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THE CONDUCTIVITY OF A HIGH FREQUENCY DISCHARGE IN HYDROGEN*

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ABSTRACT

Measurements were made of the voltage between electrodes necessary to produce a current of 100 milliamperes in the discharge for gas pressures between 0.005 mm and 1.0 mm and for frequencies of oscillation between 1.25 and 20 million cycles per second (wave-lengths between 240 and 15 meters). It was found that the discharge has its maximum conductivity when operated at a frequency of 15 million cycles (20 meters) and a pressure of 0.015 mm. A theory of the mechanism of the high frequency discharge indicates that under these conditions, an electron makes an inelastic collision with a gas molecule every electronic mean free path having been under the influence of the electric force for one half cycle. The theory also indicates that for any frequency of oscillation greater than 15 million cycles, both the electric force and the gas pressure for which the conductivity is a maximum, will increase directly with the frequency of oscillation.

INTRODUCTION

THERE are essentially two methods by which a high frequency discharge can be obtained in a rarefied gas without the use of internal electrodes. The first method consists in surrounding the discharge tube by an inductance coil through which high frequency currents are passed. In the second method, high frequency voltages are applied to external electrodes of sheet metal which are attached to the outside of the discharge tube. As for the first method, it has been shown [1,2,3] that unless the high frequency currents in the inductance coil are very large, (such as are obtained with damped oscillations), the electric forces producing the discharge are principally the electrostatic forces between the ends of the coil. In other words, for small currents the two methods of excitation are fundamentally the same. On the other hand, the second method has the advantage of greater simplicity and flexibility. For this reason, the second method was used in the work about to be described.

It is well known that a high frequency discharge is profoundly affected by changes in gas pressure and frequency of oscillation. It therefore should be of interest to discover at what pressure and frequency of oscillation the discharge will function most efficiently. One way of doing this would be to study the variation in the striking potential or the minimum maintaining potential of the discharge as the pressure and frequency of oscillation are

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[1] J. S. Townsend and R. H. Donaldson, Phil. Mag. **5**, 178 (1928).

[2] C. J. Brasefield, Phys. Rev. **34**, 1392, 1627 (1929).

[3] K. A. MacKinnon, Phil. Mag. **8**, 605 (1929).

varied. Hayman[4] has made such measurements for helium and neon while C. and H. Gutton[5] have done the same for hydrogen. In some preliminary experiments the writer attempted to reproduce Guttons' results, but found that it was practically impossible to make any reliable measurements of the striking potential of the discharge because of the difficulty in determining at just what potential the discharge struck. In an effort to obtain results which would have more significance, the problem was attacked from a different angle. It was proposed to study the conductivity of the discharge, that is, the voltage between electrodes necessary to produce a given current in the gas. Such an investigation would clearly show under what conditions the high frequency discharge operates most efficiently.

APPARATUS AND PROCEDURE

The apparatus used to produce the high frequency oscillations was essentially the same as that described in a previous paper.[6] The discharge tube was 90 cm long, 4.5 cm internal diameter while the electrodes surrounding it were of sheet copper 4 cm wide. It was found that the conductivity of the discharge could be measured with greatest accuracy when the distance between electrodes was 40 cm. If the electrodes were much farther apart than this then there was an appreciable leakage current between the high potential electrode and the standard used to support that end of the tube. On the other hand, if the electrodes were too close together, the capacity current between electrodes became appreciable. For pressures below 0.2 mm a continuous flow of pure hydrogen was maintained through the tube while above 0.2 mm the gas was stagnant.

Instead of applying the total voltage of the high frequency generator to the electrodes, one electrode was connected to ground through a 0–120 high frequency milliammeter. The experimental procedure consisted simply in measuring the voltage between electrodes which was necessary to produce a current of 100 milliamperes through the tube, for different frequencies of oscillation and different gas pressures. The voltage between electrodes, which is equal to the voltage across one of the condensers of the tank circuit, was calculated from the formula $E = I/2\pi fC$ where I is the tank current in amperes, f the frequency of oscillation and C the capacity in farads of one of the two equal tank condensers. The tank current was measured by a 0–10 Weston radio frequency ammeter (thermocouple type) which is guaranteed by the makers to be accurate within one percent of full scale deflection. The amplitude of the electromotive force between electrodes is $E(2)^{1/2}$. Measurements were made for frequencies of oscillation corresponding to 15, 20, 25, 30, 40, 50, 60, 80, 100, 120, 160, 200, and 240 meters.

RESULTS

Fig. 1 shows, for a few typical frequencies of oscillation, the relation between gas pressure, and the voltage between electrodes which is necessary to

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[4] R. L. Hayman, Phil. Mag. **7**, 586 (1929).

[5] C. and H. Gutton, Comp. Rend. **186**, 303 (1928).

[6] C. J. Brasefield, Phys. Rev. **35**, 92 (1930).

produce a current of 100 milliamperes through the tube. Fig. 2 shows the variation with frequency of oscillation of the minimum voltage necessary to produce a current of 100 milliamperes. It can be seen from Fig. 2 that the high frequency discharge in hydrogen has its maximum conductivity at frequencies of oscillation between 10 and 15 million cycles (i.e. between 20 and 30 meters) and at gas pressures between 0.035 and 0.015 mm. Operating the discharge under these conditions of maximum conductivity, it was possible to obtain a current as large as 300 milliamperes at 20, 25, and 30 meters. This current was, of course, limited only by the output of the high frequency generator.

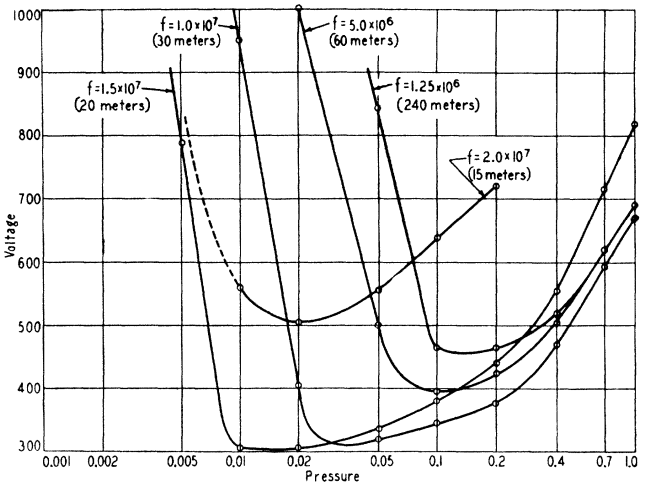

Fig. 1. Variation with gas pressure (in mm of mercury) of the voltage between electrodes necessary to produce a current of 100 milliamperes.

It can be seen from Fig. 1 that at very low pressures, the discharge is most conducting when operated at 20 meters. Using this frequency of oscillation and applying the full voltage of the high frequency generator to the electrodes it was found possible to obtain a faint white luminous discharge at pressures even as low as 10^{-6} mm. This discharge was accompanied by a blue fluorescence of the glass at the ends of the tube and by a red fluorescence of the glass at irregular spots along the tube. These spots were evidently due to bombardment of the walls of the tube by a stream of electrons for they could be shifted by moving one's finger in their neighborhood or by the presence of a magnet. The discharge itself at these low pressures seemed to be due to a beam of electrons moving along the axis of the tube, for it could be deflected by the presence of one's finger. However, no peculiarly shaped luminous bodies of gas such as Wood[7] observed using ultra high frequencies, were ever seen.

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[7] R. W. Wood, Phys. Rev. **35**, 658 (1930).

Discussion

It is not difficult to explain, qualitatively, the shape of the curves of Fig. 1. In order to maintain a stable high frequency discharge, the electrons in the discharge must satisfy two conditions. In the first place, they must produce positive ions by collision with gas molecules within a distance which is in general not greater than a few electronic mean free paths and within a time which is not greater than one-half the period of the oscillation. The second condition is, of course, that at the time of collision, the electron must have a velocity sufficient to ionize a gas molecule. Now at relatively high pressures, the mean free path of the electron is small. Hence if an electron is to attain ionizing velocity in this small distance, the applied electric force must be large. On the other hand, at low pressures the mean free path of the electron is large, so that the electric force must be large in order that the electron can make a collision with a gas molecule within one-half the period of the oscillation. Between these two extremes will be a pressure at which a smaller electric force will enable the electron to satisfy the two conditions simultaneously. These points of maximum conductivity are shown in Fig. 1.

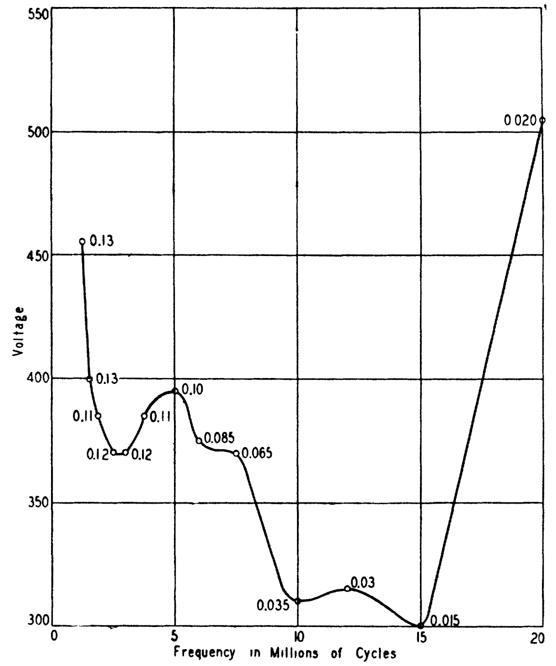

Fig. 2. Variation with frequency of oscillation of the minimum voltage necessary to produce a current of 100 milliamperes. Numbers along curve represent the pressure corresponding to the minimum voltage.

To explain the variation in the conductivity with the frequency of oscillation (Fig. 2) requires a more detailed study of the mechanism of the high frequency discharge. The motion of an electron in a high frequency electric field can be represented by the equation

$$mx'' = eE_0 \sin 2\pi ft \tag{1}$$

where E_0 is the amplitude of the electric force and f is the frequency of oscillation. Integrating, we get the velocity at any time t,

$$v_t = \frac{e}{m}\frac{E_0}{2\pi f}(1 - \cos 2\pi ft) \quad (v_0 = 0). \tag{2}$$

Integrating once again, we get the distance the electron goes in time t

$$d = \frac{e}{m}\frac{E_0}{4\pi^2 f^2}(2\pi ft - \sin 2\pi ft). \tag{3}$$

From Eq. (2) it is evident that for a given E_0 and f, the electron will have its maximum velocity after a time $t = 1/2f$, that is, after it has been under the influence of the electric force for one half cycle. If this terminal velocity is sufficient to ionize a gas molecule and if the distance which the electron has gone in one half cycle (given by Eq. (3)) is at least one mean free path, then the high frequency discharge will be operating under the most favorable conditions possible.

Let us consider the mechanism of a high frequency discharge operating at a frequency of 20 million cycles per second (15 meters). The electron velocity given by Eq. (2) must also satisfy the equation

$$\tfrac{1}{2}mv^2 = eV \tag{4}$$

where V is at least 16 volts. As a matter of fact, a previous investigation[6] has shown that to maintain a stable high frequency discharge in hydrogen, the electron velocity must probably be at least 21 equivalent volts. Assuming this value for V and substituting the corresponding value of v in Eq. (2), we find that an electric force of amplitude $E_0 = 9.7$ volts per cm acting on an electron for one half period ($t = 1/2f$) will give it enough energy to ionize a hydrogen molecule. This, then, is the minimum electric force capable of maintaining a discharge at 15 meters; moreover, a discharge will not occur for this value of E_0 unless the distance gone by the electron is at least one mean free path. Substituting $E_0 = 9.7$ in Eq. (3) we find that the distance which the electron goes in one half period is 3.39 cm. From Fig. 2 we see that the pressure corresponding to maximum conductivity at 20 million cycles is 0.02 mm. The electronic mean free path at this pressure is 3.5 cm, which compares favorably with the distance 3.39 cm which the electron goes in one-half period.

Hence we have shown that at 20 million cycles the high frequency discharge in hydrogen has its maximum conductivity at 0.02 mm pressure because at this pressure a minimum electric force of amplitude 9.7 volts per cm is sufficient to allow an electron to make an inelastic collision with a hydrogen molecule in one mean free path having been under the influence of the electric field for one half cycle. At this point of maximum conductivity the potential difference between electrodes is 505 volts. Dividing this by the distance between electrodes, 40 cm, and multiplying by $2^{1/2}$ we get an apparent electric force of amplitude 17.8 volts per cm. This is entirely compatible with the theoretical value of 9.7 volts per cm. For the potential difference between electrodes consists of two parts. The first is the drop in potential in the body of gas and the second is the drop in potential at the electrodes. Since there is at present no means of estimating the magnitude of this second factor, all that can be said is that the apparent electric force represents the upper limit of the true electric force in the gas.

Let us now consider a high frequency discharge operating at 15 million cycles per second (20 meters). Assuming as before $V=21$ volts, we find that a minimum electric force of amplitude $E_0=7.3$ volts per cm acting on an electron for one half period will give it enough energy to ionize a hydrogen molecule. Substituting this value of E_0 in Eq. (3), we find the distance which the electron goes in one half period is 4.53 cm. This compares favorably with the electronic mean free path of 4.65 cm at the pressure corresponding to maximum conductivity at 15 million cycles. Moreover, the electric force at 15 million cycles is less than the electric force at 20 million cycles. This explains why the conductivity of the discharge is greater at 15 million cycles.

At both 20 and 15 million cycles, we have seen that the mechanism of the high frequency discharge under conditions of maximum conductivity consists in an electron making an inelastic collision with a gas molecule every electronic mean free path having been under the influence of the electric force for one half cycle. If this were also the mechanism of the discharge at frequencies less than 15 million cycles, then we would expect to find that both the voltage and pressure corresponding to maximum conductivity would be less than at 15 million cycles. Since Fig. 2 shows that at frequencies less than 15 million cycles, both the voltage and pressure corresponding to maximum conductivity are greater than the voltage and pressure at 15 million cycles, we must conclude that the mechanism of the discharge at frequencies less than 15 million cycles is different from the mechanism at frequencies greater than 15 million cycles. Using Eqs. (2) and (3) we find that the difference in mechanism is due primarily to the fact that below 15 million cycles, the electron makes an inelastic collision before it has been under the influence of the electric force for an entire half period. The shape of the curve of Fig. 2., combined with a rough calculation, suggests that between 15 and 12 million cycles the electron makes an inelastic collision having been under the influence of the electric force for one third cycle; between 12 and 10 million cycles for one fourth cycle; between 10 and 7.5 million cycles for one fifth cycle; between 7.5 and 6.5 million cycles for one sixth cycle; between 6.5 and 5 million cycles for one seventh cycle; between 5 and 3 million cycles for one eighth cycle; and between 3 and 1.25 million cycles the electron makes an inelastic collision having been under the influence of the electric force for only one ninth cycle. Since the time during which the electron is under the influence of the electric field is becoming shorter, the electric force must be increased in order that the electron can attain ionizing velocity. On the other hand, it is evident that the even fractions of a cycle will be more efficient in producing ions than the odd fractions. This explains the maxima and points of inflection of Fig. 2.

If the conditions described in the last paragraph give a true picture of the mechanism of the discharge at frequencies below 15 million cycles, then the number of electronic mean free paths involved in an inelastic collision increases as the frequency of oscillation decreases. Thus for maximum conductivity at 10 million cycles, the electron makes an inelastic collision in a little more than one mean free path; at 5 million cycles in about 3 mean free paths; and at 1.25 million cycles in about 15 mean free paths.

The reason for the sharp break in the curve at 15 million cycles is probably due to the fact that 0.015 mm pressure is the lowest pressure at which sufficient ions can be produced to sustain a current of 100 milliamperes. Hence if the current through the tube were decreased, we might expect to find maximum conductivity at some frequency less than 15 million cycles and some pressure less than 0.015 mm. As yet, experiments have not been undertaken to verify this.

In conclusion, it has been shown that the conductivity of a high frequency discharge in hydrogen is greatest when the frequency of oscillation is 15 million cycles per second and the gas pressure is 0.015 mm because the discharge is operating most efficiently under these conditions. For, under these conditions, an electron is undisturbed for one complete half cycle of the electric field, and at the end of this time it has gone exactly one mean free path so that it can make an inelastic collision with its maximum velocity. Moreover, even though at 20 million cycles and 0.02 mm pressure, the mechanism of the discharge is the same (i.e. the electron goes exactly one mean free path in one half cycle) the electric force required is greater than at 15 million cycles. Finally, at frequencies of oscillation greater than 20 million cycles, if the mechanism of the discharge is the same as at 15 and 20 million cycles (which appears very probable), it is possible to predict from Eqs. (2) and (3), first, that the electric force at maximum conductivity will increase directly with the frequency of oscillation and second, that the gas pressure at maximum conductivity will also increase directly with the frequency of oscillation.

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