cm inside diameter, and is wound with 10,000 turns of No. 40 B.E.S.S. copper wire. The samples are 60 cm long and 1 mm in diameter. The magnetizing coil is supported on rubber bands in a permalloy shield 5 mm thick to protect it from mechanical and magnetic disturbances. The arrangement for changing the magnetic field slowly and uniformly is that described previously.[1] The amplifier is resistance-capacity coupled with resistances of 70,000 ohms and capacities of one microfarad. For the first stage a screened grid vacuum tube is used (Western Electric 246-A), for the next five stages 239-A tubes, and for the last stage a 104-D tube. The amplification may be varied by using different taps on the input resistance of the second stage. The space current of the last tube passes through the primary of a transformer the secondary of which is connected to the heater of a thermocouple. The impedances of the transformer match respectively the impedances of the vacuum tube in the last stage of the amplifier and the heating element of the thermocouple. The thermocouple is connected to a critically damped Moll galvanometer, G, the deflections of which are recorded on photographic paper mounted on a rotating drum. With this arrangement the deflection δ of the galvanometer, as measured on the paper, is proportional to the mean value of the power expended in the thermocouple heating element, i.e., $\delta \propto \overline{i^2}$.

The mean rate of change of induction, dB/dt, is determined by observing dB/dH and dH/dt separately. To determine dB/dH, the two search coils are connected in series aiding to the Moll galvanometer and the field is changed at a uniform rate. The galvanometer deflection, proportional to

[5] E. P. T. Tyndall, Phys. Rev. **24**, 439–451 (1924).

dB/dH, is recorded photographically as before. The calibration is made by integrating the $dB/dH - vs. - H$ curve and comparing it with the total change in B as determined in the usual way with a ballistic galvanometer. Lines indicating predetermined values of H are marked on the paper by flashing a light when the field-strength passes through these values.

The rate of change of field-strength, dH/dt, is determined by noting with a stop-watch the times at which the needle of the milliammeter in the magnetizing circuit passes through chosen positions.

The reversible permeability μ is determined by subjecting the sample to the small sinusoidal field created in a small single-layer "calibrating coil" inside the search coils and magnetizing coil. A measured current, i_s, of frequency 5 cycles/sec is passed through the calibrating coil, the search coils are connected to the amplifier in series aiding and the deflection of the galvanometer, δ_s, noted for a given value of H, the strength of the steady field produced by the magnetizing coil. The calibration for determining μ is made by removing the sample and observing the deflection δ_c, when a larger current, i_c, of the same frequency passes through the calibrating coil. The permeability may then be calculated as follows. The root mean square current passing through the heater of the thermocouple is proportional to the root mean square rate of change of flux threading the search coils. Since δ is proportional to the square of the current,

$$\delta_s^{1/2} = K i_s (\mu A_s + A_c - A_s),$$

and

$$\delta_c^{1/2} = K i_c A_c,$$

where A_s is the cross-sectional area of the sample, A_c that of the calibrating coil, and K a proportionality constant. The permeability is then given by:

$$\mu = \frac{A_c i_c \delta_s^{1/2}}{A_s i_s \delta_c^{1/2}} - \frac{A_c - A_s}{A_s}$$

and must be determined for various values of H. For the coil and samples used, $A_c = 0.083$ cm^2 and $A_s = 0.0081$ cm^2 so that

$$\mu = \frac{10.2 i_c \delta_s^{1/2}}{i_s \delta_c^{1/2}} - 9.2.$$

The maximum value of the alternating field-strength, calculated from i_s and the number of turns per unit length of the calibrating coil, was kept as low as 0.001 gauss for the more permeable materials.

The values of μ so determined were checked in several cases with those determined with a ballistic galvanometer by measuring ΔB for a change ΔH after several reversals of the field-strength between H and $H + \Delta H$, and extrapolating for the value of $\Delta B/\Delta H$ at $\Delta H = 0$.

The factor A in Eqs. (5) and (9) is the ratio of the current in the thermocouple heater to the rate of change of flux inside the search coils. This is determined by producing a known sinusoidal rate of change of flux in the calibrating coil and measuring the amplifier output i at the same time. If i_c is the root mean square current in milliamperes through the calibrating coil, f the frequency of the current, n the number of turns of wire per cm, and A_c the cross-sectional area, the root mean square value of the rate of change of flux is

$$(d\phi/dt)_c = 8\pi^2 n A_c f i_c (10)^{-4}$$

and therefore

$$A = \frac{i}{d\phi/dt} = \frac{10^4 i'}{8\pi^2 n A_c f i_c},$$

where i' is the root mean square value of i measured at the same time as i_c. Since i' is proportional to $\delta'^{1/2}$ and i is similarly proportional to $\delta^{1/2}$, we may rewrite Eq. (9) as

$$\frac{\overline{v^2}}{\bar{v}} = \frac{l\mu\delta}{16\pi(10)^9 C^2 I_s \rho dB/dt} \tag{10}$$

where

$$C = \frac{(10)^4 \delta'^{1/2}}{8\pi^2 n A_c f i_c}. \tag{11}$$

The calibrating deflection δ' is recorded on the same paper as the deflections δ caused by the Barkhausen discontinuities. Since the value of C depends on f directly as expressed by Eq. (11) and indirectly through the frequency

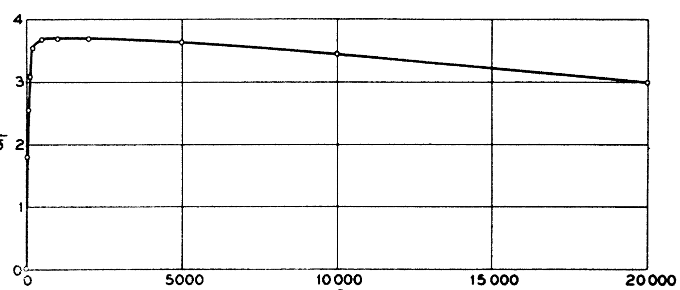

Fig. 2. Frequency characteristics of the apparatus (amplifier, transformer, thermocouple and galvanometer).

characteristics of the apparatus, the amplification was measured for a series of frequencies. At each frequency 0.7 milliampere was passed through a fixed resistance of 0.1 ohm connected to the input of the amplifier in parallel with the search coils, and the deflection of the galvanometer noted. The results are shown in Fig. 2. The range of frequencies shown is sufficient to record accurately the Barkhausen impulses in the materials examined. Calibration was made at the frequency of 60 cycles/sec., and the value of C^2 so obtained has been multiplied by 2 to take account of the lower amplification at this frequency, compared with that in the range of 300 to 5000 cycles.

To determine the length of the sample, l, in which discontinuities will affect the search coils, it is necessary to know how the intensity of a magnetic disturbance decreases with the distance from its source. This was determined experimentally for a disturbance produced by a sinusoidal current flowing in a few turns of fine wire wound directly on a specimen. The law of decay of intensity with distance once determined, it was assumed that the magnetic disturbance caused by a discontinuity in magnetization started at a point and spread along the wire according to this same law. On the basis of this assumption a calculation was made of the change in the observed Barkhausen effect, as measured by the galvanometer deflection δ, when the two search coils (connected in series opposing) were brought closer and closer together,

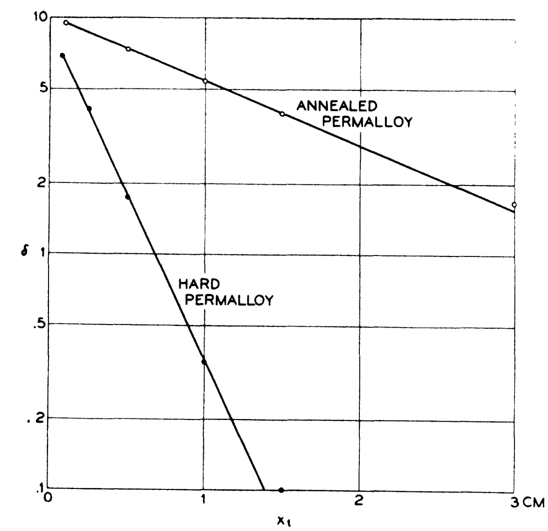

Fig. 3. Decay of magnetic disturbance with distance along wires of different permeabilities.

and this calculation was compared with observation. Ten turns of No. 40 wire were wound closely around the middle of a piece of annealed permalloy containing 81 percent Ni, and a current of 0.5 milliampere of frequency 35 cycles/sec. was passed through it. Two search coils of 0.15 cm inside diameter and 1.3 cm length were placed at equal distances on each side of the 10-turn coil and the deflection δ_1 of the galvanometer noted for various distances x_1 between the small coil and the centers of the search coils. The results (Fig. 3) show that $\delta_1 = C_1 e^{-ax_1}$, where C_1 and a are constants. Assuming that the rate of change of magnetic flux, $d\phi/dt$, at each point along the sample follows this exponential law, and remembering that $d\phi/dt$ is proportional to $\delta^{1/2}$, we have

$$d(\delta)^{1/2} = C_2 e^{-ax/2} dx \qquad (12)$$

for each point along the wire. Since the total effect is obtained by adding the value of $\delta^{1/2}$ contributed by each element along the wire, in a search coil of length l_1 we have

$$\delta_1{}^{1/2} = \int_{x_1-l_1/2}^{x_1+l_1/2} C_2 e^{-ax/2} dx \tag{13}$$

or

$$\delta_1 = \frac{4C_2}{a^2}(e^{al/4} - e^{-al/4})^2 e^{-ax_1},$$

agreeing with the experimentally determined relation in that $dln\delta_1/dx_1 = -a$. For the sample described above, $a = 0.60$.

Applying Eq. (12) to the Barkhausen effect, we get Eq. (13) for each discontinuity in magnetization which starts at a point in the sample outside of the search coil. For points inside of the search coil,

$$\delta_1'^{1/2} = \int_0^{x_1+l_1/2} C_2 e^{-ax/2} dx + \int_0^{x_1+l_1/2} C_2 e^{-ax/2} dx,$$

and integrating for all points on the sample the galvanometer deflection δ is given by

$$\delta^{1/2} = 2\int_{l_1/2}^{\infty} \delta_1{}^{1/2} dx_1 + 2\int_0^{l_1/2} \delta_1'^{1/2} dx_1,$$

or,

$$\delta = 16 C_2{}^2 l_1{}^2 / a^2. \tag{14}$$

To determine the equivalent length l of Eq. (10), used when two search coils are connected in series opposing, δ is recalculated assuming that all impulses originating at points within a length $l/2$ of the specimen affect all turns of one search coil as much as a real impulse occurring within a search coil affects the turns immediately adjacent. This substitutes for Eq. (12),

$$d(\delta)^{1/2} = C_2 dx, \tag{12'}$$

for Eq. (13)

$$\delta_1{}^{1/2} = \int_{x_1-l/2}^{x_1+l/2} C_2 dx = C_2 l_1, \tag{13'}$$

and for Eq. (14)

$$\delta^{1/2} = \int_0^{l/2} \delta_1{}^{1/2} dx_1 = C_2 l_1 l/2,$$

or,

$$\delta = C_2{}^2 l_1{}^2 l^2 / 4. \tag{14'}$$

Comparison with (14) shows that $l = 8/a$ independent of l_1, the length of each search coil. The value of a is determined experimentally as described above. The data are shown graphically in Fig. 3. A variation in the frequency of the alternating current used in this experiment between 10 and 1000 cycles per second showed only a slight change in a with frequency. The same experiment was repeated using a hard-drawn wire of the same composition and diameter and a for this sample was found to be 3.22. The ratio of the values of a for these two samples is 5.4, nearly equal to the inverse ratio of the square-roots of their permeabilities, namely $(2510/75)^{1/2}=5.8$. It is concluded that l is proportional to $\mu^{1/2}$, when the resistivity is constant. If the resistivity ρ varies, l must be proportional to $(\mu/\rho)^{1/2}$ since it is only in the ratio μ/ρ that μ and ρ appear in Maxwell's equations. Using the directly determined value of a for the sample with $\mu=2510$ and $\rho=15(10)^{-6}$ ohm-cm, we can therefore finally put

$$l=1.03(10)^{-3}(\mu/\rho)^{1/2}. \tag{15}$$

A more extensive analysis of the dependence of δ on the distance, y, between the two search coils used in measuring the Barkhausen effect, shows that a may also be determined from the relation between δ and y. Such a determination has been made for one sample and found to agree with the determination as described above.

Combining Eq. (14) with Eq. (10) and designating the ratio $\overline{v^2}/\bar{v}$ by v, we have

$$v=\frac{2.0(10)^{-14}\mu^{3/2}\delta}{C^2 I_s \rho^{3/2}(dB/dH)(dH/dt)} \tag{16}$$

in which all of the quantities are already known or can be determined as described in this section.

The Experimental Results

The following materials were examined in the form of wires 60 cm long and 0.1 cm in diameter:

Armco iron, hard drawn.

Armco iron, vacuum annealed 2 hrs. at 1000°C, cooled 300°/hr.

Large crystals of Fe[6].

Nickel, vacuum annealed 2 hrs. at 1100°C, cooled 300°/hr.

Permalloy, 80.5 percent Ni, vacuum annealed 1 hr. at 1200°C, 2 min. at 830°C, cooled about 1000°/min.

Permalloy, 78.1 percent Ni, vacuum annealed 1 hr. at 1100°C, cooled about 1000°/hr.

Permalloy, 50 percent Ni, vacuum annealed 2 hrs. at 1100°C, cooled 300°/hr.

Figs. 4 to 6 show the photographically registered curves $dB/dH-vs.-H$ for annealed iron, permalloy with 80.5 percent Ni, and permalloy with 50 percent Ni. In taking the photographs, as described in the previous section, the field-strength was varied slowly enough to enable the galvanometer to follow the gross changes in dB/dH, but not slowly enough to record the

[6] We are indebted to Dr. D. Foster of these Laboratories for these crystals. For their preparation and properties, see Phys. Rev. **33**, 1071 (1929).

individual discontinuities. It will be noticed that when the Barkhausen effect is large the curve is relatively irregular. Two photographs of each curve are reproduced to show that some irregularities are due to chance and are different in each run, and that some are permanent characteristics of the material and are reproduced during each magnetic cycle. Irregularities of both kinds are present but difficult to detect when hysteresis

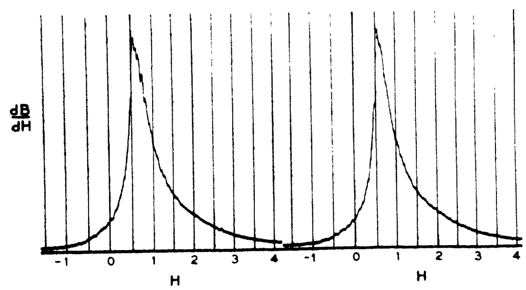

Fig. 4. Galvanometer record of dB/dH for annealed iron.

loops are determined in the usual way with a ballistic galvanometer, small variations from a smooth curve being generally indistinguishable from experimental errors.

The regular procedure was modified slightly for measurements on the large crystals. From a 1 mm wire composed entirely of long crystals, two segments were cut, each 14 cm in length, one containing a single crystal

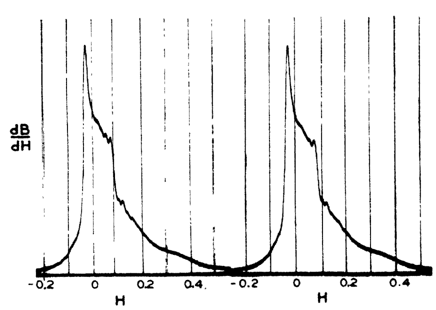

Fig. 5. Galvanometer record of dB/dH for permalloy containing 80.5 percent nickel and 19.5 percent iron.

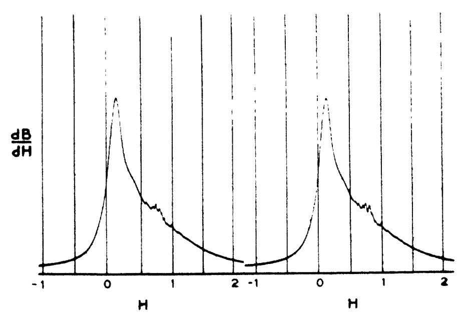

Fig. 6. Galvanometer record of dB/dH for permalloy containing 50 percent nickel and 50 percent iron.

7.2 cm long and the other a single crystal 8.6 cm long, each single crystal occupying the central portion of its segment. The remainder of the two pieces consisted of only four separate crystals, one at each end of the two longest crystals. One search coil was placed around the middle of each of the two segments, the nearest ends of which were 3 cm apart, and measurements of dB/dH, δ, and μ made without moving the specimens of coils.

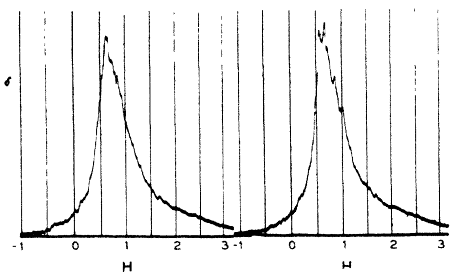

Fig. 7. Record of Barkhausen effect for annealed iron.

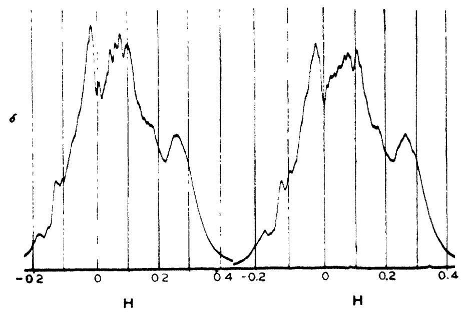

Fig. 8. Record of Barkhausen effect for permalloy containing 80.5 percent nickel and 19.5 percent iron.

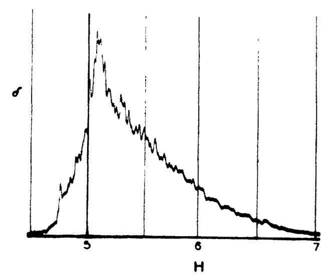

Fig. 9. Record of Barkhausen effect for hard iron.

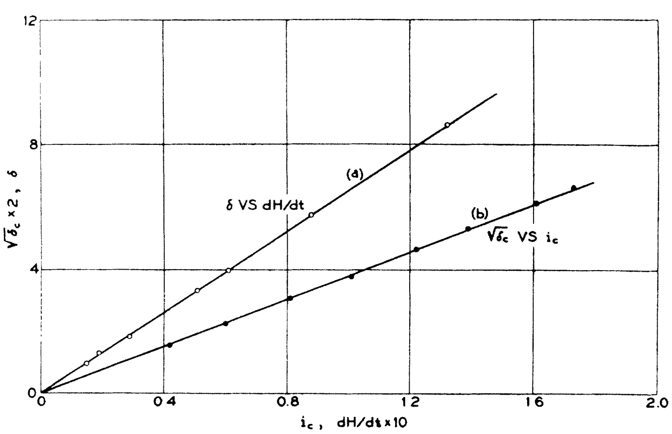

Fig. 10(a) Barkhausen effect in hard permalloy when $H=4.5$, measured by galvanometer deflection δ, as dependent upon rate of change of field-strength. The linear relation shows that the number of the discontinuities depends only on the amount of change of H, and not on dH/dt.

(b) Calibration curve showing that the deflection δ_c is proportional to the square of the calibrating current i_c.

These two crystals are so long that the only discontinuities detected by the coils occurred actually in the crystals.

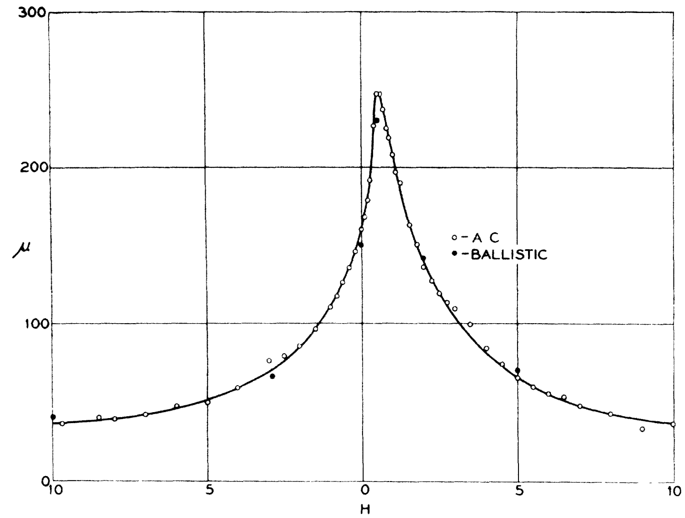

Fig. 11. Comparison of alternating current with ballistic method of determining the permeability for very small alternating fields. The sample is a 1 mm nickel wire, the frequency of the alternating current was 5 cycles/sec., the maximum field-strength during each cycle was 0.006 gauss.

Figs. 7 to 9 are reproductions of the photographically recorded Barkhausen curves, $\delta - vs. - H$, for some of the materials listed above. Duplicate

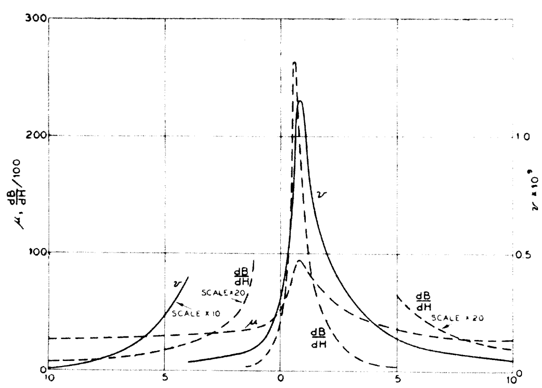

Fig. 12. The average volume, v, of the discontinuities in iron as dependent upon field-strength, and the data used in the determination.

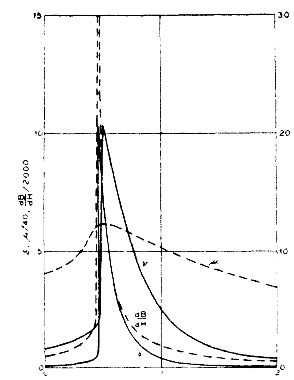

Fig. 13. The v-curve for nickel, and the data from which it was calculated.

runs for annealed iron and permalloy containing 80.5 percent nickel indicate how well the data can be repeated.

According to Eq. (16) the galvanometer deflection δ should be proportional to the rate of change of magnetic field-strength, dH/dt, when the other variables are fixed. Fig. 10(a) shows that this relation was observed, and for

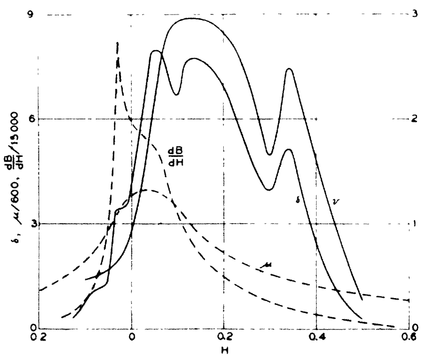

Fig. 14. The v-curve for permalloy containing 80.5 percent nickel and 19.5 percent iron, and the data from which it was calculated.

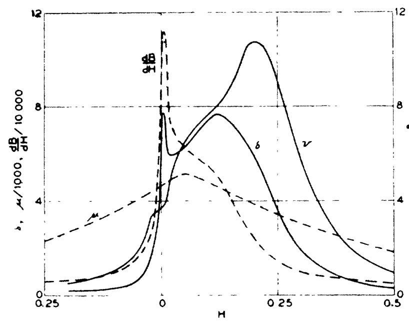

Fig. 15. The v-curve for permalloy containing 78 percent nickel and 22 percent iron, and the data from which it was calculated.

comparison Fig. 10(b) shows the linear relation between the calibrating current, i_c, and the corresponding deflection δ', which appears in Eq. (11).

Fig. 11 shows an example of a $\mu - vs. - H$ curve determined with alternating current of frequency 5 cycles/sec, and for comparison several points determined with a ballistic galvanometer.

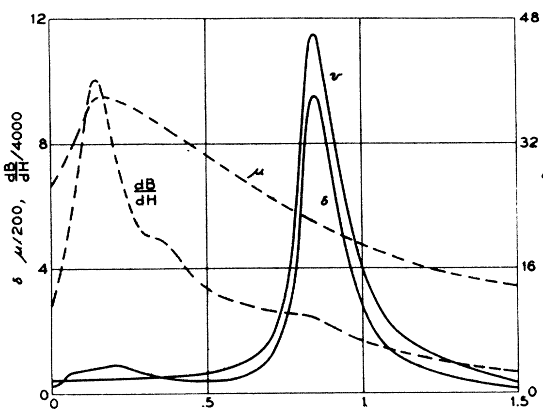

Fig. 16. The v-curve for permalloy containing 50 percent nickel and 50 percent iron, and the data from which it was calculated.

The average volume, v, of the discontinuities in magnetization was calculated in accordance with Eq. (16) for all of the materials. The curves showing v as dependent upon field-strength are shown in Figs. 12 to 18, together with the curves from which the calculations were made. In making these calculations, small irregularities in the original dB/dH and δ curves were smoothed out, as can be seen for example by comparing Figs. 4, 7, and 12. Finally, Figs. 19 to 25 show how the v–$vs.$–H curves are placed with respect to the hysteresis loops.

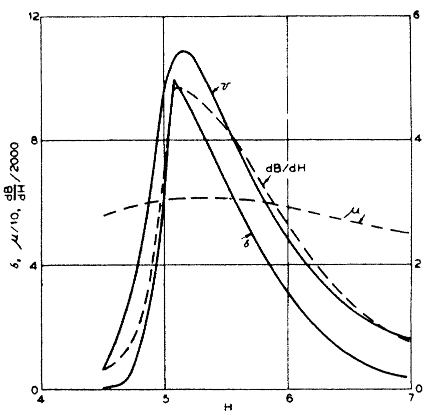

Fig. 17. The v-curve for hard iron, and the data from which it was calculated.

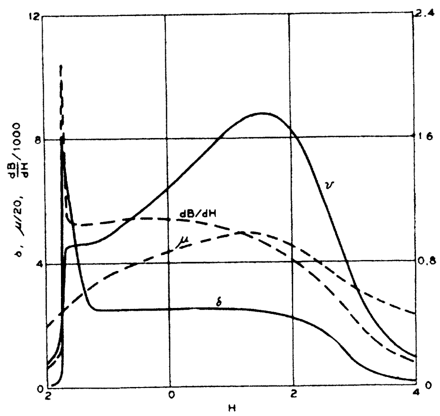

Fig. 18. The v-curve for two single crystals of iron.

Discussion of the Results

Figs. 12 to 18, as well as Figs. 19 to 25, show how the average size of the discontinuities varies from one part of the hysteresis loop to another with

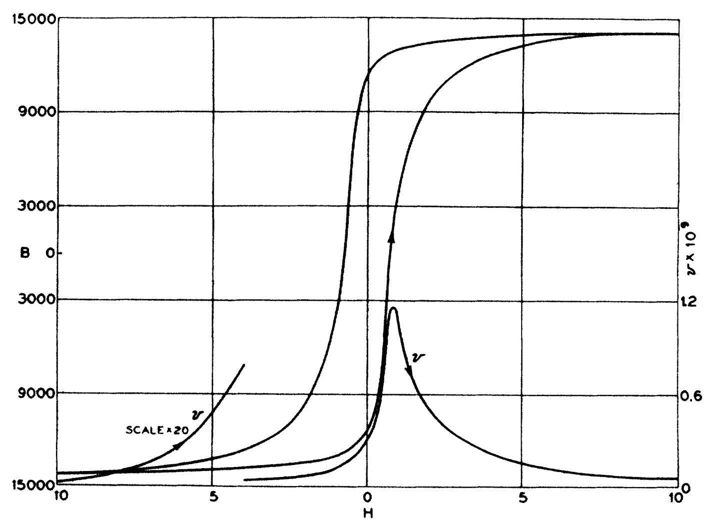

Fig. 19. Hysteresis loop and v-curve for annealed iron.

different materials. The data for annealed iron are more extensive than for the other materials and have been taken over the whole hysteresis loop, between -10 and $+10$ gauss. When the slope of the hysteresis curve is relatively small, there is some question as to whether the discontinuous changes in magnetization are due to complete magnetic reversals of small

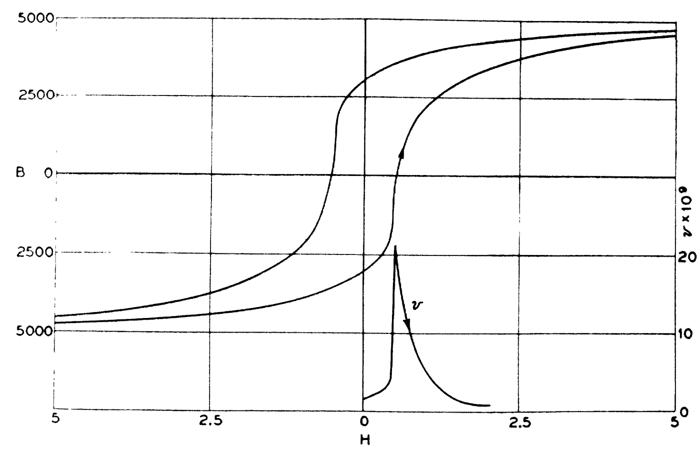

Fig. 20. Hysteresis loop and v-curve for nickel.

parts of the material or whether the reversals are only partially complete, as mentioned in the introduction. Although for this reason the determination of the mean volume of the discontinuities in this case may not be accurate,

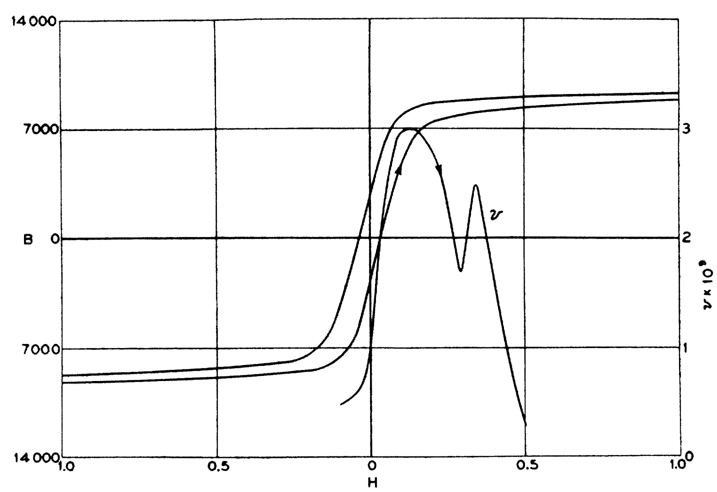

Fig. 21. Hysteresis loop and v-curve for permalloy containing 80.5 percent nickel and 19.5 percent iron.

the data do show, contrary to previous reports, that discontinuities are present throughout the whole cycle. Fig. 26 is a reproduction of the galvanometer record of the Barkhausen effect at the very beginning of the hysteresis loop for iron, and has been included to show that with sufficiently high amplification the effect is measurable even though dB/dH is as small as 40. It seems possible that the magnetization changes discontinuously in all cases, even

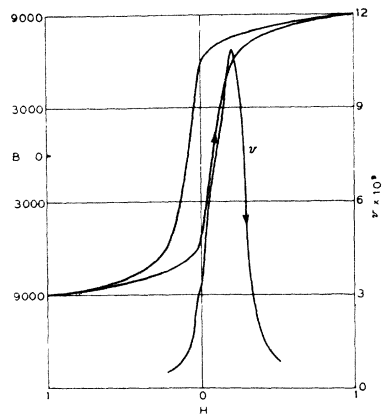

Fig. 22. Hysteresis loop and v-curve for permalloy containing 78 percent nickel and 22 percent iron.

though the material is nearly saturated, and that the discontinuities have been detected only on the steeper parts of the curve because the amplification available has not been sufficient to detect them on the parts less steep.

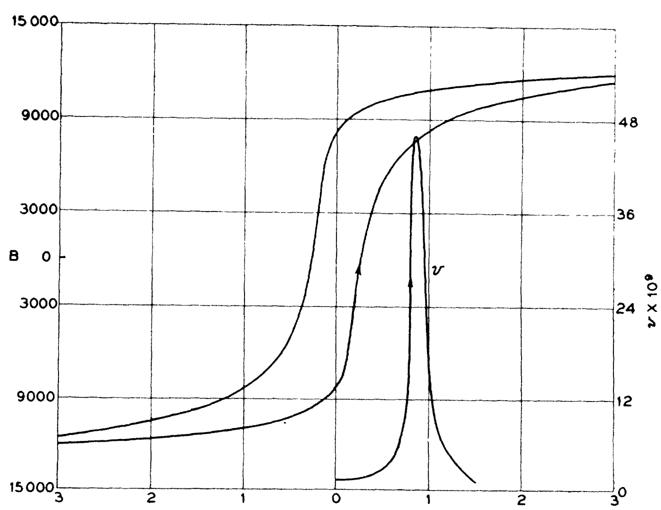

Fig. 23. Hysteresis loop and v-curve for permalloy containing 50 percent nickel and 50 percent iron.

The average volume of material in which the magnetization changes as a unit is found to have a maximum value of the order of $(10)^{-8}$ or $(10)^{-9}$ cm^3 for the materials examined. The maximum values vary only from 1.2

$(10)^{-9}$ cm^3 for iron to 45 $(10)^{-9}$ cm^3 for permalloy containing 50 percent Ni. These volumes are entirely different in order of magnitude from the volume occupied by the crystals in the "single" crystal specimen, which is much

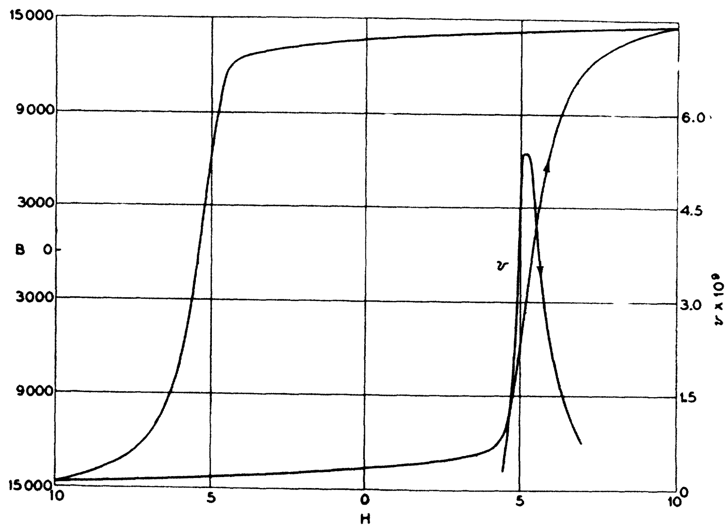

Fig. 24. Hysteresis loop and v-curve for hard drawn iron.

greater; from that occupied by the separate crystals in the hard-worked sample of iron, which is vastly smaller; and also from the volume occupied by a single atom or even by a group of atoms known to be large enough to

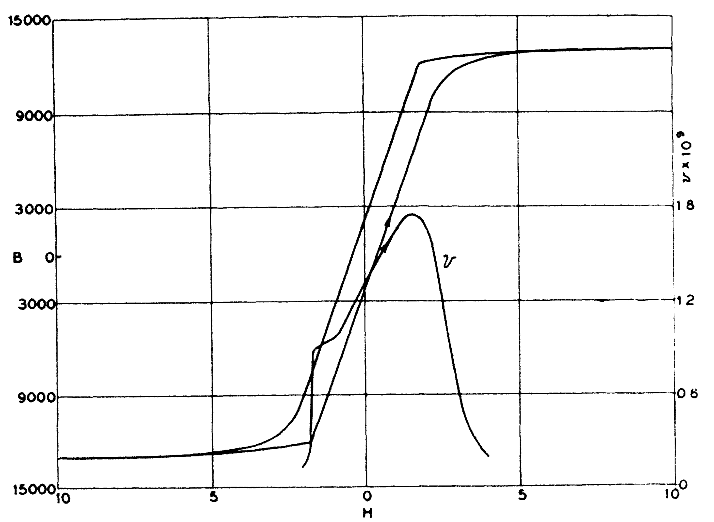

Fig. 25. Hysteresis loop and v-curve for two single crystals of iron.

be oriented by an applied magnetic field, which is very much smaller yet. We regard as a mere coincidence the fact that it is of the same order as the volume occupied by the separate crystals in many well annealed materials.

We conclude that the sizes of the Barkhausen discontinuities are independent of crystal size, and of the nature of the grouping of the atoms of ferromagnetic materials both in respect to the type of lattice upon which the crystal is built, and to the distribution of the different kinds of atoms in alloys.

It is rather surprising at first glance that the computed volumes of the groups should have so nearly the same average value for substances so different. One might expect that, if the magnetization of a group is initiated by the reversal of a single atomic magnet the energy so liberated will aid other magnets to reverse to an extent depending both upon the amount of energy liberated (proportional to the coercive force) and upon the number of other magnets which are about to reverse (proportional to dB/dH). If the coercive force is small and the sides of the hysteresis loop are steep, the initial energy liberated will be small, but there will be a comparatively

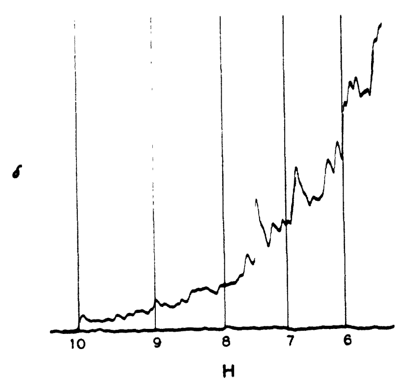

Fig. 26. Barkhausen effect at the beginning of the hysteresis loop for iron, using high amplification

large number of magnets that need but a small disturbance to turn them. If the coercive force is large, the sides of the hysteresis loop are generally less steep. We imagine that these two factors compensate each other, resulting in a relatively constant size of discontinuities in different materials.

The position of the maxima of the $v - vs. - H$ curves for hard and annealed iron and for nickel are near the coercive force, but for the three alloys are definitely displaced to higher values of H. It may be that the positions of these maxima correspond in each case to those for the thermal cooling attending magnetization, observed by Ellwood[7] for steel at magnetizations near the knee of the hysteresis loop.

Our experiments show that the number and size of the discontinuities do not depend on the rate of change of field-strength. As Fig. 10 shows, δ is accurately proportional to dH/dt and this relation implies that the number and size remain constant. For the same total discontinuous change in B, an increase in the average size of the discontinuities and a consequent decrease in their number would be attended by an increase in the ratio of

[7] W. B. Ellwood, Nature **123**, 797–798 (1929).

δ to dH/dt, in contradiction to our findings. Although this linear relation has been found to hold for all of the specimens we have examined, it may not of course hold for the fine wires examined by Heaps and Taylor[8], where the linear extent of the discontinuity is comparable with the diameter of the wire.

Pfaffenberger[9] has reported that the length of the coherence region, within which the discontinuous change is confined, is 3 mm in a steel wire 0.5 mm in diameter. The implication is that during a single discontinuous change in magnetization elementary magnets scattered throughout a volume of about 1 mm^3 are permanently reversed, but that these elementary magnets, if placed next to each other, would occupy a small part, perhaps 10^{-3}, of this volume, corresponding to our volume v. In view of the experiments on the determination of l, described above, we conclude that this 3 mm length is the extent of the region in which the eddy-currents are produced in this sample of steel by the sudden change in magnetization. If Eq. (15) holds for wires 0.5 mm in diameter, we find for Pfaffenberger's sample, using $l/2=0.3$, that $\mu=2.3$. Since Eq. (15) has been established only for wires of radius 0.5 mm, it may be that the more general equation will include some function of the radius. If, for example, the equation is

$$l=1.03(10)^{-3}(\mu/\rho)^{1/2}r^2/(0.05)^2 \qquad (15')$$

we deduce from Pfaffenberger's data that $\mu=10$, a not unreasonable figure for some kinds of steel.

The experiments described above confirm the earlier experiments[1] in showing that for the steeper parts of the hysteresis loop the volume associated with the average discontinuity is very large compared with the dimensions of the atom and seldom involves less than 10^{10} atoms. In annealed materials the average size is comparable with the size of the crystals, but this is regarded merely as a coincidence because the same sized discontinuities are found in hard-worked materials and in a single crystal. The average size is greatest at or near the steepest part of the hysteresis loop and diminishes rapidly as the ends of the loop are approached. The maximum value of the average size (the average size at or near the steepest part of the loop) is about $(10)^{-8}$ cm^3, corresponding to $(10)^{15}$ atoms, and is much the same in the different materials examined.[10]

[8] C. W. Heaps and J. Taylor, Phys. Rev. **34,** 937–944 (1929).

[9] J. Pfaffenberger, Ann. d. Physik **87,** 737–768 (1928).

[10] An interesting paper by F. Preisach, Ann. d. Physik **3,** 737–799 (Dec. 1929) has just come to our attention. His experiments support the previous report by one of us[1] that on the steeper parts of the hysteresis loop almost all of the change in magnetization is discontinuous. Our results disagree in one important respect: Preisach states that when the magnetization diminishes from its greatest value (near to saturation) at one end of the loop, no Barkhausen effect is detectable (with a three stage voice-frequency amplifier) and the change is continuous; we have shown that with sufficiently high amplification (seven stages, wide frequency range) the effect is pronounced, as shown in Fig. 26 above. Preisach has shown that the sizes of the discontinuities in fine wires can be markedly changed by straining them, and that in some cases a single discontinuity corresponds to almost the whole possible change in magnetization of the material. We have also observed such discontinuities in fine unstrained wires of permalloy containing 78 percent nickel and 22 percent iron.