ON THE REFLECTION OF THE $K\alpha$ LINE OF CARBON FROM A GLASS MIRROR

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Abstract

The author recalls previous experiments which he tried in order to determine the limiting angle of total reflection from a glass grating, for a set of radiations between 20 and 65A, making a slight correction to a recent work of E. Dershem on the reflection of the $K\alpha$ line of carbon. In the case of glass, with $\lambda = 44.9$A, if we take into account the discontinuities of absorption in the dispersive medium, the Kallman-Mark dispersion formula leads us to a result ($\delta = 1-n = 5.73 \times 10^{-3}$) of the same order as the Drude-Lorentz simplified formula. The reflected intensity from the mirror is calculated from the angle of incidence, account being taken of the absorption. The result is then compared with the experimental curve given by Dershem for the $K\alpha$ line of carbon, and shows that it is possible to determine the critical angle of total reflection from glass ($\theta_m = 6°12'$) and the refractive index ($\delta = 1-n = 5.84 \times 10^{-3}$).

In a previous work,[1] I have endeavored to determine the critical angle θ_m of total reflection for a beam of soft x-rays (wave-length between 20 and 65A) by varying the glancing angle of incidence of the beam upon a glass grating and measuring the angles $\theta_{1/2}$ at which the intensity of each line of the spectrum had decreased to one-half value. The conclusions were then: *a.* The reflected intensity does not *suddenly* decrease for a certain angle θ_m: instead of this limit in total reflection, we find for the reflected intensity a somewhat *flattened* curve, the shape of which may be computed by using the Fresnel formulae and taking into account the intense absorption of soft x-rays in the medium.[2] *b.* An increase in the flattening of the reflection curve is observed, with increase of the wave-length (from $\lambda = 45$A to $\lambda = 65$A). *c.* The $\theta_{1/2}$ angles of half-decreased intensities increase in proportion to the wave-length λ. They are equal to several degrees. $\delta = 1-n$ (n = index of refraction) therefore varies as the square of λ.

The simplified form of the Drude-Lorentz dispersion formula seems to be verified:

$$\delta = \frac{e^2}{2\pi m}\,\frac{N}{c^2}\lambda^2, \tag{1}$$

if we add a numerical coefficient 0.5 in the calculation of $\theta_{1/2}$.

Recently E. Dershem[3] has determined the curve connecting the reflected intensity with the glancing angle of incidence for the $K\alpha$ line of carbon and a glass mirror. He concludes that his measures do not confirm my previous

[1] Thibaud, Comptes rendus, **187**, 219 (1928).

[2] Cf: especially: J. Thibaud, Annales Soc. Scientifique Bruxelles, Series B, **48**, 167 (1928).

[3] E. Dershem, Phys. Rev. **34,** 1015 (1929).

results. A mistake has crept in Dershem's paper regarding his interpretation of my results. The θ_m angles in my numerical table are not connected with the complete disappearance of the reflected line (I have assumed for θ_m almost the same number as for the measured angle $\theta_{1/2}$). I insisted rather (see for instance reference 2) upon the gradual decrease of the intensity with increase of angle, and upon the great difficulty of locating with some definiteness the critical angle of total reflection.

The only difference between Dershem's results and mine is that the decrease as a function of the angle in the intensity diffracted by a grating seems more rapid than when determined from an ordinary glass mirror. Prins[4] has made a remark on this subject. A grating seems thus less fit for an accurate determination of θ_m. At another place[5] Dershem says that the shape of the curve makes it difficult to determine the limiting angle θ_m and that, on the other hand, it seems doubtful to him whether the index of refraction may be computed from the simplified Drude-Lorentz formula. I wish to discuss further these two points.

1. Kallman and Mark have changed, for the case of x-rays, the dispersion formula as quoted in Eq. (1) into another expression, which takes into account the presence of critical frequencies in the dispersive medium:

$$\delta = \frac{e^2}{2\pi m} \sum \frac{Ni}{\nu^2} = 1 + \frac{\nu_i^2}{\nu^2} 1n\left(1 - \frac{\nu^2}{\nu_i^2}\right) \tag{2}$$

and which seems to be correctly verified with ordinary x-rays.[6]

I have already pointed out[7] that the results as computed from Eq. (2) in the case of about 50A x-rays, the absorption discontinuities being taken into account, show but little difference from the numbers given by the simplified formula (1): the atomic "resonators" for the wave-length considered effect no important disturbance of index.

Let us consider with especial care the case of the reflection of the $K\alpha$ carbon line ($\lambda = 44.9$A) from an ordinary glass mirror.

If we take into account[8] the K and L discontinuities of the different constituents of glass (Si, O, Na, Ca, etc.), we find by using formula (2):

$$\delta = 5.73 \times 10^{-3},$$

whereas formula (1) would give:

$$\delta = 7.08 \times 10^{-3}$$

which is quite of the same order.

The discrepancy between the results of the two formulae becomes appreciable only for wave-lengths very near (1 or 2A) to an L, or especially an

[4] Prins, Nature, Sept. 7, 1929.

[5] Reference 3, p. 1020.

[6] A. Larsson, Dissertation. Uppsala (1929).

[7] Thibaud et Soltan. Journ. de Physique **8,** 494 (1927) and: Thibaud, Phys. Zeits. **29,** 259 (1928).

[8] The calculation will appear very soon in the Journ. de Physique.

M or N discontinuity of one of the constituent elements of the reflecting mirror. But for glass and ordinary metals, the latter case is reached only for radiations of more than 100A.

2. Reflected intensities may be computed from angles, in the case of a medium strongly absorbing for the radiation, by using the Fresnal equations relative to the components of incident and reflected radiations, which are parallel to the mirror plane. In the case of a complex index[9]

$$n' = (1 - \quad) - i\kappa$$

(κ = extinction coefficient; $\kappa = \mu\lambda/4\pi$; μ = absorption coefficient for λ in the medium). We thus arrive at an expression of the ratio A of reflected and incident intensities as a function of the angle θ,

$$A = \frac{(1+m)^2 + 2(m^2+a^2)^{1/2} + 2(1+m)\left[2(m^2)^{1/2}\right]^{1/2} + a^2 \cos\phi/2}{(1+m)^2 + 2(m^2+a^2)^{1/2} - 2(1+m)\left[2(m^2+a^2)^{1/2}\right]^{1/2} \cos\phi/2} \qquad (3)$$

in which: $\theta = (1+m)\theta_m$; $a = \kappa/2\delta = \mu\lambda/4\pi\theta_m{}^2$; $\tan \phi = -a(1-\delta)/m$; $\pi < \phi < 2\pi$. The graphic representation of (3) is given on Fig. 1 for different values of the coefficient a between 0.01 and 1.

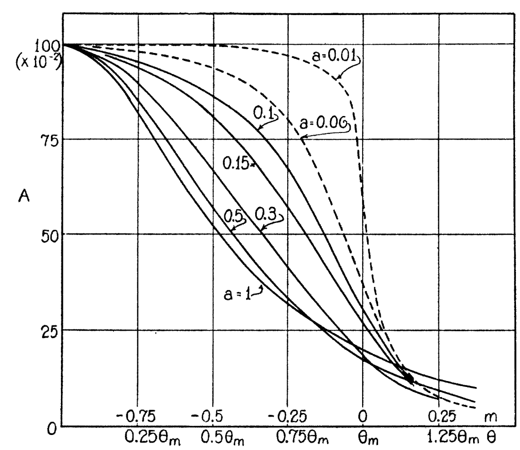

Fig. 1.

It is easy to see, with formula (3), that the reflecting power A_0 connected with the angle $\theta = \theta_m$, goes thrugh a minimum ($A_0 = 0.17$) for $a = 0.5$. Thus, for wave-lengths much longer than 45A and very strong absorptions μ, one would notice an *increase* in the reflected intensity for angles $\theta > \theta_m$.

The curves show that, in the neighborhood of $a = 0.5$, the reflecting power A_0 suffers only small variations, around 20 percent, and that for $\theta_{1/2} = 0.5\theta_m$, the reflected intensity is about one-half of incident intensity.

In the case of the reflection of the $K\alpha$ line of carbon, from a glass mirror, we have no accurate measurements of the absorption coefficient μ of glass.

[9] See also: R. Forsler, Helv. Phys. Acta **1**, 18 (1927). and J. Prins, Zeits. f. Physik **47**, 479 (1928).

The most probable value, as concluded particularly from unpublished measurements of Holweck, is $\mu = 10^5$. If we adopt the above computed value $\delta = 5.73 \times 10^{-3}$, we find: $a = 0.31$.

One may notice a close similarity between the curve $a = 0.3$ (Fig. 1) as computed from (3), with the experimental curve of reflection given by Dershem. For $a = 0.3$ we find $A_0 = 0.19$. The angle θ for which the reflected intensity is reduced to 19 percent of initial intensity is equal, on Dershem's curve to 6°12′. It is the critical angle θ_m of total reflection of the $K\alpha$ carbon line from glass. The corresponding value of the index is:

$$\delta = \frac{\theta_m{}^2}{2} = 5.84 \times 10^{-3}.$$

It is therefore possible to determine, from the experimental curves of reflection, the limiting angle and the refractive index $n = 1 - \delta$. The result is in good accord with the value as deduced from the Kallman and Mark formula (2) ($\delta = 5.73 \times 10^{-3}$). It follows moreover that the extinction coefficient, in the case of the $K\alpha$ line of carbon and glass, is approximately: $\kappa = \alpha \times 2\delta = 3.5 \times 10^{-3}$.

Lastly the curves show that the desired limiting angle θ_m is almost double the angle $\theta_{1/2}$, and thus we find the reason a 0.5 coefficient was to be introduced in my previous researches when θ_m was (arbitrarily) taken equal to $\theta_{1/2}$.