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IONIZATION IN THE UPPER ATMOSPHERE VARIATION WITH LONGITUDE*

By E. O. Hulburt
Naval Research Laboratory

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Abstract

Continuing a former paper (Phys. Rev. **34,** 1167 (1929)) the present paper gives theoretical calculations of the changes in the ionization in the upper atmosphere with longitude. The electrical conductivity of the upper atmosphere is about 1.4×10^{-5} at noon equinox at the equator and an order of magnitude less at night. The maximum density of electrons y_m varies with the latitude θ and longitude ψ, measured from noon equinox at the equator according to $y_m = 3.14\times 10^6 \cos\theta(0.18 \sin\psi + \cos\psi)$ for the daylight hours. At night the expression is more complex. The values of y_m yield skip distances of short wireless waves roughly in accord with observation in the day but somewhat too great at night. The theory puts the shortest skip distance at 40 minutes past noon and observation in temperate zones gives 2 p.m.; the agreement is good, but not perfect.

In a recent paper [11] the ionization in the upper atmosphere was worked out over the entire earth. The ionization, assumed to be caused by the ultraviolet light of the sun, was shown to explain certain facts of wireless wave propagation and of terrestrial magnetism. The variation of the ionization with latitude was described in some detail, less space being given to the variation with longitude. This paper presents the longitude calculations, and is therefore a direct continuation of the earlier paper, the nomenclature, numbering of paragraphs, equations, figures, tables and references being carried on from that paper.

33. It is assumed that the temperatures of the upper atmosphere for longitudes along the equator are given by the values of T of column 2, Table I, the degrees of latitude of column 1 being now degrees of longitude measured from high noon. For example, at 60° longitude, i.e., at 8 a.m. or 4. p.m. T is 360°K. At any point in the day hemisphere of longitude ψ and latitude θ the angular distance ζ from equatorial noon is $\cos\zeta = \cos\theta \cos\psi$ and $T = 220 + 280 \cos\zeta$. Everywhere in the night hemisphere T is 220°K. From the values of T the ionic recombination coefficient α is known at all points. The critical level z_c is nearly independent of the longitude; for example, at the equator z_c descends from the noon value of 150 km to 147 km at night, and the diurnal change at high latitudes is less.

34. In the S region the y, z curves for various longitudes were calculated for the daylight hours by the method of sections 10 and 11. The method neglected gravity diffusion of the ions and assumed that equilibrium existed between the rate of production and the rate of loss as expressed by

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[11] Hulburt, Phys. Rev. **34,** 1167 (1929).

$$q \cos \psi - \alpha n y^2 = 0. \tag{19}$$

The curves for the equator are given in Fig. 4. ("Gravity diffusion" is a short expression for "temperature diffusion of the ions to attain gravitational equilibrium.") Apart from the neglect of gravity diffusion (19) is an approximation because as the earth turns under the sun equilibrium in general does not exist. The more complete expression is

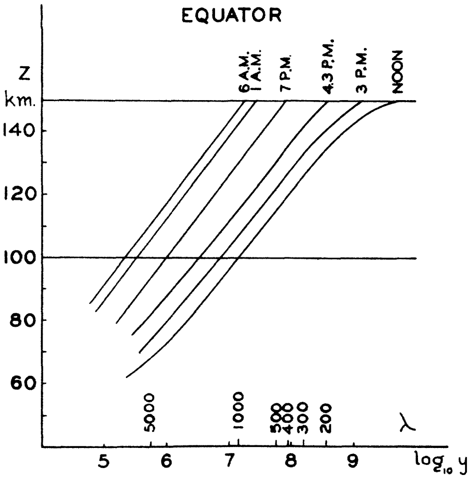

Fig. 4. Theoretical values of the density y of ion pairs in the S region of the upper atmosphere at the equator.

$$dy/dt = q \cos \omega t - \alpha n y^2, \tag{20}$$

where t is the time measured from high noon and ω is the angular velocity of rotation of the earth on its axis; $\omega = 7.28 \times 10^{-5}$ radians sec^{-1}. α and q are functions of the longitude ωt. Even if α and q are assumed to be constant with respect to t we are unable to solve (20) and must turn to approximate methods.

At noon at the equator $y = 5 \times 10^9$ for $z = 150$ km. At 1 p.m. y is 4.8×10^9 from (19). If there were no production of ions after 12 o'clock and the ions at 1 p.m. were those of noon decreased by recombination, y at 1 p.m. would be 1.5×10^9, as found from (20) with $q = 0$. Since this is less than one third the value of y (19) we see that the ions at 1 p.m. are mainly those formed by the sunlight at 1. p.m. rather than those left over from noon. Therefore, the maximum value of y at $z = 150$ km is reached within less than 20 minutes after noon. Similarly, it is found that the maximum of the ionization occurs later as z becomes less until at $z = 100$ km the maximum is near 1 o'clock. Thus, except for the small shift in the maximum, (19) gives values of y which are approximately correct during the day until about 4 p.m. After 4 p.m. (19) gives values of y which are too low; for example, it would make $y = 0$ at 6 p.m., whereas at this hour many of the ions formed during the day still exist, and indeed continue to do so all night. From (19) and from (20) with $q = 0$ it comes out that at about 4.7 p.m. the number of ions in equilibrium with the ultra-violet radiation is equal to the number left over from the earlier hours. We therefore calculate the y, z curve for 4.3 p.m. by means of (19), assume no more production of ions and calculate the y, z curves for various hours through the night from (20), with $q=0$. The night-time y, z curves for the equator are given in Fig. 4. In like manner the y, z curves throughout day and night were determined for all latitudes. The curves of Fig. 4 are probably not plotted entirely correctly, those at night being lifted up too much. Due to the cooling and contraction of the atmosphere at night and at the higher latitudes, mentioned in sections 17 and 19, the upper atmosphere is lower than at equatorial noon. The decrease in height from noon to midnight is about 25 and 15 km at 140 and 100 km, respectively. This would bring the night y, z curves closer to the noon curve than are shown in Fig. 4.

35. The height z at which a wireless wave of wave-length λ is totally reflected (refracted) at normal incidence is found by putting the refractive index μ equal to zero in the expression $\mu^2=1-2ye^2\lambda^2/\pi m$, solving for y and determining z from the y, z curve. The values of y for total reflection of waves of length 5,000, 1,000, 500, 400, 300, and 200 m are marked along the X-axis of Fig. 4. It is seen, for example, that the height reached by 1,000 m waves increases on the equator from 100 km at noon to 140 km at midnight and to 150 km in the early morning before dawn, with similar increases in the heights reached by the other long waves. The heights for total reflection at latitudes 40°, 60°, and 70° are, respectively, 5, 12, and 19 km greater than the corresponding heights at the equator. But due to the shrinkage of the atmosphere at night and at the higher latitudes the increase in the height is probably less. On the whole the heights reached by the longer waves change but little with the latitude and the time of day. This is in accord with the facts as far as they are known. It must be remembered that the observation of the height reached by wireless waves gives always an "apparent height" which is in general greater than the true height. This is due to the group velocity retardation of the wave in the ionized region of the atmosphere. Therefore, the interpretation of a height measurement is always open to some uncertainty and, in the case of the echo experiments, is often ambiguous for it is sometimes difficult to know where the echo has come from.

36. The absorption of wireless waves calculated from the y, z curves showed that in the region below the height of total reflection the intensity of 500 m waves, for example, was reduced to $1/\epsilon$'th by passage through 20, 25 and 80 km of the medium at noon at latitudes 0°, 40° and 60° respectively. These values are, if anything, below those inferred from qualitative observation. A suitable number of electrons which undoubtedly exist in this region may be called upon to give the correct absorption. The absorption of 5,000 m waves is greater than is observed if the y, z curve is assumed to continue according to the arbitrary formula of section 10 below the height where these long waves are reflected. We may conclude, as is quite reasonable, that y becomes small and that the formula is not valid in the levels below 70 km at equatorial noon.

For 500 m waves the absorption at 4 p.m., 7 p.m. and midnight was about 0.3, 0.07 and 0.01 of the noon values. The absorption thus decreases fairly rapidly at sunset, but perhaps not quite rapidly enough. It is well known that the intensity of broad-cast waves received at some distance increases rather sharply at sunset, passing within less than two hours from its low daylight value to a high night value. Although the observations are only qualitative it would seem that they might call for a more pronounced change at sunset than the y, z ion curves can give. Electrons in the S region will disappear rapidly after sunset and may account for the sharp change in the wireless phenomena. Since 1 electron is equivalent to about 10^6 ions as far as absorption is concerned and to 10^5 ions for refraction purposes, it is possible to have the absorption in the daylight S region controlled by electrons and the refraction by ions.

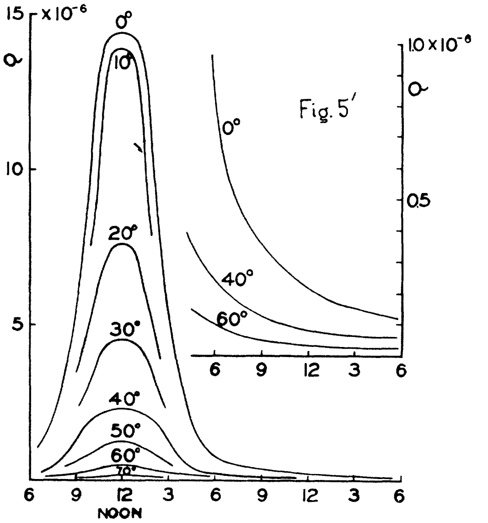

Fig. 5. Theoretical values of the electrical conductivity σ of the upper atmosphere.

In this connection an interesting difficulty appears. Experiments with long waves indicate that the polarization of the received wave varies more or less rapidly with the time during the day and the night. The explanation usually given is that the upper atmosphere contains electrons which are influenced by the earth's magnetic field and cause magnetic double refraction and absorption of the wave. Ions are too heavy to give rise to appreciable double refraction except for very long waves beyond the normal wireless range. Electrons produced by the ultra-violet light of the sun, in all probability, exist in the levels below 150 km during the day, but one would expect them to disappear very rapidly at night. For example, the rate of decrease of the electron density y_e due to attachment of the electrons to oxygen molecules of density n' is $dy_e/dt = -bn'y_e$ where $b = 7\times10^{-14}$. At 120 km at night $n' = 1.72\times 10^{11}$. Thus $\log_\epsilon y_e/y_0 = -0.012t$ and in 10^3 seconds or 17 minutes y_e becomes very small. The decrease is greater at the lower heights. The diffusion of electrons down from higher levels is inappreciable. Therefore, if the observations demand electrons in the 70 to 140 km levels at night we shall conclude that something has been left out of the present theory. It is possible that the interference of a number of rays may give rise to changes in the polarization of the received wave quite apart from magnetic double refraction effects.

37. From the y, z curves the values of the electrical conductivity σ of the S region were calculated by means of (12) and were plotted in Fig. 5 for various longitudes and latitudes. In Fig. 5′ the night values of σ are given on a larger scale. Summing up the values of σ over the light and dark hemispheres gives the ratio of the total night to day conductivity to be 1/20; this value was used in section 18. The electrical conductivity of copper is 5.9×10^{-4} and of mercury is 1.06×10^{-5} c.g.s.e.m.u. The conductivity of a 1 cm^2 column vertically upward through the S region of the atmosphere is about 1.4×10^{-5} at equatorial noon and 10^{-6} at sunset, and is therefore equivalent in these two cases to that of 14 and 1 mm of mercury, respectively.

38. Concerning the variation of the D ions with longitude there is little to add to what has already been said in sections 13 to 16. The calculations of the D ions were on the whole pretty rough because so many simplifying assumptions were made. To improve the calculations appears to be intricate and the mathematical difficulties promise to be appreciable. At equatorial noon the magnetic susceptibility of each cm^3 of the D region is about two orders of magnitude greater than that of bismuth, bismuth being the most strongly diamagnetic substance known.

39. The electrons were shown[1] to attain their maximum density y_m at a height of 190 km at summer noon. The y_e, z curve for noon at $\theta=17°$ (as at Washington, D. C., on June 22) is given in curve 1, Fig. 2, reference 1, in which $y_m=3\times10^5$. We use this curve and adopt the simplifying assumption (reference 1, page 1029) that during changes in the curve, which occur during the day and night, the electron density y_e at each point on the curve changes in the same proportion. This enables all the calculations of the y_e, z curve to be reduced to a calculation of the changes in y_m. The physical justification for this assumption lay in the fact that most of the electrons were produced somewhere above the maximum and diffused rapidly downward to build up their maximum density approximately at a height where the loss due to attachment to oxygen molecules became greater than the supply due to diffusion from above, the loss increasing and the rate of diffusion decreasing as z became less. Therefore, if the rate of production q grew less y_e diminished at all points of the curve, and the assumption is that y_e diminishes equally at all points of the curve. Using the calculations of reference 1, page 1030, and changing them to refer to the somewhat higher day temperatures of the present paper, the total loss sec^{-1} of electrons above y_m due to attachment is 1.59×10^8 or $530y_m$. The loss due to the recombination of electrons with positive ions is negligible; this is fortunate, for the recombination term involves a y_m^2 which would make the equation hard to solve. The loss due to the electrical drift downward during the day with velocity v is 250 y_m, using an average value $v=250$ cm sec^{-1} among the values of v of column 9, Table I. The loss due to gravity diffusion across the maximum is 166 y_m, but this we leave

out. For gravity diffusion can not occur in its full extent simultaneously with the electrical drift as it involves molecular collisions which interfere with the drift. Therefore the rate of loss is approximately $(530+250)\ y_m=780y_m$.

The total number of electrons above the maximum is $5.76\times10^{11}=1.95y_m$. The rate of production is $q\cos\theta\cos\omega t$ where q is the rate at equatorial noon equinox, i.e., the sun directly overhead. Then

$$dy_m/dt=q\cos\theta\cos\omega t-780y_m/1.95\times10^6. \tag{21}$$

Solving

$$y_m=\frac{q\cos\theta}{16\times10^{-8}+\omega^2}(\omega\sin\omega t+4\times10^{-4}\cos\omega t). \tag{22}$$

Putting $t=0$, $y_m=3\times10^5$, $\theta=17°$ and $\omega=7.28\times10^{-5}$ in (22) gives $q=130$, and (22) becomes

$$y_m=3.14\times10^5\cos\theta(0.18\sin\omega t+\cos\omega t). \tag{23}$$

With this expression the values of y_m were calculated over the daylight hemisphere and are plotted in the full line curves of Fig. 6. The maximum value of y_m occurs at about 40 minutes past noon. Formula (23) is not valid at night for the sunset to sunrise voltage E reverses in sign at sunset and hence the downward daytime electrical drift velocity v changes after sunset to an upward velocity. At night (18) is used. (23) gives too low values of y_m as sunset is neared, for at this time the rate of loss of the electrons grows less because the downward electrical drift diminishes due to the decreasing E and the attachment coefficient diminishes due to the cooling temperature. Therefore the night curves, calculated from (18) and plotted in the dotted lines of Fig. 6 were joined to the day curves at 5 p.m.

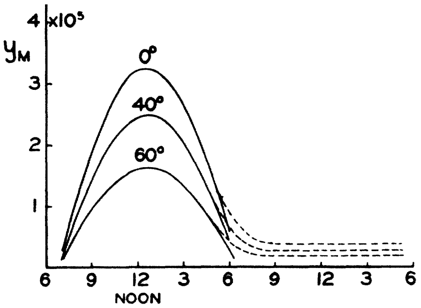

Fig. 6. Theoretical values of the maximum electron density y_m in the upper atmosphere.

40. The skip distance $2s$ for a wireless wave of length λ is given approximately by

$$2s=2h(\pi m/y_me^2\lambda^2-1)^{1/2}, \tag{24}$$

where e and m are the electronic mass and charge, respectively, and h is the value of z for y_m. h was taken to be 190 km with the sun directly overhead, to be 140 km along the sunset and sunrise longitudes and to vary between these limits over the day hemisphere according to $\cos\theta\cos\omega t$. During the night the electrons move upward with a velocity roughly 250 cm sec^{-1} or 9 km hr^{-1}. Therefore h increases 9 km each hour during the night being 140 km at nightfall and 210 km 8 hours later. With these values of h and with y_m from the curves of Fig. 6, the values of $2s$ for $\lambda 32$ meters were calculated from (24) and are plotted in Fig. 7. The skip distances for other short wireless waves calculated from (24) are approximately proportional to those for $\lambda 32$ meters. During the day the skip distance of Fig. 7 agree within 50 percent with the observed values as far as they are known; in the small hours of the night the calculated values appear to be on the whole 1.5 to 2 times too great.

Equation (24) is not exact for it leaves out of account the magnetic field and the curvature of the earth; the neglect of the first gives too long and of the second too short skip distances, the error due to the approximations being in general less than 30 percent. (24) also assumes sharp reflection of the wave at a height h and therefore neglects refraction by ions and electrons below y_m. The inclusion of this refraction will lessen the calculated skip distances. The skip distances of Fig. 7 are a minimum at 40 minutes past noon, the time when y_m is a maximum. Observation in temperate latitudes puts the shortest skip distance at about 2 o'clock. The agreement between theory and experiment is therefore good but not perfect. It is difficult to give a satisfactory explanation of the slight discrepancy; perhaps the value of the attachment coefficient is a little high, or perhaps there are shifts in the distribution of the ionization below the maximum which cause a warping of the ray paths.

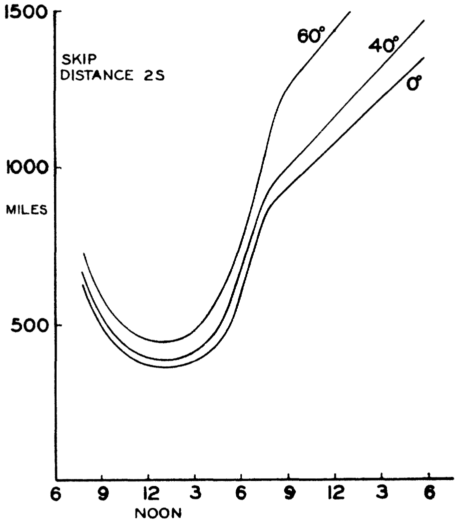

Fig. 7. Calculated values of the skip distance $2s$ for a 32 meter wireless wave.

41. In conclusion, it may be stated that an ionization produced by the ultra-violet light of the sun has been worked out which accounts in a general way for wireless wave propagation phenomena over the earth and for the part of the earth's magnetic field of external origin and its diurnal variations. We can not regard, however, the ionization which has been obtained as a unique solution of the problem of the ionization of the upper atmosphere. The ionization can be modified within limits in a number of ways without disturbing

the present conclusions, and probably some modification may be desirable for the numbers of ions and electrons which have been used may be slightly too great.

With the sun directly overhead D is 1.5×10^{16} atomic nitrogen ion pairs from 150 to 180 km, or the ion density is 10^{10}. From the relation[12]

$$\mu^2 = 1 - ye^2\lambda^2/(1 - He\lambda/2\pi cm)\pi m, \tag{25}$$

it is seen that 10^{10} atomic nitrogen ions give the same refraction for 40 meter waves as 3.2×10^5 electrons. Therefore at equatorial noon either the ions or the electrons will account for the wireless skip distances equally well, but both the 3.2×10^5 electrons and the 10^{10} ions are unnecessary, although to be sure there is no objection to having both if they are at different levels. Early in the morning and late in the afternoon the refraction of the ions falls below that of the electrons, as calculated from the values of y and y_e of the foregoing paragraphs, and at night it rises above the electronic refraction. If the observational facts of short wireless waves call for a refraction predominately electronic there are several ways in which the refractive effects of the ions may be decreased without disturbing the present development. One may keep the same number of ions and suppose that, say, one-half are molecular ions. This will decrease their refraction, change the drift currents and modify the calculations throughout, but will not change any of the values by as much as 100 percent. Or, one may decrease the number of ions and assume higher noon temperatures. In illustration, let us suppose that the number of ions were 1/2 and the noon temperatures 2 times the respective values which have been used. This will multiply the day values of σ and E by 1/4 and 3/2, respectively. The currents in the D and S regions for day and night, instead of being, respectively, about 11.6, 8.7, 2.3 and 0.6, become 6, 3, 2.5 and 0.5 millions of amperes. The steady current around the earth and the diamagnetism are unchanged.

On the other hand one can not be entirely certain that it is the ions which should be reduced. Actually the ions will give very closely the same skip distance-wave-length relation as the electrons. Therefore if the refraction of short wireless waves were due to ions rather than to electrons the skip distance theory[12] would be unchanged. In this case it would be an "ion limitation" theory rather than an "electron limitation" theory. A sufficient number of electrons could be assumed such that by their magnetic double refraction and absorption they would account for the dip in the wireless range curve at λ200 meters[12] and the polarization phenomena. It is doubtful whether at the present time the observational data permit a final decision among the various possibiliiies. We may express the view, based on the complexity of wireless wave behavior, that some complex modification may be the one which will best account for all of the facts. That is, that perhaps the ions and electrons are present in such numbers that their refractions are about equal, and that during certain hours of the day or night and at certain latitudes the refraction of wireless waves is controlled by electrons and at other times and latitudes by ions.

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[12] Taylor and Hulburt, Phys. Rev. **27,** 189 (1926), equation 2.