ON THE ANOMALOUS ROTATION OF THE SUN*

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Abstract

The requirements of Jeans' theory of the anomalous solar rotation are not in accord with the facts of solar magnetism. A new theory is worked out which attributes the anomaly to atmospheric motions arising from the interaction of ions with the solar magnetic field and an assumed electric field. Observations of the solar atmosphere are made only in those regions where the gaseous pressure is such that the ion free paths are long. Ions of both kinds which execute long free paths in crossed electric and magnetic fields are swept in the same direction and give rise to a mass motion. This superimposed drift of the solar atmosphere is shown to account for the rotational anomaly and the calculated variation of the angular velocity with latitude is of the observed form. The theory requires that the sun possess a radial electric field having a sign and distribution similar to that observed on the earth.

It HAS been known for nearly 100 years that the measured rotational period of the sun depends upon the latitude of the point selected for observation and that it varies from 25 days at the equator to over 30 days at the pole. Moreover, the value of the period at any given place seems to depend somewhat on the level of the point observed. It has been found that in the solar reversing layer the measured angular velocities at low latitudes, expressed in degrees per day, were given approximately by the relation

$$W = 14°.54 - 3°.50 \sin^2 \lambda \tag{1}$$

where λ is the latitude.

Jeans[1] has proposed a theory to account for the rotation phenomena and he attributes the effects to the braking action of the radiation which filters through the outer solar layers. The final form of his theory requires that the central regions of the sun rotate from ten to fifty times faster than the surface layers and that these layers play only a minor part in the transfer of the radiated angular momentum. It is difficult to establish the existence of high rotational velocities inside the sun from theoretical considerations and it now appears that such a rotation may be improbable. The essential characteristics of the solar magnetic field are known from the work of Hale and his collaborators at Mt. Wilson[2,3] Observatory. This work established the fact that the distribution of the solar magnetic field at any given level was similar

* Released by the Navy Department.

[1] Jeans, Astronomy and Cosmogony, Cambridge Press, 1928.

[2] Hale, Astrophys. J. **38**, 31 (1913).

[3] Hale, Sears, van Maanen and Ellerman Astrophys., J. **49**, 1 (1918).

to that of the earth; that the field was limited radially and that the magnetic pole did not coincide with the geographic pole but rotated about it with a period of 31.4 days. These magnetic facts suggest that Jeans' theory of the anomalous solar rotation is incorrect, for if we assume that the rotational velocity changes from layer to layer as we go outward from the core of the sun, then the motion of the magnetic pole with respect to the axis of rotation will set up electrical eddy currents in the moving layers which will tend to bring them into rotational synchronism with the magnetic field. This difficulty would vanish if the magnetic and geographic poles coincided. A still more serious difficulty arises from the fact that on almost any theory of the permanent magnetic field of the sun and earth, as for example, a theory which attributes the field to electrical currents[4,5], the magnetic pole would be expected to rotate with the same period as the entire mass. Superimposed on this might be a slow precession but it is unlikely that the solar fields should behave in a manner essentially different from that observed in the case of the earth whose magnetic poles rotate with the period of the entire mass. It thus seems clear that the rotational period of the entire solar mass is 31.4 days and not one-tenth of this as is required by Jeans' theory. The objections to Jeans' theory are of such a nature that the problem has been re-examined. It has been found possible to account for the observations in a simple manner by considering the electromagnetic reactions of the ions in the solar reversing layer.

Mt. Wilson researches demonstrated that the solar magnetic field is essentially similar to that of a uniformly magnetized sphere whose exterior field is limited by some special mechanism. It has been shown that this limitation of the solar field is adequately explained by the diamagnetic properties[6] of the solar atmospheric ions; for the effective permeability of an ion gas approaches zero under precisely the conditions that exist in the solar reversing layer. It has been argued recently that boundary effects may annul the diamagnetism of the reversing layer, but this view can not be supported. The effect of the boundary has been considered by van Leeuwen[7] for the case of a metal, who found that the current sheet at the boundary is able to compensate for the volume diamagnetic effect under certain special conditions. She considered the case of short free paths and assumed that the free electrons making up the electron gas were reflected specularly when they collided with the boundary. To meet this condition it is necessary that the boundary be sharp and that the component of the electron momentum normal to the boundary be reversed on impact while the tangential component is preserved. The requirement is satisfied when an electron collides with a heavy ion but when an ion collides with a boundary ion or molecule of its own mass, the normal momentum of the colliding ion is not reversed and boundary currents cannot cancel the volume diamagnetism. Ions play an important part in solar dia-

[4] Gunn, Phys. Rev. **34**, 335 (1929).
[5] Gunn, Phys. Rev. **34**, 1621 (1929).
[6] Gunn, Phys. Rev. **33**, 614 (1929).
[7] van Leeuwen, Jour. d. Physique **2**, 361 (1921).

magnetism and it is clear that van Leeuwen's calculations do not apply there. Moreover, we must note that the sun's atmosphere has no outer boundary and the inner boundary is not sharp so that complete compensation is impossible and we are left with the primary effect—diamagnetism. This general conclusion was pointed out clearly enough in the writer's first paper.[8] Chapman[9] has proposed an alternative theory of the radial limitation of the magnetic field but the author has shown[10] that the effect he invokes to explain the phenomena is largely cancelled by another effect which arises from the inhomogeneity of the impressed magnetic field. Moreover, Chapman's theory encounters a serious qualitative difficulty in that the gravitational drift currents of the type he postulates become very small at high solar latitudes where the magnetic field intensity becomes large and nearly parallel to the gravitational acceleration. Thus his theory leads to the conclusion that the magnetic field at a given level near the poles should be much larger than is observed and that a large unobserved stray magnetic field should exist. Diamagnetism on the other hand is equally effective at the poles as at the equator and predicts the same fractional reduction of the initial field at a given ion pressure. This is in accord with observation.

The existence of diamagnetism in the reversing layer and direct spectroscopic evidence both show that the ion free paths in the reversing layer must be longer than the critical free path; that is, they are longer numerically than the radius of the helix generated by the ion as it is caused to spiral about the impressed magnetic field. When this condition is satisfied, other important motions of the nature of ion drifts arise from a combination of an impressed magnetic field with other types of field. These drifts may result from a magnetic field crossed with a gravitational field[11,12], an inhomogeneous magnetic field,[10] or an electric field.[8] The ion drifts and their possible effects have been investigated elsewhere; it is only of importance to note here that the first two effects give rise to ion drifts which are opposite in direction for the positive and negative ions and hence constitute a current while the electric field crossed with a magnetic field acts in such a way as to urge both ions in the same direction and hence is of the nature of a mass movement. When the free paths of the ions and electrons are both longer than their respective critical free paths both drift in the same direction with the same velocity. In certain unobserved regions of the solar atmosphere, however, the free paths of the electrons are longer than their critical radius while the free paths of the ions are less than their own critical value. Thus in this special region electric currents flow which may tend to magnetize or demagnetize the sun according to the direction of the impressed electric field. Such currents may be important in theories of sunspots or of the limitation of the solar magnetic field,

[8] Gunn, Phys. Rev. **32**, 133 (1928).
[9] Chapman, Monthly Notices **89**, 57 (1928).
[10] Gunn, Phys. Rev. **33**, 832 (1929).
[11] Hulburt, Phys. Rev. **33**, 412 (1929).
[12] Chapman, Monthly Notices **89**, 57 (1928).

but it is to be noted that the conductivity of this narrow region is moderately large and any electric field must necessarily be small. Our greatest interest in the present paper is in the mass motions of the atmosphere and we therefore assume the sun to have electric as well as magnetic fields and will consider their joint mechanical effects. At the solar equator the direction of the magnetic field is tangential and that of any electric field must be radial. These crossed fields give rise to an equatorial ion drift which is in the same direction for both types of ions and is independent of the charge or mass of the ion. Were neutral molecules present they would presumably be swept along by occasional collisions with ions. It thus seems possible that the observed anomalous rotation of the sun is an effect arising from the movement of the solar atmosphere and is not due to deep seated circulation which seems to be required in earlier ideas regarding the phenomena.

The problem of the electrical state of a rotating heavenly body has been considered by several writers[13,14,15] and the electrical charge distribution worked out under special assumptions. For example, Rosseland[13] has pointed out that in the stars loss of charge by radioactive processes might be expected. In addition we may have electrical fields arising from a separation of charge due to (*a*) gravitational fields[16] (*b*) temperature gradients (*c*) radiation pressure (*d*) pressure gradients[13] (*e*) motion in a magnetic field.[15] It seems probable that in the reversing layer (*a*), (*b*), and (*c*) are small compared to the electric fields arising from other causes. A study of the electric fields ar sing from the motion of the earth in its own magnetic field has been made by Page[15] who assumed that the earth was a uniformly magnetized conducting sphere. Upon working out the consequences of his assumptions he found that electric fields are set up in the high atmosphere which have values such that the ions are swept to the westward with a velocity of the same order of magnitude as the peripheral velocity of the earth. An observer outside the earth watching the ions in the high atmosphere would thus observe an apparent rotational period greater than 24 hours. The entire calculation may equally well be applied to the sun and correction made for the change of magnitude of the quantities due to a larger field and a different type of magnetic distribution. This readjustment does not greatly alter the situation and if the sun is assumed to be unchanged as a whole and free of electric fields in the reversing layer, except those arising from the solar rotation, then an observer on the earth watching an equatorial point in the reversing layer would observe a period of rotation for the layer which is greater than the period of rotation as indicated by the motion of the magnetic pole. This is not in accord with observation and we must conclude that radial electric fields other than that due to rotation in its own magnetic field exist in the solar reversing layer. We shall not enter into a discussion as to the origin of this electric field but simply point out that observation demands its existence. Moreover, as we

[13] Rosseland, Monthly Notices **84,** 720 (1924).
[14] Rosseland, Astrophys. J. **62,** 387 (1925).
[15] Page, Phys. Rev. **33,** 823 (1929).
[16] Pannekoek, Bull. Astro. Inst. Netherlands 19, (1922).

shall see presently, the electric field required is in the same direction, is much smaller, and varies radially in substantially the same manner as the observed electric field of the earth. It is thus consistent with such astrophysical data as are available at the present time.

The mean drift velocity $\boldsymbol{u}$ imposed on both kinds of ions which execute long free paths in crossed electric and magnetic fields is given by

$$\boldsymbol{u}=\frac{\boldsymbol{E}\times\boldsymbol{H}}{H^2}=\frac{E}{H}\sin\beta. \tag{2}$$

Where E and H are in e.m.u. and β is the angle between E and H. We will take H positive northward in the reversing layer at the equator since it coincides in direction with the rotational velocity of the sun and E radially outward. Under this condition a positive value of $\boldsymbol{u}$ corresponds to a westward velocity relative to the surface. According to Mt. Wilson data[2,3] the magnetic field H at a given atmospheric level and latitude λ is given by

$$H=H_0(1+3\sin^2\lambda)^{1/2} \tag{3}$$

where H_0 is the value of the equatorial field at the level considered. Thus combining (1) and (2) the superposed angular velocity ω at a given level which arises from the velocity of drift is

$$\omega=\frac{E\sin\beta}{H_0R[(1+3\sin^2\lambda)(1-\sin^2\lambda)]^{1/2}} \tag{4}$$

where R is the radius of the sun. The diamagnetic layer of the solar atmosphere distorts the permanent magnetic field and the value of β is not readily determined. It is convenient to note that due to diamagnetism the magnetic field is nearly parallel to the surface of the sun in a region 30° on each side of the equator (say) and that the electric field must be radial, so that within the range specified an expansion of Eq. (4) is valid and we may also set $\sin\beta=-1$. Making this substitution, expanding (4) and retaining only the first two terms we have

$$\Omega=\Omega_0-(E/H_0R)(1-\sin^2\lambda) \tag{5}$$

where Ω is the resultant (observed) angular velocity, Ω_0 the angular velocity as measured by the rotation of the magnetic pole and the last two terms represent the contribution due to the atmospheric drift. In order to account for faster rotation at the equator we must now assume that the required value of E is radially inward or negative. If we set

$$\Omega_0+(E/H_0R)=\Omega_0' \tag{6}$$

then (5) becomes

$$\Omega=\Omega_0'-(E/H_0R)\sin^2\lambda \tag{7}$$

and agrees in form with the observed relation given in Eq. (1). By selection of the correct value for E; Eq. (7) gives the exact relation. Thus from Eq. (1) and (7)

$$(E/H_0R)=7.06\times10^{-7}\ \text{rad/sec}$$

and taking $R=6.95\times10^{10}$ cm and $H=25$ gauss we find $E=0.013$ volts/cm.

There are no data available which contradict the assumption of a radial electric gradient of 0.01 volt/cm in the observed regions of the reversing layer and unless the atmosphere of the sun rotates slower than the magnetic poles we must assume that some kind of an electric field exists which is directed radially inward. The conductivity of the layer in the direction of the electric field has been shown to approach zero[8] when the electric and magnetic fields are nearly perpendicular as they actually are in the equatorial zone we have considered. There is therefore no reason to suppose that electric charges can readily flow in such a manner as to neutralize the field. Near the poles the electric and magnetic fields approach parallelism and the conductivity increases; thus the electric field distribution in this region is quite different from that at the equator.

The previous calculation of the equatorial electric field was made for an observed level where the magnetic field was taken as 25 gauss. At different values of the magnetic field we might expect different values for the drift velocity. Observations show that deviations do exist but they are not so large that it would be legitimate to assume the electric field constant at all altitudes. A better approximation is suggested by the fact that the drift velocity is observed to be nearly constant with superposed regular variations. On this assumption we have

$$(E/H)=K_1=5\times10^4\ \text{cm/sec} \tag{8}$$

which requires that E increase with increasing ion pressure in the same manner that H is known to increase. This relation undoubtedly breaks down in the deeper layers of the solar atmosphere. The origin of the electric field is unknown and the distribution cannot now be calculated from independent considerations. The required distribution can be worked out, however, by use of Eq. (8) and earlier results obtained from diamagnetic considerations[10] which gave an approximate relation between the ion density and the magnetic field in the reversing layer. This relation is:

$$N=H^2/4\pi kT \tag{9}$$

where N is the ion density, H the magnetic field intensity, k the Boltzmann constant and T the absolute temperature. By the use of observed values for H it was found that the distribution of N was a logarithmic function of the altitude. Making the further assumption that the solar atmosphere was in gravitational equilibrium it was found that the distribution calculated from magnetic data agreed with that required by a gravitational equilibrium distribution when the mean atomic weight of the particles in the reversing layer was taken as 3.3. A correction for the presence of an electric field was also made but the calculation is not now believed to be valid. In view of the recent work of Unsöld[17] it hardly seems legitimate to talk of equilibrium in the solar atmosphere but if the values of the ion pressure are averaged over a fairly long period the usual expression probably well represents the distribution. On this assumption the ion density n at any level r is given by

$$n = n_0 \exp\left(\frac{-zm_H g(r-R)}{kT}\right) \tag{10}$$

where n_0 is the number of ions per cm^3 at the "surface" of the sun, z the mean atomic weight of the solar ions and R the radius of the sun. We will follow earlier work and assume that the surface of the sun is located at a level where the free paths of the ions are just equal to the radius of the helix generated by an electron as it is caused to spiral about the impressed magnetic field. Extrapolation has shown that the ion pressure at the surface so defined was nearly one half an atmosphere and that the magnetic field intensity was roughly 12,000 gauss. Below this surface the magnetic field is no longer modified by diamagnetism and the electric conductivity is so large that electric fields are negligibly small compared to those outside. Combining (9) and (10) we have

$$H = H_0 \exp\left(\frac{-zm_H g(r-R)}{2kT}\right) \tag{11}$$

where H_0 is the surface magnetic field intensity. We are now able to calculate the difference of potential between the solar surface and outer space. Let this potential difference be ϕ, then from Eq. (8) we have

$$\phi = \int_R^\infty E dr = K_1 \int_R^\infty H dr. \tag{12}$$

The integration is carried out along a path radially outward at the equator which gives by aid of Eq. (11)

$$\phi = \frac{2K_1 H_0 kT}{zm_H g}. \tag{13}$$

Taking therefore $K_1 = 5\times10^4$ cm/sec; $H_0 = 1.2\times10^4$ gauss; $Z = 3.3$; $m_H = 1.66\times10^{-24}$ gm; $g = 2.7\times10^4$ cm/sec^2; $k = 1.37\times10^{-16}$ and $T = 6\times10^3$ we get $\phi = 6.6\times10^{15}$ e.m.u. or 6.6×10^7 volts. We have pointed out that in the special region where the electron free path is long and the ion free path is short (according to our special definition) the conductivity is moderately large and the electric field must therefore be small. If we assume that the electric field in this region is zero and integrate from the outer edge of the region where H_0 is 280 gauss[4] out to infinity we find the potential difference to be 1.5×10^6

[17] Unsöld, Astrophys. J. **69**, 209 (1929).

volts. This is nearly the same potential difference as is observed between the earth and free space. While this calculation cannot be exact since it involves a questionable extrapolation of what may be considered an empirical relation it probably does give a good approximation to the potential difference required to maintain an electric field of the assumed type and distribution.

Special attention should be called to the striking similarity between the the solar magnetic and electric fields and the same fields on the earth. Spectroscopic data show that the solar magnetic field is not unlike that of the earth except for the distortion of the field due to diamagnetic effects in the reversing layer. The requirements in regard to the electric field as deduced in this paper from the observed movements of the solar atmosphere are almost identical with the observed features of the earth's electric field. The earth's electric field in the region of poor electrical conductivity is radially inward, its value drops off with decreasing pressure fairly rapidly in a manner precisely as required in the sun. Moreover the observed potential difference between the surface of the earth and its outer layers is almost identical with that calculated for the sun. The equality of the potentials of the sun and earth seems significant, for conditions on the sun and earth are greatly different and such equality could hardly be expected unless the electric fields arise from the same fundamental mechanism.